MA 102 (Multivariable Calculus)

Quiz-2

Date: March 28, 2013 **Time:** 50 minutes **Maximum Marks:** 20

Answer ALL questions

1. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be C^1. Show that
$$N = \frac{\left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, -1\right)}{\sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 + 1}}$$
is a unit normal vector to the graph of f. **2 marks**

2. Find the normal line and the tangent plane to the surface $z = xe^y$ at the point $(1, 0, 1)$. **4 marks**

3. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be given by $f(x, y) := \dfrac{x^2 y}{x^2 + y^2}$ if $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$. Show that f is continuous at $(0, 0)$. Show that the directional derivative $\mathrm{D}_u f(0, 0)$ exists for all $u \in \mathbb{R}^2$ and determine $\mathrm{D}_u f(0, 0)$. Is f differentiable at $(0, 0)$? **6 marks**

4. Let $f : \mathbb{R}^n \to \mathbb{R}$ be such that $f(tx) = t^m f(x)$ for all $x \in \mathbb{R}^n$ and $t \in \mathbb{R}$, where m is a nonnegative integer. If f is differentiable then show that $\langle x, \nabla f(x) \rangle = m f(x)$. **2 marks**

5. Find maxima, minima and saddle point, if any, of the function $f(x, y) := 4xy - 2x^2 - y^4$ **6 marks**

**** End ****