

1. Write your answer NEATLY in the space provided.

[12marks]

QUESTION	ANSWER
(i) Consider the one parameter family of curves $x^2 = ce^y$ where $c$ is an arbitrary constant. The trajectory through the point $x = 0, y = 0$ which is orthogonal to the given family also passes through the point $x = 2$ and	$y = -1$
(ii) Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be differentiable at $(0,0)$ . Further, for $u := (3/5, 4/5)$ and $v := (1/\sqrt{2}, 1/\sqrt{2})$ , the directional derivatives are given by $D_u f(0,0) = 12$ and $D_v f(0,0) = -4\sqrt{2}$ . Then	$f_x(0,0) = -92$ $f_y(0,0) = 84$
(iii) Let $f : \mathbb{R}^3 \rightarrow \mathbb{R}$ be $C^2$ and $F : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be a $C^2$ vector field. Then	$\text{curl}(\nabla f) = 0$ $\text{div}(\text{curl } F) = 0$
(iv) If the equation $xy - z \log y + e^{xz} = 1$ can be solved locally as $y = f(x, z)$ around the point $(0, 1, 1)$ then $\nabla f(0, 1)$ equals	$(2, 0)$
(v) Let $\Gamma$ be the circle $x^2 + y^2 = 9$ oriented positively. Then by Green's theorem $\oint_{\Gamma} \left( (3y - e^{\sin x})dx + (7x + \sqrt{y^4 + 1})dy \right)$ equals	$36\pi$
(vi) Let $D(r) := \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq r^2\}$ . Then $\lim_{r \rightarrow \infty} \iint_{D(r)} e^{-(x^2+y^2)} dA$ equals	$\pi$