MA 201
COMPLEX ANALYSIS
ASSIGNMENT–3

(1) Find the values of z such that (a) $e^z \in \mathbb{R}$ and (b) $e^z \in i\mathbb{R}$.

(2) Prove that $\sinh(\text{Im}z) \leq |\sin(z)| \leq \cosh(\text{Im}z)$. Deduce that $|\sin(z)|$ tends to ∞ as $|\text{Im}z| \to \infty$.

(3) Find all the complex numbers which satisfy the following:
(i) $\exp(z) = 1$ (ii) $\exp(z) = i$ (iii) $\exp(z - 1) = 1$.

(4) Evaluate the following:
(i) $\log(3 - 2i)$ (ii) $\text{Log}\, i$ (iii) $(i)^{(-i)}$

(5) If γ is the boundary of the triangle with vertices at the points 0, $3i$ and -4 oriented in the counterclockwise direction then show that $\left| \int_\gamma (e^z - \overline{z})\ dz \right| \leq 60.$

(6) Evaluate $\int_\gamma |z|\ \overline{z}\ dz$ where γ is the circle $|z| = 2$.

(7) Use ML-inequality to obtain the following upper bounds:
(a) $\left| \int_{|z-1|=2} \frac{1}{z} dz \right| \leq 4\pi$
(b) $\left| \int_{|z|=10} \frac{z-1}{z+1} dz \right| \leq \frac{220}{9}\pi$
(c) $\left| \int_\gamma \frac{1}{z^4+1} dz \right| \leq \frac{3}{80}\pi, \gamma(t) = 3e^{it}, t \in [0, \pi]$