Classical Mechanics (PH211 + PH403)

Tutorial VI-B

1. Find (a) the inertia tensor, (b) principal moments of inertia and (c) semi-axes of inertia ellipsoid, for a solid cube of size a and density ρ. The cube has one of its corners at the origin, and three adjacent edges along the axes of the coordinate frame.

   (a) Find the orientation of the principal axes w.r.t the original axes.

   (b) Find the rotation matrix that relate the two sets of axes.

   (c) Find the kinetic energy of the cube if it is rotated with an angular velocity ω (a) about the z-axis (b) about the face diagonal through the origin that lies on the XZ-plane (c) about the body axis though the origin.

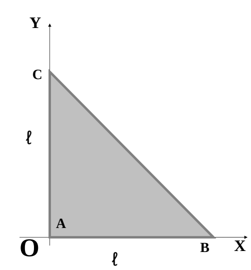
2. A solid sphere with its center at the origin of the coordinate axis is cut along XY, YZ and XZ planes into eight equal pieces. Calculate the inertia tensor for the piece in the 1st octant of the coordinate axes ($x \geq 0; y \geq 0; z \geq 0$). Find the kinetic energy of the solid when rotated about an axis passing through (1, 1, 1) and the origin of the coordinate axes. The density of the sphere is ρ and radius is R.

3. A thin lamina of mass M is in the shape of a 45^o right triangle $\mathbf{ABC}$, with $\angle A = 90^o$ and sides $AB = AC = l$. Vertex A is at the origin of the coordinate system and vertices B and C lie on the X and Y-axes respectively (see figure).

   (a) Find the inertia tensor of the lamina with respect to the coordinate axes (X, Y, Z).

   (b) Find the inertia tensor of the lamina about a coordinate axes (X', Y', Z'), which is parallel to (X, Y, Z) but with origin at the center of mass of the lamina.

   (c) Find the inertia tensor of the lamina with respect to the (X, Y, Z) axes after being rotated about the Z-axis by an angle 30^o.



4. A rigid body of mass M and having principal moments, I_1, I_2, I_3 about its centre of mass is suspended by a point on its x_1 axis. The body is set to oscillatory motion in plane perpendicular to the x_3 axis. Find the period of small angle oscillations of the body using Euler equations. Take l as the distance of the point of suspension from the centre of mass. Find the value of l for which the period of oscillation a minimum?