Classical Mechanics (PH211 + PH403)

Tutorial IV (on 06/09/2010)

Note: Neglect friction unless otherwise stated. You need not have to solve the differential equations unless specifically asked to.

1. Setup the Lagrangian and obtain the Euler-Lagrange equations for the following systems. Obtain the generalized momenta and forces. Determine the number of constants of motion, and try to identify them in each of the cases.

   (a) A pendulum bob of mass m, connected to a point "O" by inextensible string of length l, is oscillating in the $X - Y$ plane. The point of suspension "O" is moving vertically with time as, $O(t) = O_0 + a\,t + b\,t^2$, where a and b are constants.

   (b) A mass m is constrained to move on a parabolic wire ($y = A\,x^2$) under gravity $F = -mg\hat{z}$. The wire is rotated about the z-axis at an angular velocity ω.

   (c) A mass m is constrained to move on the surface of a sphere of radius R under gravity $F = -mg\hat{z}$.

   (d) Two masses m_1 and m_2 are connected by a light inextensible rod of length l. The masses are moving on the $X - Y$ plane such that, the first one is constrained to move on the X-axis while the other on the Y-axis.

2. Using Euler-Lagrange's equations obtain the equation of motion of a stone falling under gravity. The air drag on the stone is given by $\mathbf{F} = -k\mathbf{v}$.

3. A circular hoop of mass M and radius R that *rolls* down (without sliding) an incline under gravity. The angle of the incline with the horizontal is α. Obtain the equation of motion of the hoop using,

   (a) Euler-Lagrange equations for holonomic constraints.

   (b) the method of Lagrange's undetermined multipliers.

4. Show that the geodesics (that is the shortest path between a pair of points on the surface) of,

   (a) a plane is a straight line.

   (b) a cylinder is a helix.

   (c) a sphere is the great circle (the plane of which contain the center of the circle).