Classical Mechanics (PH211 + PH403)

Tutorial I

1. The position vector of a moving particle at any time, t, is given by $\bar{r}(t) = (2t^2 - 3)\hat{i} + (4t + 4)\hat{j} + (t^3 + 2t^2)\hat{k}$. Find (i) the distance of the particle from the origin at t=0, (ii) the velocity of the particle at t=1 and (iii) the acceleration of the particle at t=2.

2. Show that for a particle of constant mass the kinetic energy T satisfies, $\frac{dT}{dt} = \bar{F}.\bar{v}$ and if the mass varies, $\frac{d(mT)}{dt} = \bar{F}.\bar{p}$.

3. Show that the centre of mass $\mathbf{R}$ of a system of particles about an arbitrary origin satisfies,

$$M^2R^2 = M\sum_i m_i r_i^2 - 1/2\sum_{i,j} m_i m_j r_{ij}^2$$

where M is the total mass, $r_{ij} = |\mathbf{r_i} - \mathbf{r_j}|$ and R = $|\mathbf{R}|$.

4. The trajectory of an insect in plane polar coordinates is given by, $r = b\exp(\Omega t)$ and $\theta = \Omega t$. Show that the velocity and acceleration of the particle at any instant satisfies, $\mathbf{v} = r\Omega(\hat{r} + \hat{\theta})$ and $\mathbf{a} = 2\Omega^2 r\hat{\theta}$. Find the angle between the $\mathbf{v}$ and $\mathbf{a}$.

5. Three atoms located at (0.0,0.0,0.0), (0.0, $2^{1/6}\sigma$, 0.0) and (σ, 0.0, 0.0) interact with each other through Lennard-Jones potential,

$$V = 4\epsilon\left(\left(\frac{\sigma}{r_{ij}}\right)^{12} - \left(\frac{\sigma}{r_{ij}}\right)^{6}\right).$$

where $r_{ij} = |\mathbf{r_i} - \mathbf{r_j}|$ and σ is a constant. Find

(i) an expression for the force between a pair of atoms,

(ii) calculate the total force on the atom at the origin, and

(iii) the total potential energy of the system.