$\boxed{\text{KK } 12.11}$

By velocity addition formula

$$u_x = \frac{u_x' + V}{1 + \frac{V u_x}{c^2}}$$

Taking differentials (dropping subscripts)

$$du = \frac{du'}{1 + \frac{V}{c^2}u'} - \frac{u' + V}{(1 + \frac{V}{c^2}u')^2}\frac{V}{c^2}du'$$

We also have

$$dt = \gamma\left(dt' + V\frac{dx'}{c^2}\right) = \gamma\, dt'\left(1 + \frac{u'V}{c^2}\right)$$

Dividing

$$\frac{du}{dt} = \frac{du'}{dt'}\cdot\frac{1}{\gamma}\frac{1}{(1 + \frac{V}{c^2}u')^2}\left[1 - \frac{u' + V^2/c^2}{(1 + \frac{V}{c^2}u')^2}\right]$$

In the rest frame

$$\frac{du'}{dt'} = a_0 \qquad \text{and} \quad u' = 0$$

Hence $a_x = a_0 \frac{1}{\gamma}\cdot\left[1 - \frac{V^2}{c^2}\right]$

$$= a_0/\gamma^3$$

12.12 KK

Using the results of the last problem, with $a' = a_0$ and denoting the instantanious velocity of the ship in the initial rest system by v, we have

$$\frac{dv}{dt} = a_0 \left(1 - \frac{v^2}{c^2}\right)^{3/2}$$

$$\int_0^{v_f} \frac{dv}{\left(1 - \frac{v^2}{c^2}\right)^{3/2}} = \int_0^t a_0 \, dt = a_0 t$$

using $\int \frac{dx}{(1-x^2)^{3/2}} = \frac{x}{\sqrt{1-x^2}}$

we obtain

$$c \left. \frac{v/c}{\sqrt{1 - v^2/c^2}} \right|_0^v = \frac{v}{\sqrt{1 - v^2/c^2}} = a_0 t$$

$$\Rightarrow \quad v = \frac{a_0 t}{\gamma}$$

$$v^2 = (a_0 t)^2 \left(1 - \frac{v^2}{c^2}\right) = \frac{(a_0 t)^2}{\left(1 + \left(\frac{a_0 t}{c}\right)^2\right)}$$

(b) Now

$$v = v_0 / \sqrt{1 + (v_0/c)^2}$$

$$= v_0 \left(1 + 10^{-6}\right)^{-\frac{1}{2}}$$

$$= v_0 \left(1 + 5 \times 10^{-7}\right)$$ and so on

③ We have $E = \hbar\omega - W$

E – Electron kinetic energy

The minima occurs when the kinetic energy is zero

$$\omega_0 = \frac{W}{\hbar} = 6.08 \times 10^{15} \ \frac{\text{rad}}{s}$$

At $\omega = 2\omega_0$

$$E = 2W - W = W$$

Since $E = \frac{1}{2}mv^2$, we find the velocity

$$v = \sqrt{\frac{2W}{m}} = 1.19 \times 10^6 \ \frac{m}{s}$$

④ $E^2 = p^2c^2 + m^2c^4$

$$cp = \sqrt{E^2 - m^2c^4}$$

that De Broglie wave length

$$\lambda = \frac{h}{p} = \frac{hc}{\sqrt{E^2 - m^2c^4}} = \frac{hc}{\sqrt{E^2 - (mc^2)^2}} \quad \text{— ①}$$

Rest mass energy of proton is 938 MeV

$\Rightarrow \frac{mc^2}{E}$ is vanishingly small

$$\lambda = \frac{hc}{E} = 3.54 \times 10^{-19} \ m$$

Since we ignored the mass term, this is the same wavelength light would have at energy.

(5) $$\psi = A\, e^{-\lambda|x|}\, e^{-i\omega t}$$

$$\psi^* = A\, e^{-\lambda|x|}\, e^{i\omega t}$$

Since A, λ, and ω are real positive constant

$$1 = \int_{\infty}^{\infty} |\psi|^2\, dx = \int_{-\infty}^{\infty} \psi^* \psi\, dx$$

$$= A^2 \int_{-\infty}^{\infty} e^{-2\lambda|x|}\, e^{-i\omega t + i\omega t}\, dx$$

$$= A^2 \int_{-\infty}^{\infty} e^{-2\lambda|x|}\, dx$$

$$= 2A^2 \int_{0}^{\infty} e^{-2\lambda x}\, dx$$

$$= 2A^2 \left(\frac{e^{-2\lambda x}}{-2\lambda} \right) \Big|_0^{\infty} = \frac{A^2}{\lambda}$$

$$\Rightarrow A = \sqrt{\lambda}$$

So $$\psi(x,t) = \sqrt{\lambda}\, e^{-\lambda|x|}\, e^{-i\omega t}$$