PH101: Tutorial 10 **Due: Oct 30, 2012** **DKG**

1. Length of a spaceship is measured to be exactly half of it's proper length. What is the speed of the spaceship relative to the observer's frame? What is the ratio of times noted in the spaceship and observer's frames?

2. A muon has lifetime of 2×10^{-6} s in its rest frame. It is created 100 km above the earth and moves towards it at a speed 2.97×10^{8} m/s. At what altitude does it decay? According to the muon, how far did it travel in its brief life?

3. The earth and sun are 8.3 light-minutes apart. Ignore their relative motion for this problem and assume they live in a single inertial frame, the Earth-Sun frame. Events A and B occur at t = 0 on the earth and at 2 minutes on the sun respectively. Find the time difference between the events according to an observer moving at u = 0.8c from Earth to Sun. Repeat if observer is moving in the opposite direction at u = 0.8c.

4. A light beam is emitted at an angle θ_0 with respect to x' axis in S' frame. Find the angle θ that the beam makes with respect to x-axis in S. Let v be the relative velocity of the two frames.

5. Problem **K&K 12.6**

6. Problem **K&K 12.9**

TUTORIAL 09 DUE: OCT 30, 2012 DKG

Note Title 19-Oct-12

[1] The contracted length

$$l_{obs} = l_0 \sqrt{1 - (v/c)^2}$$

Here $l_{obs} = \frac{l_0}{2}$

$$\Rightarrow \quad \frac{l_0}{2} = l_0 \sqrt{1 - (v/c)^2}$$

$$\Rightarrow \quad v = \sqrt{\frac{3}{4}}\, c$$

Considering time dilation

$$t_{obs} = \frac{t_0}{\sqrt{1 - (v/c)^2}}$$

$$\Rightarrow \quad \frac{t_{obs}}{t_0} = \frac{1}{2}$$

(2) The time dilation factor is $\gamma = 1/\sqrt{1-(v/c)^2}$

Muon's life time τ in the rest frame corresponds to $\gamma\tau$ in the laboratory frame.

That means muon travels a distance

$$d = v\gamma\tau$$

$$= \frac{(2.97 \times 10^{8}\, m/s)(2\times 10^{-6}\, s)}{\left[1 - \left(\frac{2.97\times 10^{8}\, m/s}{3\times 10^{8}\, m/s}\right)^2\right]^{\frac{1}{2}}}$$

$$= 4.2 \text{ km}$$ before it decay!

According to muon, it has only traveled

$$\frac{d}{\gamma} = v\tau = (2.97\times 10^{8}\, m/s)(2\times 10^{-6}\, s)$$

$$= 590 \text{ m.}$$

---

(3) According to the formula for a Lorentz transformation

$$\Delta t_{obs} = \gamma\left[\Delta t_{Earth\text{-}Sun} - \frac{u}{c^2}\Delta x_{Earth\text{-}Sun}\right]$$

with $\gamma = \frac{1}{\sqrt{1-(u/c)^2}}$

$$\Delta x_{Earth\text{-}Sun} = 8.3 \text{ light-minutes}$$

$$= 8.3\, c$$

$$\Delta t_{obs} = \frac{2\ \text{min} - \frac{0.8c\,(8.3c\,\text{min})}{c^2}}{\sqrt{1-0.8^2}}$$

$$= -7.7\ \text{min}$$

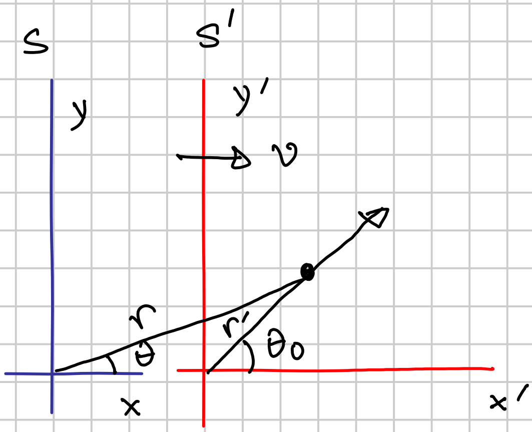
Which means that according to the observer, event B happened before event A!

If we reverse the sign of u then

$$\Delta t_{obs} = \frac{2\,\text{min} + 0.8\,(8.3\ \text{min})}{\sqrt{1-0.8^2}} = 14\ \text{min}$$

---

(4)



S' frame

$x' = r' \cos\theta_0$
$y' = r' \sin\theta_0$
$r' = ct'$

S frame

$x = r\cos\theta$
$y = r\sin\theta$
$r = ct.$

From Lorentz transformation equations,

$$x = \gamma\,(x' + vt') \quad \text{—①}$$

$$t = \gamma\left(t' + \frac{vx}{c^2}\right) \quad \text{—②}$$

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}}$$

From ①

$$ct\cos\theta = \gamma\,(ct'\cos\theta_0 + vt')$$

$$= \gamma c\left(\frac{v}{c} + \cos\theta_0\right)\left(1 - \frac{vx}{c^2 t}\right)$$

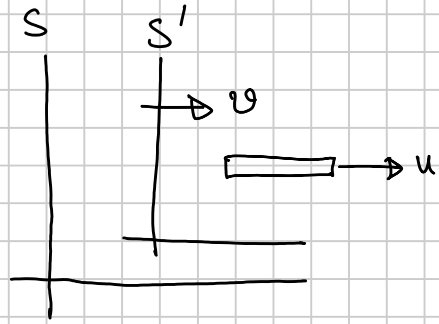
So,

$$\cos\theta = \frac{\gamma^2\,(\cos\theta_0 + v/c)}{\left[1 + \gamma^2\left(\frac{v^2}{c^2} + \frac{v}{c}\cos\theta_0\right)\right]}$$

$$= \frac{\left(\frac{v}{c} + \cos\theta_0\right)}{\left[1 - \frac{v^2}{c^2} + \frac{v^2}{c^2} + \frac{v}{c}\cos\theta_0\right]}$$

$$= \left[\frac{\frac{v}{c} + \cos\theta_0}{1 + \frac{v}{c}\cos\theta_0}\right]$$

(KK 12·6)



speed of the rod in S' is

$$V = \frac{u - v}{1 - \frac{uv}{c^2}}$$

Using Lorentz contraction

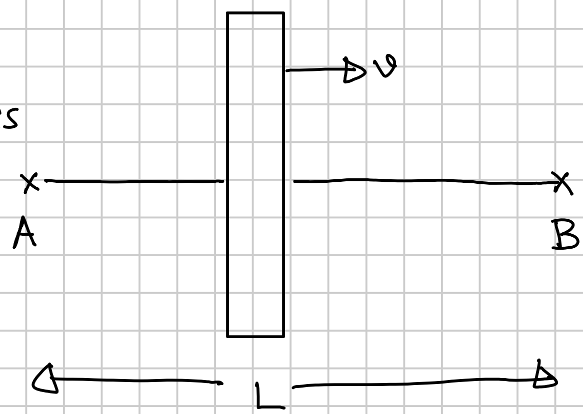
$$l' = l_0\sqrt{1 - V^2/c^2}$$

$$= \frac{l_0}{c}\sqrt{c^2 - \left(\frac{u-v}{1 - \frac{uv}{c^2}}\right)^2}$$

$$= \left(\frac{l_0}{c^2 - uv}\right)\left[(c^2 - u^2)(c^2 - v^2)\right]^{\frac{1}{2}}$$

$$l' = l_0 \frac{\left[(c^2 - u^2)(c^2 - v^2)\right]^{\frac{1}{2}}}{c^2 - uv}$$

(KK 12·9)



Let t_0 = time spent outside the glass

t_1 = " " through " "

To find the time for the passage of light through the glass, Consider two events in the system moving with the glass

(a) light enters at $t_a' = 0$, $x_a' = 0$

(b) light leaves at $t_b' = \frac{nD}{c}$; $x_b' = D$

In the lab system, light enters at $t_a = 0$, $x_a = 0$ and leaves at

$$t_1 = t_b = \gamma\left(t_b' + x_b' v/c^2\right) = \gamma\left(\frac{nD}{c} + \frac{vD}{c^2}\right) = \gamma \frac{D}{c}\left(n + \frac{v}{c}\right)$$

The distance light travels while passing through glass in lab system

$$x_b = \gamma\left(x_b' + v t_b'\right) = \gamma\left(D + n\frac{v}{c}D\right) = \gamma D\left[1 + n\frac{v}{c}\right]$$

Since the distance traveled in free space is $L - x_b$

so $\quad t_0 = (L - x_b)/c$

$$T = (L - x_b)/c + t_1 = \frac{L}{c} - \gamma\frac{D}{c}\left(1 + n\frac{v}{c}\right) + \gamma\frac{D}{c}\left(n + \frac{v}{c}\right)$$

$$= \frac{L}{c} + \gamma\frac{D}{c}\left[n - 1 + \frac{v}{c}(1 - n)\right] = \frac{L}{c} + \frac{D}{c}\frac{1}{\sqrt{1 - v^2/c^2}}(n-1)\left(1 - \frac{v}{c}\right)$$

$$\Rightarrow T = \frac{L}{c} + \frac{D}{c}(n-1)\sqrt{\frac{1 - v/c}{1 + v/c}}$$

Now if $v = 0$,

$$T = \left[L + (n-1)D\right]/c$$

and if $v = c$

$$T = \frac{L}{c}$$