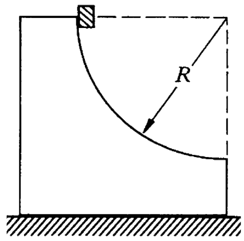
PH 101 **Tutorial 04** **Due:Aug 28,2012** **DKG**

Note Title 23-Aug-12

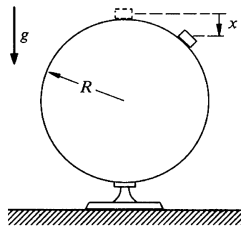
4.4 A small cube of mass m slides down a circular path of radius R cut into a large block of mass M, as shown at right. M rests on a table, and both blocks move without friction. The blocks are initially at rest, and m starts from the top of the path.

Find the velocity v of the cube as it leaves the block.

Ans. clue. If $m = M$, $v = \sqrt{gR}$



4.6 A small block slides from rest from the top of a frictionless sphere of radius R (see at right). How far below the top x does it lose contact with the sphere? The sphere does not move. *Ans.* $R/3$



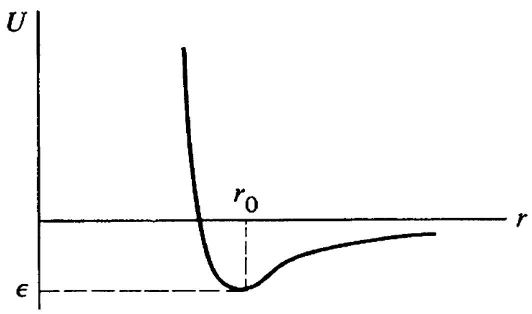
4.13 A commonly used potential energy function to describe the interaction between two atoms is the Lennard-Jones 6,12 potential

$$U = \epsilon\left[\left(\frac{r_0}{r}\right)^{12} - 2\left(\frac{r_0}{r}\right)^{6}\right].$$

a. Show that the radius at the potential minimum is r_0, and that the depth of the potential well is ϵ.

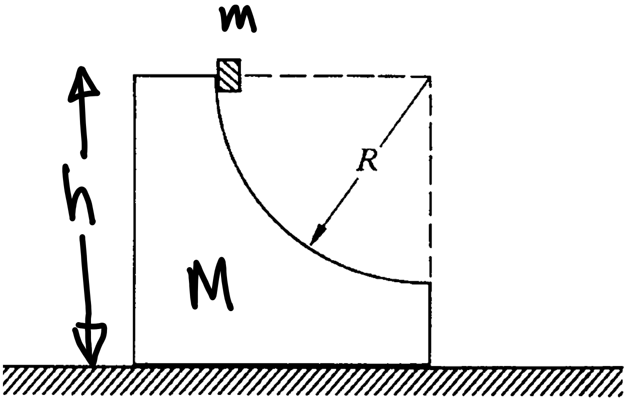
b. Find the frequency of small oscillations about equilibrium for 2 identical atoms of mass m bound to each other by the Lennard-Jones interaction.

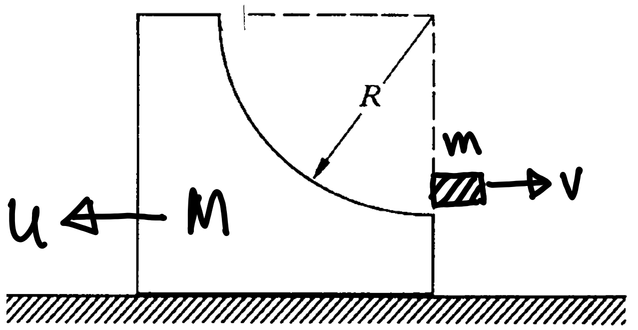
Ans. $\omega = 12\sqrt{\epsilon/r_0{}^2m}$



Practice problems for the students (not to be discussed in the Tutorial class)
4.7, 4.14, 4.20

KK 4.4





Total energy

$$E_i = mgh$$

Total Energy

$$E_f = mg(h-R) + \frac{1}{2}mv^2 + \frac{1}{2}Mu^2$$

Hence,

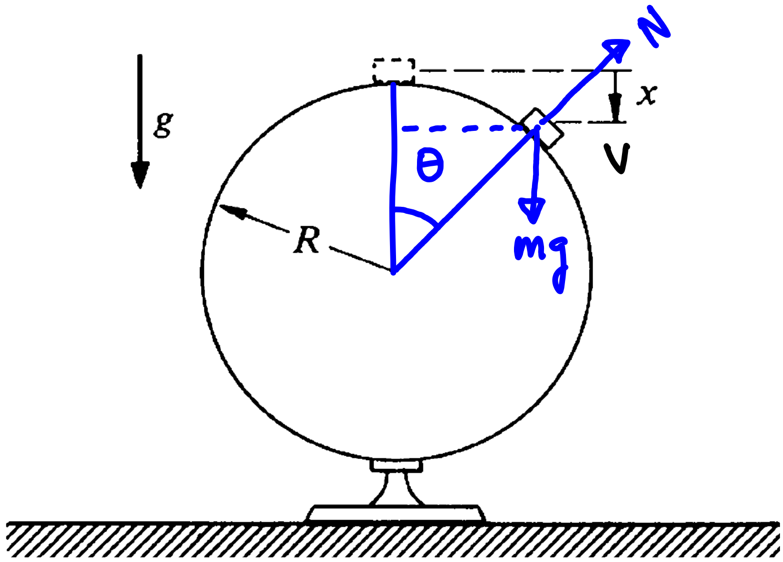
$$0 = \frac{1}{2}mv^2 + \frac{1}{2}Mu^2 - mgR$$

By conservation of momentum

$$Mu = mv$$

$$\Rightarrow \quad v = \left[\left(\frac{M}{m+M}\right)2gR\right]^{1/2}$$

$\boxed{\text{KK } 4.6}$



Here,

$$N - mg\cos\theta = -\frac{mv^2}{R}$$

Contact is lost if

$$N = 0$$

$$\Rightarrow \quad g\cos\theta = \frac{v^2}{R} \quad \text{— (1)}$$

From energy considerations

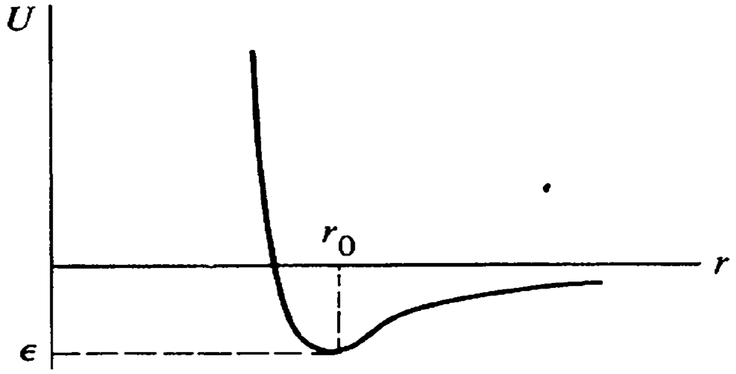
$$mgR = mgR\cos\theta + \frac{1}{2}mv^2 \quad \text{— (2)}$$

Solving (1) & (2)

$$\cos\theta = 2/3$$

Therefore, $x = R(1-\cos\theta) = R/3$

$\boxed{\text{KK } 4.13}$



$$U = \varepsilon\left[\left(\frac{r_0}{r}\right)^{12} - 2\left(\frac{r_0}{r}\right)^{6}\right]$$

ⓐ
$$\frac{dU}{dr} = 0 = \varepsilon\left[-\frac{12\,r_0^{12}}{r^{13}} + \frac{12\,r_0^{6}}{r^{7}}\right]$$

$$\Rightarrow \quad \left(\frac{r_0}{r}\right)^6 = 1 \Rightarrow \quad r = r_0$$

$$U(r_0) = \varepsilon\left[\left(\frac{r_0}{r_0}\right)^{12} - 2\left(\frac{r_0}{r_0}\right)^{6}\right] = -\varepsilon$$

ⓑ
$$\omega = \sqrt{\frac{k}{\mu}} \; ; \quad \mu = \frac{m}{2}$$
$$= \sqrt{\frac{2k}{m}}$$

Ref. section 4.10 (KK).
Discussed in the class

$$k = \left.\frac{d^2U}{dr^2}\right|_{r=r_0} = \varepsilon\left[\frac{(12)(13)\,r_0^{12}}{r_0^{14}} - \frac{(12)(7)\,r_0^{6}}{r_0^{8}}\right]$$

$$= \frac{72}{r_0^2}\,\varepsilon$$

$$\Rightarrow \quad \omega = 12\sqrt{\varepsilon/m r_0^2}$$