Energy ?? ~ 1930

Nuclear reaction | Niels Bohr

Beta Decay ! | Pauli

| Fermi

$F \longrightarrow \bigcirc$ → changes velocity

Particle

<u>1D</u>

$F \rightarrow \bigcirc$ Speeding up

$\vec{F} = \text{const}$ (x-direction

Constant accl^n $\qquad a = \frac{F}{m}$

$$v^2 = v_0^2 + 2 \cdot a \cdot d.$$

$$v_2^2 = v_1^2 + 2 \cdot \frac{F}{m} \cdot d$$

$$\underbrace{\frac{1}{2} m v_2^2}_{} - \underbrace{\frac{1}{2} m v_1^2}_{} = F \cdot d$$

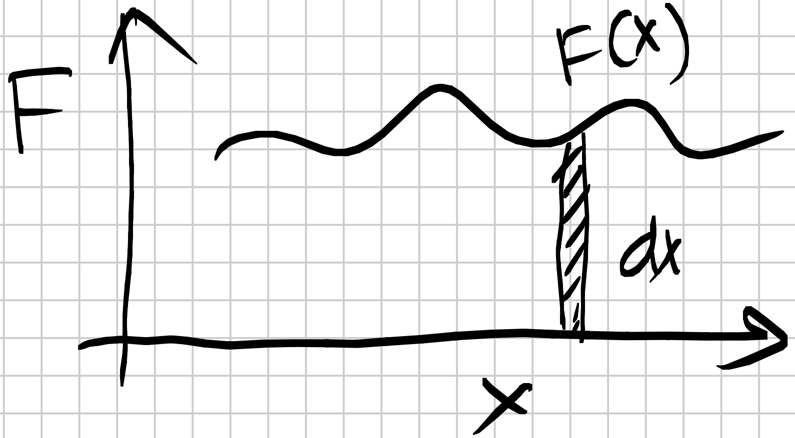
$$K.E_2 - K.E_1 = F.d$$

ΔK = Work done ! $\qquad N \cdot m = J$

$$\frac{\Delta K}{\Delta t} = F \cdot \frac{\Delta x}{\Delta t} = \text{Power} = F.v$$

J-sec = Watt.

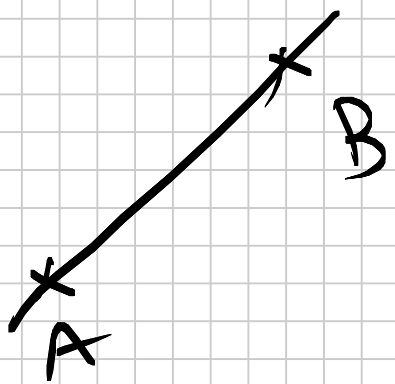
If force is not constant



$$\frac{1}{2} m v_1^2 - \frac{1}{2} m v_2^2 = \int_{x_0}^{x} F(x)\, dx$$

Work-energy theorem

<u>works</u>



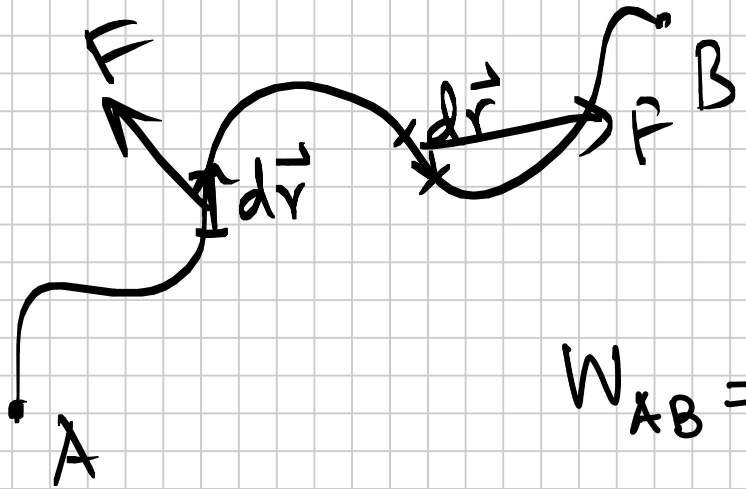
$$W_{AB} = \int_A^B F \cdot dx$$

$$= \frac{1}{2} m v_B^2 - \frac{1}{2} m v_A^2$$

$$= \left. \begin{matrix} +ve \\ -ve \\ 0 \end{matrix} \right\}$$

<u>3D</u> Point A to point B

$$W_{AB} = \int_A^B \vec{F} \cdot d\vec{r}$$



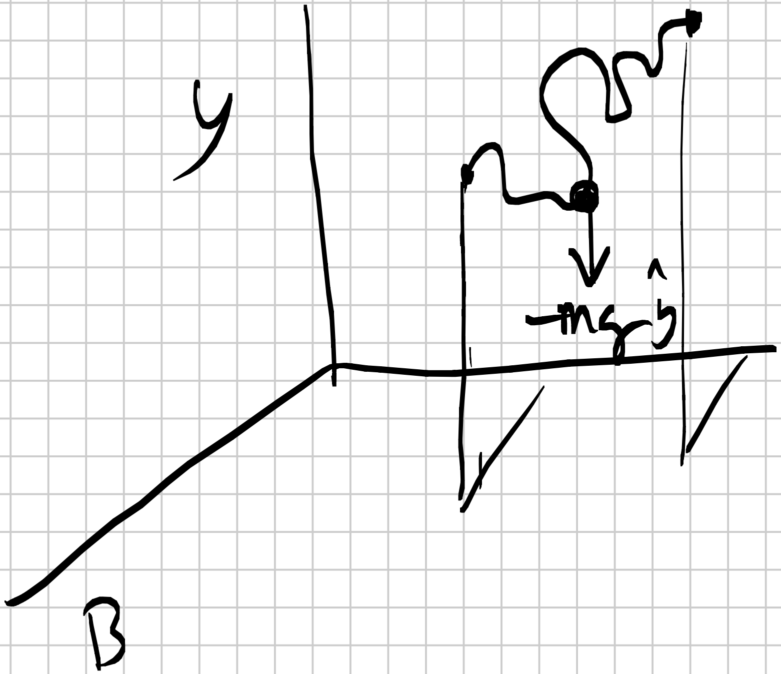
$$\vec{F} = F_x \hat{x} + F_y \hat{y} + F_z \hat{z}$$

$$d\vec{r} = dx \, \hat{x} + dy \, \hat{y} + dz \, \hat{z}$$

$$W_{AB} = \int_A^B dW = \int_A^B F_x dx + \int_A^B F_y dy + \int_A^B F_z dz$$

$$= \frac{1}{2} m \left(v_{B_x}^2 - v_{A_x}^2 \right) + \frac{1}{2} m \left(v_{B_y}^2 - v_{A_y}^2 \right) + \frac{1}{2} m \left(v_{B_z}^2 - v_{A_z}^2 \right)$$

$$= \frac{1}{2} m \left(v_B^2 - v_A^2\right)$$



$y_B - y_A = h$

<u>work done by gravity</u>

$$W_{AB} = \int_A^B \vec{F} \cdot d\vec{r} = \int_A^B F_y \, dy = -mg\,(y_B - y_A)$$

$$= -mgh$$

Gravity conservative force !

workdone - independent of path.

$$-mgh = -mg(y_B - y_A) = KE_B - K.E_A$$

$$\boxed{mgy_B + KE_B = mgy + KE_A}$$

$mgy \rightarrow$ gravitational potential energy! (U)

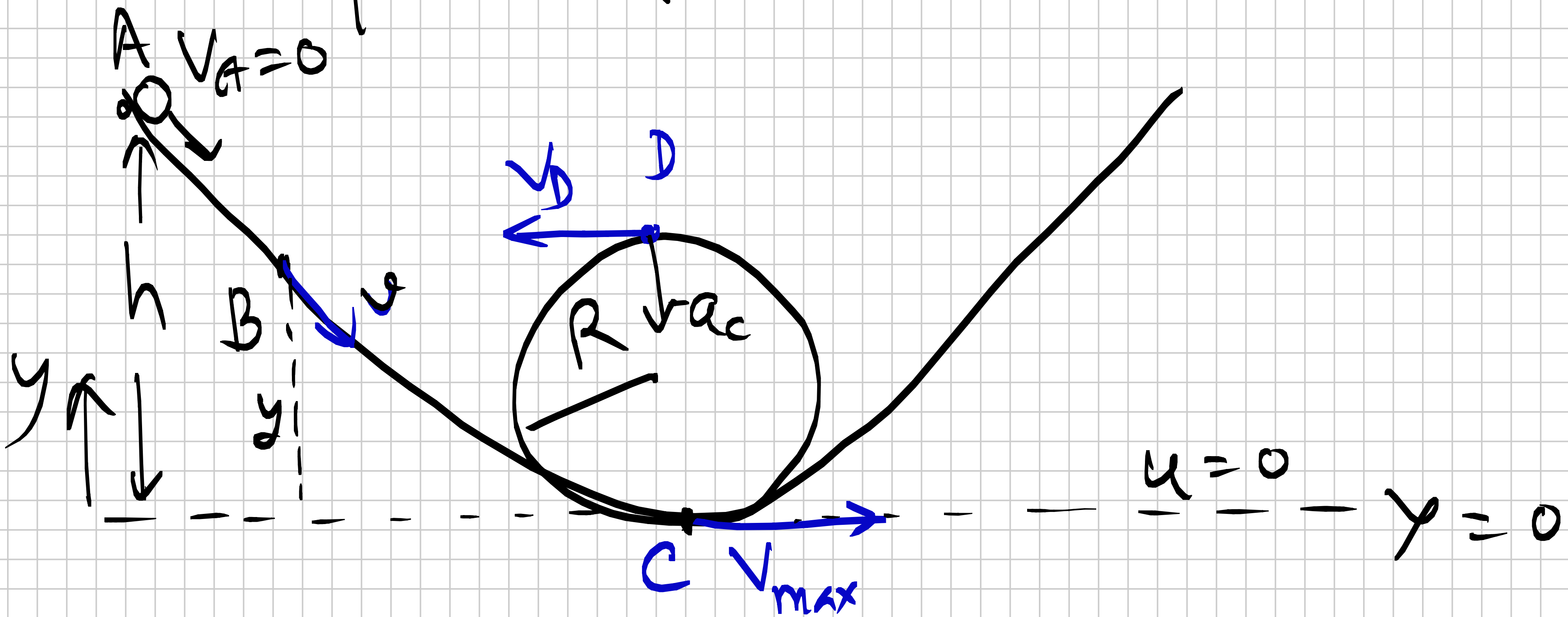
$$\boxed{U_B + KE_B = U_A + KE_A}$$

Mechanical energy conserved

if force is conservative

$\longrightarrow$ Friction not conservativ.

Consequence of conservation of energy!



$$u_A + KE_A = u_B + KE_B = u_C + KE_C = u_D + KE_D$$

$$mgh = mgy + \frac{1}{2}mv^2 \Rightarrow v^2 = 2g(h-y) \quad \text{--- } ①$$

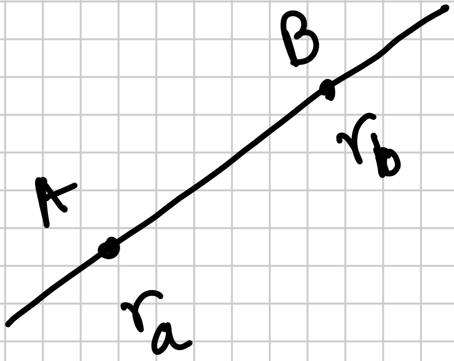
$$a_c = \frac{v^2}{R} \geq g \quad - (2)$$

$$2g(h - 2R) \geq gR$$

$$2h - 4R \geq R$$

$$\boxed{h \geq \frac{5}{2} R}$$

Potential Energy !



Conservative force $\Rightarrow$ work done does not depend on path but the position

$$\int_{r_a}^{r_b} \vec{F} \cdot d\vec{r} = f(r_b) - f(r_a)$$

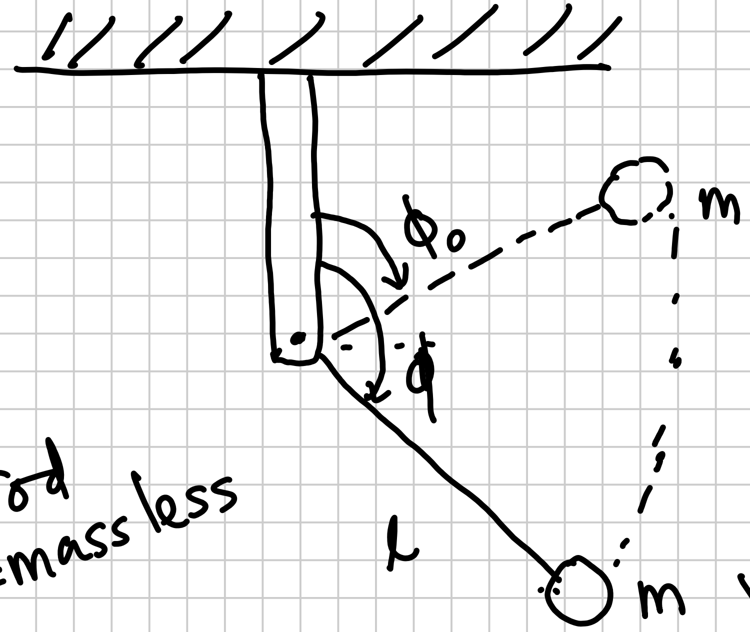
$$= -U(r_b) + U(r_a)$$

Work energy theorem !

$$W_{AB} = KE_B - KE_A$$

$$KE_A + U_A = KE_B + U_B = E \text{ (total energy)}$$

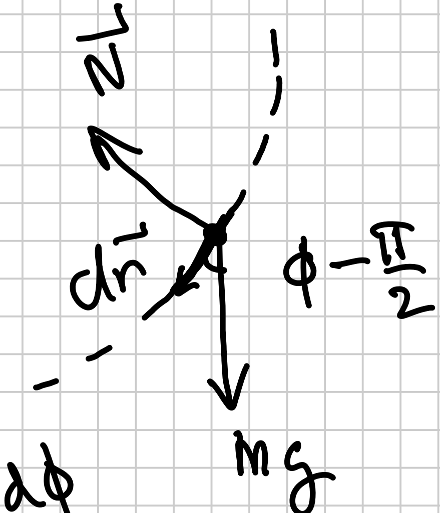
$U \rightarrow$ Potential Energy !

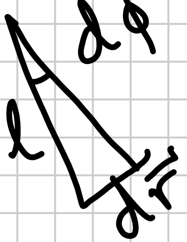


rod = massless

Release!
$v_0 = 0$

Velocity = ?





$|d\vec{r}| = l\,d\phi$

Work energy theorem

$$\frac{1}{2} m v(\phi)^2 - \frac{1}{2} m v_0^2 = W_{\phi_0 \phi}$$

$$v(\phi) = \left(\frac{2 W_{\phi_0 \phi}}{m} \right)^{\frac{1}{2}}$$

$$W_{\phi_0 \phi} = ?$$

Force!

$$\vec{N} \cdot d\vec{r} = 0$$

No work done by this force!

Only gravity works

Work done by gravity

$$m\vec{g} \cdot d\vec{r} = mgl \cos(\phi - \pi/2)\, d\phi$$

$$= mgl \sin\phi \quad d\phi$$

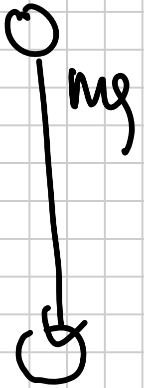
$$W_{\phi,\phi_0} = \int_{\phi_0}^{\phi} mgl \sin\phi \, d\phi$$

$$= -mgl \cos\phi \Big|_{\phi_0}^{\phi} = mgl\left[\cos\phi_0 - \cos\phi\right]$$

$$v(\phi) = \left[2gl(\cos\phi_0 - \cos\phi)\right]^{\frac{1}{2}}$$

max^m velocity $\Rightarrow$ $\phi = 0$ to $\phi = \pi$

$$v_{max} = 2(gl)^{\frac{1}{2}}$$



$$v^2 = u^2 + 2gd$$

$$d = 2l$$

$$v_{max}^2 = 2g\,2l$$

$$v_{max} = 2(gl)^{\frac{1}{2}}$$

Same as before!

Consider – central force! $\Rightarrow$ radial force!

$$\vec{F} = f(r)\,\hat{r}$$ moving from r_a to r_b

Consider motion in plane

$$d\vec{r} = dr\,\hat{r} + r\,d\theta\,\hat{\theta}$$

$$W_{ab} = \oint_a^b \vec{F} \cdot d\vec{r}$$

$$= \oint_a^b f(r)\,\hat{r} \cdot \left(dr\,\hat{r} + r\,d\theta\,\hat{\theta} \right)$$

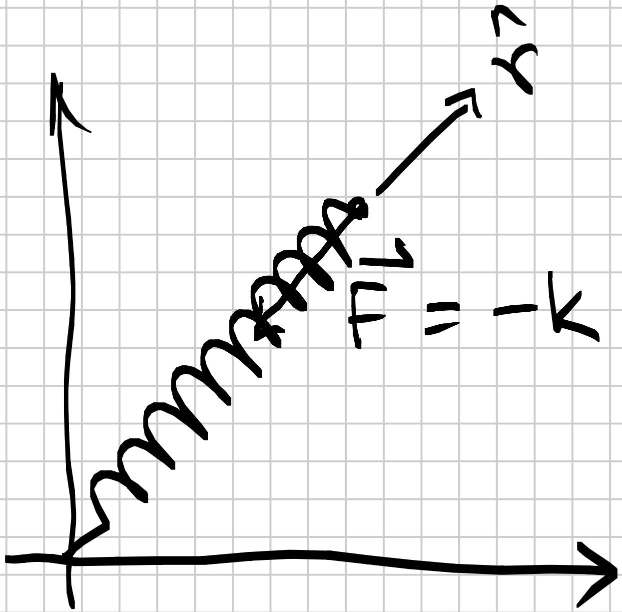
$$= \int_a^b f(r)\,dr$$

$\theta \rightarrow$ vanishes ! Path independent

Spring Force!

(Conservative)

(restoring force)



$$\vec{F} = -k\,(r - r_0)\,\hat{r}$$

r_0: equilibrium length

$$\vec{F} = -k\,(r - r_0)\,\hat{r}$$

$$U(r) - U(a) = -\int \vec{F} \cdot d\vec{r}$$

$$U(r) - U(a) = -\int_a^r (-k)\,(r - r_0)\,dr$$

$$= \frac{1}{2} k\,(r - r_0)^2 \Big|_a^r$$

$$U(r) = \frac{1}{2} k (r - r_0)^2 + C \quad (C\text{- Constant})$$

Conventionally $\quad U(r_0) = 0$

$$C = 0$$

$$U(r) = \frac{1}{2} k (r - r_0)^2$$