

1. A modified infinite well potential is given by

$$V(x) = \begin{cases} \epsilon x & \text{if } 0 \leq x \leq b \\ \infty & \text{otherwise} \end{cases} \quad (1)$$

Obtain approximate energy eigenvalues to the first order in ϵ . Also find the second order correction to the ground state energy.

2. The bottom of an infinite well is changed to have the shape

$$V(x) = \epsilon \sin \frac{\pi x}{b} \quad 0 \leq x \leq b \quad (2)$$

Calculate the energy shifts for all the states to first order in ϵ .

3. A simple harmonic oscillator is perturbed by a constant force. Find the energy eigenvalues. Calculate second order perturbation correction to the energy values and compare with the exact answer.
4. A simple harmonic oscillator is perturbed by a potential $V(x) = \lambda x^4$. Show that the perturbation can be written as

$$V(x) = \lambda \left(\frac{\hbar}{2m\omega} \right)^2 \left(a^4 + a^2(4\hat{n} - 2) + (6\hat{n}^2 + 6\hat{n} + 3) + a^{\dagger 2}(4\hat{n} + 6) + a^{\dagger 4} \right) \quad (3)$$

where $\hat{n} = a^\dagger a$. Obtain the first order correction to the energies. Discuss validity of this approximation for states with large \hat{n} eigenvalues.

5. The Hamiltonian of a rigid rotator in magnetic field is of the form $A\mathbf{L}^2 + BL_z + CL_y$, if quadratic terms are neglected. Obtain exact eigenvalues and eigenfunctions of this Hamiltonian. Assuming $B \gg C$, use second order perturbation theory to obtain approximate values of energy and compare with the exact answers.
6. A charged particle is constrained to move on a spherical shell in a weak uniform electric field. Obtain the energy spectrum to the second order in the field strength. (Is it ok to use non-degenerate perturbation theory?)
7. In hydrogenic atoms, assume that the nucleus is uniformly charged sphere of radius R . Calculate the energy shift for $n = 1$ and $n = 2$ states.
8. Find out the energy shifts due to linear Stark effect in $n = 3$ state of hydrogen atom.
9. Consider a Hamiltonian of the form

$$\mathbf{H} = \begin{pmatrix} E_0 & 0 \\ 0 & -E_0 \end{pmatrix} + \lambda \begin{pmatrix} \alpha & U \\ U^* & \beta \end{pmatrix} \quad (4)$$

Find the energy shift to first and second order in λ . Compare your results with exact eigenvalues.

10. The perturbing hamiltonian for Hydrogen atom in constant magnetic field is given by

$$V = -\frac{e}{2mc} \mathbf{B} \cdot (\mathbf{L} + \mathbf{S}) \quad (5)$$

Show that, in this case a level of given total angular momentum quantum number j splits in $(2j + 1)$ levels according to the formula $E_{jm}^{(1)} = -g_j \mu_B m$ where g_j is Lande's factor.