1. Find the displacement $u(x,t)$ for a string whose ends are fixed at $x = 0$, L, given the following initial conditions. Also find the maximum speed attained by any point.
   (a) $u(x,0) = 2hx/L$ is $x < L/2$ and $u(x,0) = 2h(L-x)/L$ if $x > L/2$ and $\dot{u}\,(x,0) = 0$.
   (b) $u(x,0) = a\sin(3\pi x/L)$, $\dot{u}\,(x,0) = 0$.
   (c) $u(x,0) = 0$, $\dot{u}\,(x,0) = a\sin(4\pi x/L)$.
   (d) $u(x,0) = a\sin(\pi x/L)$, $\dot{u}\,(x,0) = b\sin(\pi x/L)$.

2. A string with ends at $x = 0$, L, initially at rest is set into vibrations by giving a transverse impulse P at a distance d from the $x = 0$ end. Find the displacement $u(x,t)$ at later time. [Hint: Consider initial conditions, $u(x,0) = 0$ and $\dot{u}(x,0) = v_0$ if $x \in [d-\epsilon, d+\epsilon]$, and 0, otherwise. Then take a limit $a \to 0$,$v_0 \to \infty$ and $2amv_0 \to P$.]

3. Show that for any functions f and g,
$$u(r,t) = \frac{1}{r}f(r-ct) + \frac{1}{r}g(r+ct)$$
is a general *spherically symmetric* solution of the wave equation.

4. Obtain the Dirichlet Green's function for the Helmholtz operator $\nabla^2 + k^2$ for a volume bounded by $x = 0, a$, $y = 0, b$ and $z = 0, c$ planes.

5. Find the Dirichlet Green's function for the Helmholtz operator $\nabla^2 + k^2$ in the region $0 \le r \le a$. Verify that in $a \to \infty$ limit, the Green's function goes to free space Green's function. Also, verify that in $k \to 0$ limit, the Green's function correctly goes to the Green's function of the Laplace operator.

6. A plane wave (wavelength λ) is incident normally on a circular apperture (radius a) as shown in the figure. Find the intensity of Frounhofer diffraction pattern.

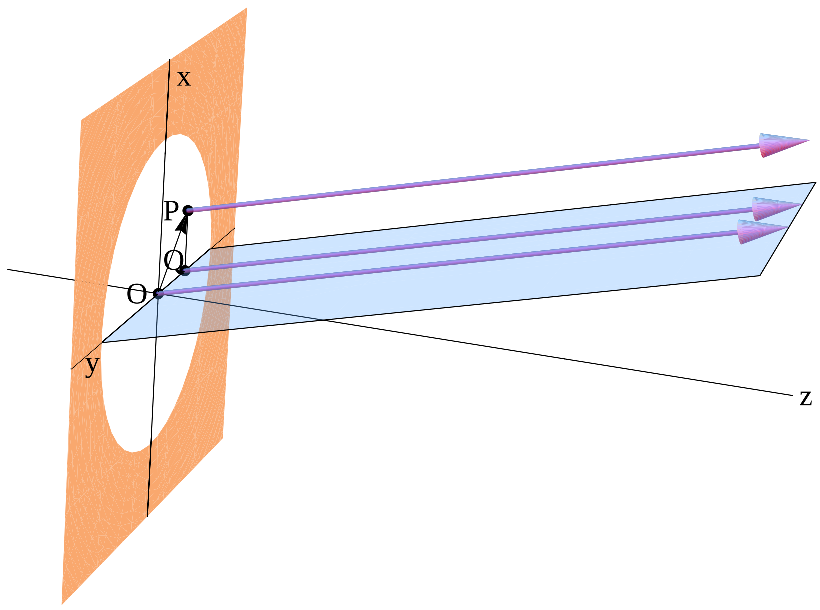

Figure 1: Problem 6