1. Prove by mathematical induction that

$$j_n(x) = (-1)^n x^n \left(\frac{1}{x}\frac{d}{dx}\right)^n \left(\frac{\sin x}{x}\right).$$

2. A quantum particle of mass m is trapped in a square well of radius a. The potential of the well is given by

$$V(r) = \begin{cases} -V_0 & 0 \le r < a \\ 0 & r > a. \end{cases}$$

Let $E(< 0)$ denote the energy eigenvalue. Show that the radial part of the wave function is given by $j_l(q_1 r)$ inside the well and $k_l(q_2 r)$ outside, where l is angular momentum quantum number, $q_1^2 = 2m(E + V_0)/\hbar^2$ and $q_2^2 = -2mE/\hbar^2$. Write down the boundary condition at $r = a$.

3. Heat flows in a semi-infinite rectangular plate, the end $x = 0$, being kept at temperature θ_0 and the long edges $y = 0$ and $y = a$ at zero temperature. Find the temperature at a point in the plate.

4. Consider a region $V = \left\{(r, \theta, z) \,|\, a < r < b,\, 0 \le \theta \le \frac{\pi}{2}\right\}$. The value of electrostatic potential along the surface $r = b$ is $\theta\,(\pi/2 - \theta)$, and along all other surfaces the potential is zero. Find the electrostatic potential in V.

5. A circular wire of radius a charged with line density λ surrounds a grounded concentric spherical conductor of radius c. Determine the electrical charge density on the surface of the conductor.

6. Two halves of a long hollow conducting cylinder of inner radius b are separated by a small lengthwise gaps on each side, and are kept at different potentials V_1 and V_2. Show that the potential inside is given by

$$\Phi(s, \phi) = \frac{V_1 + V_2}{2} + \frac{V_1 - V_2}{2} \tan^{-1}\left(\frac{2bs}{b^2 - s^2}\cos\phi\right)$$

where ϕ is measured from a plane perpendicular to the plane through the gap. Calculate the surface charge density on each half of the cylinder.

7. A hollow right circular of radius b has its axis coincident with the z axis and its ends at $z = 0$ and $z = L$. The potential on the end faces is zero, while the potential on the cylindrical surface is given by

$$\Phi(b, \theta, z) = \begin{cases} V & \text{for } -\frac{\pi}{2} < \theta < \frac{\pi}{2} \\ -V & \text{for } \frac{\pi}{2} < \theta < \frac{3\pi}{2} \end{cases}$$

Find the potential inside the cylinder.