1. Find two linearly independent power series solutions of the equation
$$y'' - xy' + y = 0.$$
For which values of x do the series converge?

2. Find a series solution for
$$(1 + x^2)y'' + y = 0$$
about $x = 0$. [Is $P(x) = (1 + x^2)^{-1}$ analytic everywhere?]

3. The equation
$$(1 - x^2)y'' - xy' + \alpha^2 y = 0$$
where α is a constant, is called the *Chebyshev equation.*
   (a) Compute two linearly independent series solutions for $|x| < 1$.
   (b) Show that for every non-negative integer $\alpha = n$ there is a polynomial solution of degree n. When appropriately normalized these are called *Chebyshev polynomials.*

4. The equation
$$y'' - 2xy' + 2\alpha y = 0$$
where α is a constant, is called *Hermite equation.*
   (a) Find two linearly independant series solutions for $-\infty < x < \infty$.
   (b) Are these solutions convergent for all x? Find the behaviour of these solutions for large x.
   (c) Show that there is a polynomial solution of degree n for every $\alpha = n$ non-negative integer.

5. Find the singular points of the following equations and determine those which are regular singular points:
   (a) $x^2y'' + (x + x^2)y' - y = 0$
   (b) $(1 - x^2)\, y'' - 2xy' + l(l + 1)y = 0$ (Legendre)
   (c) $x^2y'' - 5y' + 3x^2y = 0$
   (d) $(x^2 + x - 2)\, y'' + 3(x + 2)y' + (x - 1)y = 0$
   (e) $(1 - x^2)\, y'' - xy' + n^2y = 0$ (Chebyshev)
   (f) $y'' - 2xy' + 2\alpha y = 0$ (Hermite)
   (g) $x^2y'' + xy' + (x^2 - n^2)y = 0$ (Bessel)
   (h) $xy'' + (1 - x)y' + ay = 0$ (Laguerre)

6. The equation
$$xy'' + (1 - x)y' + ay = 0$$
where α is a constant, is called the *Laguerre equation.* Find two linearly independent series solutions about $x = 0$. Check convergence.

7. The interaction between two nucleons may be described by a mesonic potential
$$V(x) = \frac{Ae^{-ax}}{x}$$
where A and a are constants.Find the first few nonvanishing terms of the solution of 1D Schrodinger equation.

8. Find the series solutions of Legendre equation

$$\left(1-x^2\right)y'' - 2xy' + l(l+1)y = 0$$

about $x = 1$. Show that there are polynomial solutions for non-negative l.