

1. A ring of mass  $M$  hangs from a thread, and two beads of mass  $m$  slide on it without friction. The beads are released simultaneously from the top of the ring and slide down opposite sides. Show that the ring will start to rise if  $m > 3M/2$ , and find the angle at which this occurs.
2. A block shown in the drawing is acted on by a spring with spring constant  $k$ , and a weak friction force of constant magnitude  $f$ . The block is pulled distance  $x_0$  from the equilibrium and released. It oscillates many times before coming to a halt.

- (a) Show that the amplitude decreases by same amount in each cycle of oscillation.
- (b) Find the number of cycles  $n$ , the mass oscillates before coming to rest.

3. Find the forces for the following potential energies.

- (a)  $U = Ax^2 + By^2 + Cz^2$
- (b)  $U = A \ln(x^2 + y^2 + z^2)$
- (c)  $U = A \cos \theta / r^2$  (Plane Polar Coordinates)

4. A particle of mass  $m$  moves in a horizontal plane along the parabola  $y = x^2$ . At  $t = 0$  it is at point  $(1, 1)$  moving in the direction shown with speed  $v_0$ . Apart from the force of constraint holding it to the path, it is acted on by the following external forces

$$\begin{aligned}\mathbf{F}_a &= -Ar^3\hat{\mathbf{r}} \\ \mathbf{F}_b &= B(y^2\mathbf{i} - x^2\mathbf{j})\end{aligned}$$

- (a) Are the forces conservative?
  - (b) What is the speed  $v_f$  of the particle when it arrives at the origin?
5. The potential energy function for a particular two dimensional force field is given by  $U = Cxe^{-y}$ , where  $C$  is a constant.
    - (a) Sketch constant energy lines.
    - (b) Show that along the constant energy lines  $d\mathbf{r} = dx(\mathbf{i} + \mathbf{j}/x)$ .
    - (c) Using b, show explicitly that  $\nabla U$  is perpendicular to the constant energy line.

6. How much work is done around the path that is shown by the force  $\mathbf{F}_b = A(y^2\mathbf{i} + 2x^2\mathbf{j})$ , where  $A$  is a constant and  $x$  and  $y$  are in meters? Find the answer by evaluating the line integral as well as by the surface integral (Stokes' Theorem)
7. If a force field is given by  $\mathbf{F} = 3x^2y\mathbf{i} + (x^3 + 3y^2)\mathbf{j}$ , find the potential energy.

