Homework # 2

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Problems: Prove that each of the following problems is NP-complete by giving a poly-time reduction from a known NP-complete problem (either proved in the class or in Sipser's book).

1. Given a graph G and integer k, does G have a cycle, with no repeated nodes, of length at least k?

2. Given m equations
$$\Sigma_{i=1}^{n} a_{ij} x_j = b_i,\ i = 1, \ldots, m$$
in n variables with integer coefficients a_{ij} and b_i, does the system have a solution in which all x_j's are either zero or one?

3. Given a directed graph G with a positive lengths d_{ij} on each edge (i, j), two nodes 1 and n, and an integer k, is there a path from 1 to n, not repeating any node, with total length k or more?

4. Given a family of sets $\{S_1, S_2, \ldots, S_n\}$ and an integer b is there a set H with b or fewer elements such that H intersects all sets in the family?

5. Given a family $\mathcal{F}$ of subsets of a universe U and an integer k are there k sets in $\mathcal{F}$ whose union equals U?

6. The problem is to schedule n tasks on two machines, with the following conditions:
   - Both machines have the same speed.
   - Each task can be executed on either machine.
   - There are no restrictions on the order of task execution.

   Given the execution times $a_1, \ldots, a_n$ of the tasks and a deadline D, all in binary, can all the tasks be completed within their deadline?