Distributed Clock Skew and Offset Estimation from Relative Measurements in Mobile Networks with Markovian Switching Topology ⋆

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Abstract

We analyze a distributed algorithm for estimation of scalar parameters belonging to nodes in a mobile network from noisy relative measurements. The motivation comes from the problem of clock skew and offset estimation for the purpose of time synchronization. The time variation of the network was modeled as a Markov chain. The estimates are shown to be mean square convergent under fairly weak assumptions on the Markov chain, as long as the union of the graphs is connected. Expressions for the asymptotic mean and correlation are also provided.

Key words: sensor networks; mobile networks; time synchronization; distributed estimation; cooperative control; Markovian switching.

1 Introduction

We consider the problem of estimation of variables in a network of mobile nodes in which pairs of communicating nodes can obtain noisy measurement of the difference between the variables associated with them. Specifically, suppose the u-th node of a network has an associated *node variable* $x_u \in \mathbb{R}$. If nodes u and v are neighbors at discrete time index k, then they can obtain a measurement $\zeta_{u,v}(k)$ where

$$\zeta_{u,v}(k) = x_u - x_v + \epsilon_{u,v}(k). \qquad (1)$$

The problem is for each node to estimate its node variable from the relative measurements it collects over time, without requiring any centralized information processing or coordination. We assume that at least one node knows its variable. Otherwise the problem is indeterminate up to a constant. A node that knows its node variable is called a *reference node*. All nodes are allowed to be mobile, so that their neighbors may change with time.

The problem of time synchronization (also called clock-synchronization) through clock skew and offset estimation falls into this category, and provides the main motivation for the study. Time synchronization in ad-hoc networks, especially in wireless sensor networks, has been a topic of intense study in recent years. The utility of data collected and transmitted by sensor nodes depend directly on the accuracy of the time-stamps. In TDMA based communication schemes, accurate time synchronization is required for the sensors to communicate with other sensors. Operation on a pre-scheduled sleep-wake cycle for energy conservation and lifetime maximization also requires accurate knowledge of a common global time. We refer the interested reader to the review papers [1–3] for more details on time synchronization.

The relationship between local clock time $\tau_u(t)$ of node u and global time t is usually modeled as $\tau_u(t) = \alpha_u t + \beta_u$, where the scalars α_u, β_u are called its *skew* and *offset*, respectively [1,3]. A node can determine the global time t from its local clock time by using the relationship $\hat{t} = (\tau_u(t) - \hat{\beta}_u)/\hat{\alpha}_u$ as long as it can obtain estimates $\hat{\alpha}_u, \hat{\beta}_u$ of the skew and offset of its local clock. Hence the problem is clock synchronization in a network can be alternatively posed as the problem of nodes estimating their skews and offsets. It is not possible for a node to measure its skew and offset directly. However, it is possible for a pair of neighboring nodes to measure the difference between their offsets and logarithm of skews by ex-

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changing a number of time stamped messages. Existing protocols to perform so-called *pairwise synchronization*, such as [4–6], can be used to obtain such relative measurements. The details will be described in Section 2.1. The problem of clock offset and skew estimation can therefore be cast as a special case of the estimation from relative measurements described above. If an algorithm is available to solve the scalar node variable estimation problem, nodes can execute two copies of this algorithm in parallel to estimate both skew and offset. Therefore we only consider the scalar case. In the context of time synchronization, the existence of a reference node means that at least one node has access to the global time t. This is the case when at least one node is equipped with a GPS receiver, in which case that node has access to the UTC (Coordinated Universal Time). If no node has a GPS receiver, then one node has to be elected to be the reference so that it's local clock time is considered the global time that everyone has to synchronize to.

1.1 Related work

Time synchronization in sensor networks can be classified into pairwise synchronization and global synchronization methods. In pairwise synchronization, a pair of nodes try to synchronize their clocks to each other. In practice this is often achieved by one of the nodes estimating its relative offset and/or skew with respect to the other node, so that the local time of the other node serves as a reference [4–7]. Precise definitions of relative offset and relative skew are postponed till Section 2.1. In global synchronization, also called network-wide synchronization, all nodes synchronize themselves to a common time.

A common approach for global synchronization in sensor networks is to first elect a root node and construct a spanning tree of the network with the root node being the "level 0" node. Every node thereafter synchronizes itself to a node of lower level (higher up in the hierarchy) by using a pairwise synchronization method. Examples of such spanning-tree based protocols include Timing-Sync Protocol for Sensor Networks (TPSN) [8] and Flooding Time Synchronization Protocol (FTSP) [9]. Change in the network topology due to node mobility or node failure requires recomputing the spanning tree and sometimes even re-election of the root node. This adds considerable communication overhead. The situation gets worse if nodes move rapidly.

Recently, a number of fully distributed global synchronization algorithms have been proposed that do not need spanning tree computation. Distributed protocols are therefore more readily applicable to mobile networks than tree-based protocols. Among the distributed synchronization protocols proposed, some are based on estimation of the skew and/or offset of each clock with respect to a reference clock (called *absolute time synchronization*). The algorithms proposed in [10–14] belong to this category. Another class of protocols estimate a common global time that may not be related to the time of any clock in the network. The algorithms proposed in [15–17] belong to this category, which we call *virtual time synchronization*.

1.2 Contribution

In this paper we consider the problem of distributed estimation of skews and offsets with respect to a reference clock in a mobile network for global absolute time synchronization, where the network changes with time due to nodes' motion. The common thread among virtual time synchronization methods mentioned earlier is the use of consensus-type algorithms to construct virtual skew and offsets that every node agrees to. In many applications, absolute time synchronization is preferable over virtual time synchronization. This occurs when the user of the sensor network is interested in the time of an event that is measured in an absolute reference time, such as UTC provided by a GPS unit on a base station. Therefore, in this paper we consider only absolute time synchronization.

We analyze an algorithm for estimating absolute skews and offsets from noisy pairwise relative measurements of skews and offsets, which is a slight modification of the algorithms proposed in [18,10,12]. Though the algorithm is adopted from these earlier papers, the analysis in those papers were limited to static networks. Thus, little is known about how such an algorithm will perform in a mobile network.

The main contribution is that we analyze the convergence of the algorithm when the network topology changes due to the motion of the nodes, as well as random communication failure. We model the resulting time-varying topology of the network as the state of a Markov chain. Techniques for the analysis of jump linear systems from [19] are used to study convergence of the algorithm. We show that under fairly weak assumptions on the Markov chain, the proposed algorithm is mean square convergent if and only if the union of the graphs that occur is connected. Mean square convergence means the expected value and the variance of the estimates obtained by each node converges to fixed values that do not depend on the initial conditions. When the relative measurements are unbiased, then limiting mean is the same as the true value of the variable, meaning the estimates obtained are asymptotically unbiased. Formulas for the limiting mean and variance are obtained by utilizing results from jump linear systems.

The algorithm we analyze bears a close resemblance to consensus algorithms. In fact, the estimation error dynamics turns out to be a leader-follower consensus algorithm, where the leader states - corresponding to the estimation error of the reference nodes - are always 0.

However, existing results from consensus cannot be directly used to analyze the scenario examined in this paper. Consensus literature almost always treats the problem where all nodes participates in the consensus algorithm, i.e., "leaderless consensus", while ours is a "leader-follower" consensus since the reference nodes error state stays at 0. One may expect analysis of this case would be easier, but that turns out to be not the case. Even though the literature on consensus is extensive, the topic of consensus with both time-varying graph topology and additive measurement noise is considered only in a limited number of papers, e.g. [20–23]. There are several differences between the consensus algorithms studied in [20–23] and the error dynamics examined in this paper, which preclude using their results to perform the analysis. These include requirement of symmetry or balance in graphs/matrices, preassignment of time-varying gains that must be synchronized among all nodes, etc. None of these restrictions are imposed in our analysis. In addition, use of Markov jump linear systems theory allows us to provide formulas for limiting mean and correlation. In [24], we show that when nodes move according to the Random Waypoint Mobility model [25], the resulting switching dynamics can indeed be modeled by a Markov chain. This provides justification for assuming the switching of topology is Markovian when analyzing mobile networks. An more detailed discussion on the relation between this work and existing results on consensus with switching networks can be found in Remark 4 of [24].

A preliminary version of this paper was presented in [26]. Compared to that paper, we make several additional contributions. While the paper [26] provided only sufficient conditions for mean square convergence, here we provide both necessary and sufficient conditions. An assumption of symmetry of certain matrices were made in [26], which is removed in the present paper.

The rest of the paper is organized as follows. Section 2 describes the connection between the problem of estimation from relative measurements and the problem of skew/offset estimation, and then states the problem precisely. Section 3 describes the proposed algorithm and states the main result (Theorem 3). It also discusses the relevance of the Markovian switching topology model. Section 4 is devoted to the proof of the theorem. Simulation studies are presented in Section 5.

2 The estimation problem

We consider the problem of estimating the scalar parameters (called *node variables*) x_u, $u = 1, \dots, n_b$, where n_b is the number of nodes in the network that do not know their node variables. We assume that there are n_r additional nodes that knows their node variables, where $n_r \geq 1$. These define a node set $\mathcal{V} = \{1, \dots, n\}$, where $n = n_b + n_r$ is the total number of nodes. For later reference, we define $\mathcal{V}_b := \{1, \dots, n_b\}$ and $\mathcal{V}_r = \{n_b+1, \dots, n_b+n_r\}$, so that $\mathcal{V} = \mathcal{V}_b \cup \mathcal{V}_r$. Note that $n = n_b + n_r$. Time is measured by a discrete time-index $k = 0, 1, \dots$. The mobile nodes define a time-varying undirected *measurement graph* $\mathcal{G}(k) = (\mathcal{V}, \mathcal{E}(k))$, where $(u, v) \in \mathcal{E}(k)$ if and only if u and v can obtain a relative measurement of the form (1) during the time interval between the time indices k and $k+1$. Specifically, for each $(u, v) \in \mathcal{E}(k)$, there is a measurement $\zeta_{u,v}(k) = x_u - x_v + \epsilon_{u,v}(k)$ that is available to both u and v at time k. In practice, one of the two nodes computes this measurement from sensed information. We assume that if u computes the measurement $\zeta_{u,v}$, it then sends this measurement to v so that v also has access to the same measurement. We follow the convention that the relative measurement between u and v that is obtained by the node u is always of $x_u - x_v$ while that used by v is always of $x_v - x_u$. Since the same measurement is shared by a pair of neighboring nodes, if v receives the measurement $\zeta_{v,u}$ from u, then it converts the measurement to $\zeta_{v,u}$ by assigning $\zeta_{v,u}(k) := -\zeta_{u,v}(k)$. We assume, without any loss of generality, that between a pair of nodes u and v, the node with the lower index obtains the relative measurement between them first, and then shares with the node with the higher index.

The *neighbors* of u at k, denoted by $\mathcal{N}_u(k)$, is the set of nodes that u has an edge with in the measurement graph $\mathcal{G}(k)$. We assume that if $v \in \mathcal{N}_u(k)$, then u and v can also exchange information through wireless communication at time k. Therefore, if one prefers to think of a communication graph, we assume that it is the same as the measurement graph.

The task is to estimate the node variables x_u for $u = 1, \dots, n$ by using the relative measurements $\zeta_{u,v}(k), (u, v) \in \mathcal{E}(k)$ that becomes available over time $k = 0, 1, \dots$. In addition, the algorithm has to be distributed in the sense that each node has to estimate its own variables, and at every time k, a node u can only exchange information with its neighbors $\mathcal{N}_u(k)$. Note that the estimation problem is indeterminate unless $n_r > 0$.

2.1 Relation to skew and offset estimation

To see the connection between skew/offset estimation and the problem of estimation from noisy relative measurements introduced in the previous section, we first discuss the notion of pairwise synchronization between a pair of neighboring nodes u and v. By exchanging a number of time-stamped messages, it is possible for node u to estimate the so-called *relative skew* $\alpha_{u,v}$ and *relative offset* $\beta_{u,v}$ between itself and v, where

$$\tau_u(t) = \alpha_{u,v} \tau_v(t) + \beta_{u,v}. \tag{2}$$

That is, the parameters $\alpha_{u,v}$ and $\beta_{u,v}$ relate the local time of u to the local time of v at the same global time t.

A number of methods are available that allows pairwise synchronization between a node pair from time-stamped messages [4,5,27,13,6]. The parameters $\alpha_{u,v}$ and $\beta_{u,v}$ are also referred to as the *skew and offset of node u with respect to node v* [7].

The relationship between the absolute skew and offset $\alpha_u, \beta_u, \alpha_v, \beta_v$ and relative skew and offset $\alpha_{u,v}, \beta_{u,v}$ is given by

$$\alpha_{u,v} := \frac{\alpha_u}{\alpha_v} \qquad \beta_{u,v} := \beta_u - \beta_v \frac{\alpha_u}{\alpha_v}. \tag{3}$$

This relationship is obtained by expressing the local time $\tau_u(t)$ of node u at global time t in terms of the local time $\tau_v(t)$ at node v at the same time t by using (1):

$$\tau_u(t) = \alpha_u(\frac{\tau_v(t) - \beta_v}{\alpha_v}) + \beta_u = \frac{\alpha_u}{\alpha_v}\tau_v(t) + \beta_u - \beta_v\frac{\alpha_u}{\alpha_v},$$

and comparing with (2). Suppose a node u obtains noisy estimates $\hat{\alpha}_{u,v}, \hat{\beta}_{u,v}$ of the parameters $\alpha_{u,v}, \beta_{u,v}$ by using a pairwise synchronization protocol.

(1) We model the noisy estimate of $\alpha_{u,v}$ as

$$\hat{\alpha}_{u,v} = \exp(\epsilon^s_{u,v})\alpha_{u,v} \tag{4}$$

where $\exp(\cdot)$ is exponential function and $\epsilon^s_{u,v}$ is a random variable. If the estimation error is small, then $\epsilon^s_{u,v}$ is close to 0. Taking log, we get

$$\log\hat{\alpha}_{u,v} = \log\alpha_{u,v} + \epsilon^s_{u,v} = \log\alpha_u - \log\alpha_v + \epsilon^s_{u,v}. \tag{5}$$

Eq. (5) can be rewritten as $\zeta^s_{u,v} = x^s_u - x^s_v + \epsilon^s_{u,v}$, with the definitions $\zeta^s_{u,v} := \log\hat{\alpha}_{u,v}$ and $x^s_i := \log\alpha_u$, which makes $\zeta^s_{u,v}$ a noisy relative measurement of the node variables x^s_u and x^s_v; cf. (1). It is important to notice that $\zeta^s_{u,v}$ is a measured quantity – since $\hat{\alpha}_{u,v}$ is measured – while the variables x^s_u, x^s_v, which are logarithms of the skews, are unknown.

(2) Similarly, the noisy estimate $\hat{\beta}_{u,v}$ of $\beta_{u,v}$ with random estimation error $e^o_{u,v}$ can be written as

$$\hat{\beta}_{u,v} = \beta_{u,v} + e^o_{u,v} = \beta_u - \beta_v + \epsilon^o_{u,v}, \tag{6}$$

where $\epsilon^o_{u,v} := \beta_v(1 - \alpha_u/\alpha_v) + e^o_{u,v}$. Again, (6) can be rewritten as $\zeta^o_{u,v} = x^o_u - x^o_v + \epsilon^o_{u,v}$, with the definitions $\zeta^o_{u,v} := \hat{\beta}_{u,v}$ and $x^o_u := \beta_u$, which makes $\zeta^o_{u,v}$ a noisy relative measurements of the node variables x^o_u and x^o_v; cf. (1). In this case the node variables are the clock offsets β_u's. The noise $\epsilon^o_{u,v}$ in the offset measurement is in general biased even if the measurement of the relative offset $\beta_{u,v}$ is unbiased.

This discussion shows that the estimates of the relative skew and the relative offset between a pair of neighboring nodes, which can be obtained by existing algorithms for pairwise synchronization, can be expressed as a noisy relative measurement of node variables by appropriate redefinitions. The node variables are log-skews and offsets. Once node u obtains estimates $\hat{x}^s_u$ and $\hat{x}^o_u$ of its two node variables x^s_u and x^o_u, it can estimate its skew and offset as $\hat{\alpha}_u := \exp(\hat{x}^s_u)$ and $\hat{\beta}_u := \hat{x}^o_u$. Thus, the problem of estimating the skews and offsets of all the clocks in a network can be transformed to an estimation from relative measurements problem, where relative measurements are of the form (1).

Remark 1 *From this point on, we only consider the estimation problem involving scalar node variables. This entails no loss of generality since estimation of the two scalar variables, skew and offset, can be performed in parallel. In the skew estimation problem, log-skews take the role of node variables and* $\log\hat{\alpha}_{u,v}$*'s obtained from pairwise synchronization take the role of relative measurements. In the offset estimation problem, node variables are the offsets and relative measurements are the* $\hat{\beta}_{u,v}$*'s obtained from pairwise synchronization. The assumption on the existence of the reference node is equivalent to at least one node knowing the global time. This can be achieved by either one or more nodes having access to GPS time, or by arbitrarily electing a node as a reference and choosing its local time as the global time.*

3 Algorithm and results

3.1 Algorithm for distributed estimation from relative measurement

The algorithm we consider is adopted from [10,12,18], with minor modification to make it applicable to time varying networks. Each node u maintains in its local memory an estimate $\hat{x}_u(k)$ of its node variable $x_u \in \mathbb{R}$. Every node - except the reference nodes - iteratively updates its estimate as we'll describe now. The estimates can be initialized to arbitrary values. In executing the algorithm at iteration k, node u communicates with its current neighbors to obtain measurements $\zeta_{u,v}(k)$ and their current estimates $\hat{x}_v(k)$, $v \in \mathcal{N}_u(k)$. Since obtaining measurements require exchanging time-stamped messages, the current estimates can be easily exchanged during the process of obtaining new measurements. Node u then updates its estimate according to

$$\hat{x}_u(k+1) = \begin{cases} \frac{w_{uu}(k)\hat{x}_u(k) + \sum_{v\in\mathcal{N}_u(k)} w_{vu}(k)(\hat{x}_v(k)+\zeta_{u,v}(k))}{w_{uu}(k)+\sum_{v\in\mathcal{N}_u(k)} w_{vu}(k)}, & u \in \mathcal{V}_b \\ x_u(k), & u \in \mathcal{V}_r, \end{cases} \tag{7}$$

where the *weights* $w_{vu}(k)$ and $w_{uu}(k)$ are arbitrary positive numbers. The update law is well-defined even at times when u has no neighbors. Nodes continue this iterative update unless they see little change in their local estimates, at which point they can stop updating. The update procedure in each node $u \in \mathcal{V}_b$ is specified in Algorithm 1.

Algorithm 1 Distributed update at node u

Initialize estimate $\hat{x}_u(0) \in \mathbb{R}$ and local iteration counter $k = 0$
while u is performing iteration **do**
  if $\mathcal{N}_u(k_u) \neq \emptyset$ **then**
    for $v \in \mathcal{N}_u(k)$ **do**
      if u does not have $\zeta_{u,v}(k)$ **then**
        1.u and v perform pairwise synchronization: u obtains $\zeta_{u,v}(k)$, v obtains $\zeta_{v,u}(k)$;
        2.u and v exchange their current estimates: u saves $\hat{x}_v(k)$; v saves $\hat{x}_u(k)$;
      else
        u does not communicate with v;
      end if
    end for
    u updates $\hat{x}_u(k+1)$ using (7);
  else
    $\hat{x}_u(k+1) \leftarrow \hat{x}_u(k)$;
  end if
  k=$k+1$;
end while

Each node u is allowed to vary its local weights $w_{uv}(k)$ with time and use distinct weights for distinct neighbors to account for the heterogeneity in measurement quality. Between two neighbors p, q of node u at time k, the relative measurement $\zeta_{u,p}(k)$ between u and p may have lower measurement error than the relative measurement $\zeta_{u,q}(k)$ between u and q. This occurs, for example, if u and p were able to exchange more time stamped messages than u and q before computing the relative measurements [6,7]. In this case, node u should choose its local weights at k so that $w_{pu}(k) > w_{qu}(k)$. Due to the denominator in (7), it is only the ratios among the weights that matter, not their absolute values.

3.1.1 Asynchronous implementation

The description so far is in terms of a common global iteration index k. In practice, nodes do not have access to such a global index. Instead, each node keeps a local *iteration index*. After every increment of the local index, the node tries to collect a new set of relative measurements with respect to one or more of its neighbors within a pre-specified time interval. At the end of the time interval, whether it is able to get new measurements or not, it updates its estimate according to the update law (7) and increments its local iteration counter. Now the index k in (7) has to be interpreted as the local iteration index. The process then repeats. It follows from (7) that if a node is unable to gather new measurements from any neighbors, then its updated estimate is precisely the previous estimate.

The global iteration index is useful to describe the algorithm from the point of view of an omniscient spectator. Let T the time interval, say, in seconds, between two successive increments of the global index k. The parameter T is arbitrary, as long as is small enough so that no node updates its local estimate more than once with the time interval T. In that case, one of only two events are possible for an arbitrary node u at the end of the time interval when the global counter is increased from k to $k+1$: (i) u either increases its local index by one, or (ii) u does not increases its local index. If a node increases its local index, both the local and global indices increase by one. A node does not increase its local iteration index if it is not able to gather new measurements. In the omniscient spectator's view, the node's neighbor set is empty at this time index; so according to (7), the next estimate of the node's variable is the same as the previous one. Thus, a node's local asynchronous state update can be described in terms of the synchronous algorithm (7); the latter being more convenient for exposition. We therefore consider only the synchronous version in the sequel.

3.2 Convergence analysis with Markovian switching

In this paper we model the sequence of measurement graphs $\{\mathcal{G}(k)\}_{k=0}^{\infty}$ that appear as time progresses as the realization of a (first order) Markov chain, whose state space $\mathbb{G} = \{\mathcal{G}_1, \ldots, \mathcal{G}_N\}$ is the set of graphs that can occur over time. The Markovian switching assumption on the graphs means that $P(\mathcal{G}(k+1) = \mathcal{G}_i | \mathcal{G}(k) = \mathcal{G}_j) = P(\mathcal{G}(k+1) = \mathcal{G}_i | \mathcal{G}(k) = \mathcal{G}_j, \mathcal{G}(k-1) = \mathcal{G}_\ell, \ldots, \mathcal{G}(0) = \mathcal{G}_p)$ where $\mathcal{G}_i, \mathcal{G}_j, \mathcal{G}_\ell, \ldots, \mathcal{G}_p \in \mathbb{G}$ and where $P(\cdot)$ denotes probability. We assume that the Markov chain is homogeneous, and denote the transition probability matrix of the chain by $\mathcal{P}$, in which p_{ij} is the (i,j)-th entry of $\mathcal{P}$.

Remark 2 *The examination of the applicability of the Markovian model of graph switching is provided in [24, Appendix A]. We examine the Random Waypoint Mobility model by using an empirical conditional entropy based method suggested in [28]. The analysis in [24] reveals that the graph switching process can be modeled as a (first order) Markov chain.*

Let $e_u(k) := \hat{x}_u(k) - x_u$ be the estimation error at node u. Since $\zeta_{u,v}(k) = x_u - x_v + \epsilon_{u,v}(k)$, the update law (7) can be rewritten as

$$e_u(k+1) = \begin{cases} \frac{w_{uu}(k)e_u(k) + \sum_{v \in \mathcal{N}_u(k)} w_{vu}(k)(e_v(k) + \epsilon_{u,v}(k))}{w_{uu}(k) + \sum_{v \in \mathcal{N}_u(k)} w_{vu}(k)}, & u \in \mathcal{V}_b \\ 0, & u \in \mathcal{V}_r. \end{cases} \quad (8)$$

The right hand side of (8) is a weighted average of estimation errors of x_u and measurement noise. If the measurement noise $\epsilon_{u,v}(k)$ is zero-mean and the initial estimates are unbiased, i.e. $\mathrm{E}[e_u(0)] = 0$, $\forall u \in \mathcal{V}_b$, then $\mathrm{E}[e_u(k)] = 0$ for all k, where $\mathrm{E}[\cdot]$ denotes expectation.

The main result of the paper - on the mean square convergence of (8) - is stated below as a theorem. In the statement of theorem, $\mathbf{e}(k) := [e_1(k), \ldots, e_{n_b}(k)]^T$ is the estimation error vector. Moreover, $\boldsymbol{\mu}(k) := \mathrm{E}[\mathbf{e}(k)]$ is the mean and $\mathbf{Q}(k) := \mathrm{E}[\mathbf{e}(k)\mathbf{e}(k)^T]$ is the correlation matrix of the estimation error vector. We say that a stochastic process $\mathbf{y}(k)$ is *mean square convergent* if $\mathrm{E}[\mathbf{y}(k)]$ and $\mathrm{E}[\mathbf{y}(k)\mathbf{y}^T(k)]$ converges as $k \to \infty$ for every initial condition. The *union graph* $\hat{\mathcal{G}}$ is defined as follows:

$$\hat{\mathcal{G}} := \cup_{i=1}^{N} \mathcal{G}_i = (\mathcal{V}, \cup_{i=1}^{N} \mathcal{E}_i), \tag{9}$$

where $\mathcal{E}_i$ is set of edges in $\mathcal{G}_i$. We assume that the measurement noise $\epsilon_{u,v}(k)$ affecting the measurements on the edge (u, v) is a wide sense stationary process. We also assume that the measurement noise sequence $\epsilon_{u,v}(k)$ and the initial condition $\hat{x}_u(0)$, for any u, v, k is independent of the Markov chain that governs the time-variation of the graph.

Due to technical reasons, we make an additional assumption that there exists a time k_0 after which the edge-weights do not change. The choice of weights during the transient period (up to k_0) will affect initial reduction of the estimation errors but will not change the asymptotic behavior.

Recall that $\mathbf{e}(k)$ is the estimation error vector for the nodes who do not know their node variables, the main theorem is as follows:

Theorem 3 *Assume that the temporal evolution of the communication graph $\mathcal{G}(k)$ is governed by an N-state homogeneous Markov chain that is ergodic, and $p_{ii} > 0$ for $i = 1, \ldots, N$. The estimation error $\mathbf{e}(k)$ is mean square convergent if and only if $\hat{\mathcal{G}}$ is connected.*

Remark 4 *The formulas for computing the limiting values $\lim_{k\to\infty} \boldsymbol{\mu}(k)$ and $\lim_{k\to\infty} \mathbf{Q}(k)$ are provided in Lemma 6 (Section 4). It follows directly from the formulas (see Lemma 6), that if additionally all the measurements are unbiased, then $\lim_{k\to\infty} \boldsymbol{\mu}(k) = 0$.*

The implication of the theorem is that as long as nodes are connected in a "time-average" sense characterized by $\hat{\mathcal{G}}$ being connected, the estimates of the node variables will converge to random variables with a constant mean and variance, irrespective of the initial conditions. Thus, after a sufficiently long time, the nodes can turn off the synchronization updates without much loss of accuracy. The assumption of ergodicity of the Markov chain ensures that there is an unique steady state distribution and that the steady state probability of each state is nonzero [19]. This means every graph in the state space of the chain occurs infinitely often. Since their union graph is connected, ergodicity implies that information from the reference node(s) will flow to each of the nodes over time. None of the graphs that ever occur is required to be a connected graph. The assumption $p_{ii} > 0$ means $P(\mathcal{G}(k+1) = \mathcal{G}_i | \mathcal{G}(k) = \mathcal{G}_i) > 0$. This can be assured if the nodes move slowly enough.

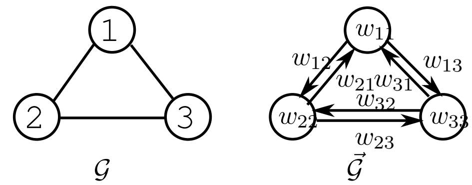


Fig. 1. A measurement graph $\mathcal{G}$ and the corresponding weight graph $\vec{\mathcal{G}}$.

4 Proof of Theorem 3

We consider a *weighted directed* graph $\vec{\mathcal{G}}(k) = (\mathcal{V}, \vec{\mathcal{E}}(k), W(k))$ associated with undirected measurement graph $\mathcal{G}(k) = (\mathcal{V}, \mathcal{E}(k))$. In particular, there exists an undirected edge (u,v) in $\mathcal{G}(k)$, then there exist two directed edges (u, v) and (v, u) in $\vec{\mathcal{G}}(k)$. The *weight matrix* $W(k)$ defined as

$$W_{uv}(k) := \begin{cases} w_{vu}(k) > 0 \text{ for } (u, v) \in \vec{\mathcal{E}}(k) \\ w_{uu}(k) > 0 \text{ for } v = u \\ \qquad 0 \qquad \text{o.w.} \end{cases} \tag{10}$$

Thus, given a measurement graph $\mathcal{G}(k)$ and $W(k)$, $\vec{\mathcal{G}}(k)$ is specified. See Figure 1 for an example of an undirected measurement graph and an associated directed weighted graph. The square *non-negative* matrices, $D(k)$, $M(k)$ and $N(k)$ is defined as follows: $D(k)$ is a $n \times n$ diagonal matrix made up of the diagonal entries of $W(k)$, and $N(k) := W(k) - D(k)$. $M(k)$ is a $n \times n$ diagonal matrix with entry $M_{uu}(k) = \sum_{u \neq v} W_{uv}(k)$. Furthermore, we define the $n_b \times n_b$ *basis matrix* $D_b(k)$, $M_b(k)$ and $N_b(k)$ as the principle submatrix of $D(k)$, $M(k)$ and $N(k)$ obtained by removing those rows and columns corresponding to the reference nodes. Now, (8) can be compactly expressed as

$$\mathbf{e}(k+1) = J_b(k)\mathbf{e}(k) + B_b(k)\boldsymbol{\epsilon}(k), \tag{11}$$

where

$$\begin{aligned} &J_b(k) := (M_b(k) + D_b(k))^{-1}(N_b(k) + D_b(k)), \\ &B_b(k) := (M_b(k) + D_b(k))^{-1} A_b(k), \\ &\boldsymbol{\epsilon}(k) := [\bar{\epsilon}_1(k)^T, \ldots, \bar{\epsilon}_{n_b}(k)^T]^T, \\ &\bar{\epsilon}_u(k) := [\epsilon_{u,1}(k), \ldots \epsilon_{u,n}(k)]^T, \\ &A_b(k) := diag(\bar{N}_1(k), \ldots, \bar{N}_{n_b}(k)), \\ &\bar{N}_u(k) := [N_{u1}(k), \ldots, N_{un}(k)], \end{aligned} \tag{12}$$

where vector $\bar{\epsilon}_u(k)$ and $\bar{N}_u(k)$ do not contain $\epsilon_{u,u}(k)$ and $N_{uu}(k)$ respectively. Note that diagonal matrix $M_b(k)+D_b(k)$ is always non-singular because diagonal entries in $D_b(k)$ are always positive as $W_{uu}(k) > 0$ for all u, and $M_b(k)$ is nonnegative. When $N_{uv}(k) = 0$, the corresponding $\epsilon_{u,v}(k)$ is taken to be an arbitrary random variable with mean and variance such that the stationary assumption is satisfied. Since these noise terms are multiplied by 0, this entails no loss of generality. Moreover, recall that $\epsilon_{u,v}(k) = -\epsilon_{v,u}(k)$.

As a result of the assumption that there exists a time k_0 after which the weight between two nodes do not change, the graph $\mathcal{G}(k)$ uniquely determines the weight matrix $W(k)$ for $k > k_0$. Since there are N distinct graphs in $\mathbb{G}$, a set $\mathbb{W} := \{W_1, \dots, W_N\}$ is also defined, with W_i associated with $\mathcal{G}_i$. As a result, for $k \geq k_0$, if $\mathcal{G}(k) = \mathcal{G}_i$ then $W(k) = W_i$. Therefore, $D_i, M_i, N_i, D_{bi}, M_{bi}, N_{bi}, J_{bi}$ are uniquely defined by $\{\mathcal{G}_i, W_i\}$.

With these choices stated above, the state of the following system is identical to that of (11) for the same initial conditions:

$$\mathbf{e}(k+1) = J_{b\ \theta(k)}\mathbf{e}(k) + B_{b\ \theta(k)}\boldsymbol{\epsilon}(k), \quad k \geq k_0 \tag{13}$$

where $\theta : \mathbf{Z}^+ \to \{1, \dots, N\}$ is the switching process that is governed by the underlying Markov chain $\mathcal{G}(k)$. The reason for the qualifier $k \geq k_0$ is that weights are not limited to the set $\mathbb{W}$ before k_0, so technically the matrices $J_{b\ \theta(k)}$ and $B_{b\ \theta(k)}$ are uniquely determined by the Markov chain only for $k \geq k_0$. The error dynamics (13) is a Markov jump linear system (MJLS) [19]. To proceed with the analysis of the mean square convergence of (13), we need some terminology.

$$\gamma := \mathrm{E}[\boldsymbol{\epsilon}(k)], \qquad \Gamma := \mathrm{E}[\boldsymbol{\epsilon}(k)\boldsymbol{\epsilon}^T(k)], \tag{14}$$
$$\boldsymbol{\mu}(k) := \mathrm{E}[\mathbf{e}(k)], \qquad \mathbf{Q}(k) := \mathrm{E}[\mathbf{e}(k)\mathbf{e}^T(k)]. \tag{15}$$

Furthermore, for a set of matrices $X_i \in \mathbb{R}^{\ell_1 \times \ell_2}$, $Y_{ij} \in \mathbb{R}^{\ell_1 \times \ell_2}$, $i,j = 1, \dots, N$, denote the $\ell_1 N \times \ell_2 N$ block diagonal matrix $\mathrm{diag}[X_i] = \mathrm{diag}\{X_1, \dots . X_N\}$ and

$$[Y_{ij}] := \begin{bmatrix} Y_{11} & \dots & Y_{1N} \\ \vdots & \ddots & \vdots \\ Y_{N1} & \dots & Y_{NN} \end{bmatrix}_{\ell_1 N \times \ell_2 N}.$$

Now, define the matrices

$$J_i := (M_i + D_i)^{-1}(N_i + D_i) \in \mathbb{R}^{n \times n},$$
$$J_{bi} := (M_{bi} + D_{bi})^{-1}(N_{bi} + D_{bi}) \in \mathbb{R}^{n_b \times n_b},$$
$$F_i := J_i \otimes J_i \in \mathbb{R}^{n^2 \times n^2},\ F_{bi} := J_{bi} \otimes J_{bi} \in \mathbb{R}^{n_b^2 \times n_b^2} \tag{16}$$

where $\otimes$ denotes the Kronecker product. Furthermore, define the matrices

$$\mathcal{D} := \left(\mathcal{P}^T \otimes I\right) diag[F_i] = [p_{ji}F_j] \in \mathbb{R}^{Nn^2 \times Nn^2}, \tag{17}$$
$$\mathcal{D}_b := (\mathcal{P}^T \otimes I) diag[F_{bi}] = [p_{ji}F_{bj}] \in \mathbb{R}^{Nn_b^2 \times Nn_b^2}, \tag{18}$$
$$\mathcal{C}_b := (\mathcal{P}^T \otimes I) diag[J_{bi}] = [p_{ji}J_{bj}] \in \mathbb{R}^{Nn_b \times Nn_b},$$

where I is an identity matrix of appropriate dimension. Recall that $\mathcal{P}$ is the transition probability matrix of the Markov chain.

The key to establish Theorem 3, is the following technical result. The proof is provided in [24, Appendix B] since it requires introduction of considerable new terminology.

Lemma 5 *When the temporal evolution of the graph $\mathcal{G}(k)$ is governed by a homogeneous ergodic Markov chain whose transition probability matrix $\mathcal{P}$ has the property that its diagonal entries are strictly positive, then $\rho(\mathcal{D}_b) < 1$ if and only if the union graph $\hat{\mathcal{G}}$ defined in* (9) *is connected, where $\mathcal{D}_b$ is defined in* (18) *and $\rho(\cdot)$ denotes the spectral radius. If $\hat{\mathcal{G}}$ is not connected, $\rho(\mathcal{D}_b) = 1$.*

The following definitions and terminology from [19] will be needed in the sequel. Let $\mathbb{R}^{\ell_1 \times \ell_2}$ be the space of $\ell_1 \times \ell_2$ real matrices. Let $\mathbb{H}^{\ell_1 \times \ell_2}$ be the set of all N-sequences of real $\ell_1 \times \ell_2$ matrices, so that $V \in \mathbb{H}^{\ell_1 \times \ell_2}$ means $V = (V_1, V_2, \dots, V_N)$ where $V_i \in \mathbb{R}^{\ell_1 \times \ell_2}$ for $i = 1, \dots, N$. The operators φ and $\hat{\varphi}$ is defined to create a tall vector by stacking together columns from these matrices, as follows: let $(V_i)_j \in \mathbb{R}^{\ell_1}$ be the j-th column of $V_i \in \mathbb{R}^{\ell_1 \times \ell_2}$, then

$$\varphi(V_i) := [(V_i)_1^T, \dots, (V_i)_n^T]^T \in \mathbb{R}^{\ell_1 \ell_2} \tag{19}$$
$$\hat{\varphi}(V) := [\varphi(V_1)^T, \dots, \varphi(V_N)^T]^T \in \mathbb{R}^{N\ell_1 \ell_2}. \tag{20}$$

Similarly, the inverse function $\hat{\varphi}^{-1} : \mathbb{R}^{N\ell_1\ell_2} \to \mathbb{H}^{\ell_1 \times \ell_2}$ is defined so that it produces an element of $\mathbb{H}^{\ell_1 \times \ell_2}$ given a vector in $\mathbb{R}^{N\ell_1\ell_2}$.

Lemma 6 *Consider the jump linear system* (13) *with an underlying homogeneous and ergodic Markov chain. The state vector $\mathbf{e}(k)$ of the system* (13) *converges in the mean square sense if and only if $\rho(\mathcal{D}_b) < 1$, where $\mathcal{D}_b$ is defined in* (18)*. When mean square convergence occurs, then $\boldsymbol{\mu}(k) \to \boldsymbol{\mu}$ and $\mathbf{Q}(k) \to \mathbf{Q}$, where*

$$\boldsymbol{\mu} := \sum_{i=1}^{N} q_i, \quad \mathbf{Q} := \sum_{i=1}^{N} Q_i, \tag{21}$$

where

$$[q_1^T, \dots, q_N^T]^T = q := (I - \mathcal{C}_b)^{-1}\psi \quad \in \mathbb{R}^{Nn_b},$$
$$(Q_1, \dots, Q_N) = Q := \hat{\varphi}^{-1}\left((I - \mathcal{D}_b)^{-1}\hat{\varphi}(R(q))\right) \in \mathbb{H}^{n_b \times n_b},$$

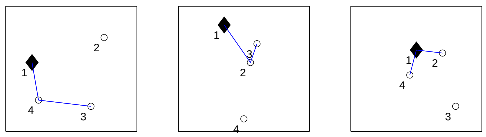


Fig. 2. The three graphs $\mathcal{G}_1, \mathcal{G}_2, \mathcal{G}_3$ that comprises $\mathbb{G}$. Node 1 is the reference.

and ψ, $R(q)$ *are given by*

$$\psi := [\psi_1^T, \dots, \psi_N^T]^T \in \mathbb{R}^{Nn_b}, \ \psi_j := \sum_{i=1}^{N} p_{ij} B_{bi} \gamma \pi_i \in \mathbb{R}^{n_b},$$
$$R(q) := (R_1(q), \dots, R_N(q)) \in \mathbb{H}^{n_b \times n_b},$$
$$R_j(q) :=$$
$$\sum_{i=1}^{N} p_{ij} (B_{bi} \Gamma B_{bi}^T \pi_i + J_{bi} q_i \gamma^T B_{bi}^T + B_{bi} \gamma q_i^T J_{bi}^T)) \in \mathbb{R}^{n_b \times n_b}.$$

Moreover, $\mathbf{Q}$ *is positive semi-definite.*

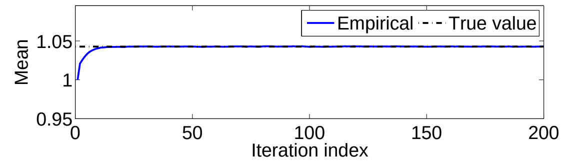
Proof: It follows from Theorem 3.33, Theorem 3.9, and remark 3.5 of [19] that mean square convergence of (13) is equivalent to $\rho(\mathcal{D}_b) < 1$. The expressions for the mean and correlation, as well as the fact that $\mathbf{Q} \geq 0$, also follow from [19, Proposition 3.37,3.38]. The existence of the steady state distribution π (that appear in the formulas) follows from the ergodicity of the Markov chain. □

Now we are ready to prove Theorem 3.

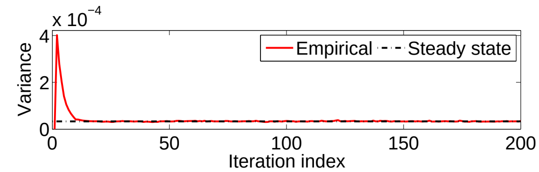
Proof of Theorem 3 (Sufficiency): It follows from the hypotheses and Lemma 5 that we have $\rho(\mathcal{D}_b) < 1$. It then follows from Lemma 6 that the state converges in the mean square sense. **(Necessity)**: If the union of graph is not connected, we have from Lemma 5 that $\rho(\mathcal{D}_b) = 1$. This shows that (due to Lemma 6) convergence will not occur. □

5 Simulation studies

As discussed in Section 2.1, skew and offset estimation are special cases of the problem of estimation of scalar node variables from relative measurements. Therefore simulations are conducted only for scalar node variable estimation. In all simulations, node variables are chosen arbitrarily, a single reference node is present, and the value of the its node variable is 0. The noise on each measurement is a normally distributed random variable. All the edge weights are assigned a value of unity at every time.



(a) Mean



(b) Variance

Fig. 3. Mean and variance of the estimate of node 3's node variable as a function of time. The empirical estimate of mean and variance is computed from 1000 Monte Carlo experiments. In (b), the "steady-state" corresponds to the limiting standard deviation predicted by Lemma 6.

5.1 Four-node network with Markovian switching

In this scenario the nodes move in such a way that the graph $\mathcal{G}(k)$ can be one of only 3 graphs shown in Figure 2. The graphs change according to a Markov chain whose transition probability matrix is

$$\mathcal{P} = \begin{bmatrix} 0.3 & 0 & 0.7 \\ 0.1 & 0.5 & 0.4 \\ 0 & 0.5 & 0.5 \end{bmatrix}. \quad (22)$$

Notice that none of the graphs is a connected graph, though the union of the graphs in $\mathbb{G}$ is connected. Also, $\mathcal{P}$ is ergodic. The mean and variance of measurement noise on every edge are chosen as 0 and 10^{-4}, respectively. The limiting means and variances of the estimates therefore can be computed from the predictions of Lemma 6. Monte-Carlo experiments are conducted to empirically estimate the mean and variance of the estimation error, by averaging over 1000 sample runs. Figure 3(a) and Figure 3(b) show the empirically estimated mean and variance of node 3's estimate of its node variable. As predicted by Theorem 3, the mean of the estimate converges to the true value, since the measurement noise is 0 mean. The variance also converges to the theoretical steady state variance as predicted by Lemma 6.

5.2 A 100-node network with RWP mobility model

Here 100 nodes move in a 1000 $m \times 1000$ m square according to the widely used Random Waypoint (RWP) mobility model [25]. It has been justified in the Appendix A of [24] that the graph switching process in this mobility model can be reasonably modeled as a (first order) Markov chain. The parameters maximum/minimum speed and pause time are

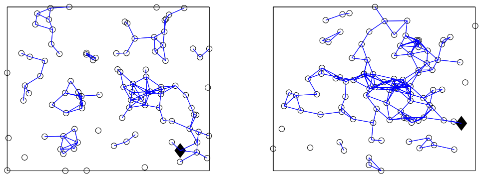


Fig. 4. Two graphs that occur during a simulation with 100 nodes moving according to the random waypoint mobility model.

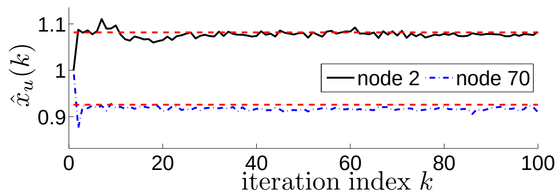


Fig. 5. The estimates of two nodes in one of the numerical experiments involving the 100-node mobile network.

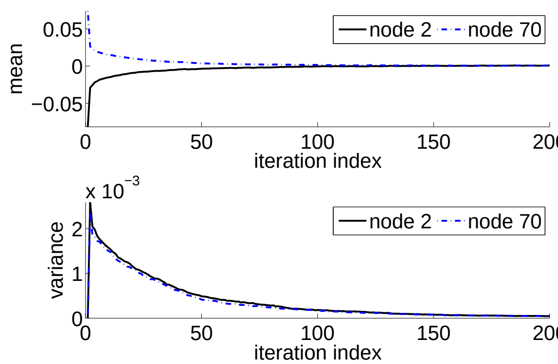


Fig. 6. Empirically estimated mean and variance of the estimation error for one of the nodes in the 100-node mobile network.

$v_{min} = 10\ m/s$, $v_{max} = 50\ m/s$, and $t_p = 0.1s$. The communication range is chosen as 100 m, and a link failure probability of 0.1 is used. The mean and variance of the measurement noise are chosen as 0 and 10^{-4}. Figure 4 shows two snapshots of the network during one of the simulations. Figure 5 shows the time trace of the estimates of two nodes in one of the simulations. The mean and variance of the estimation error was empirically computed from 1000 Monte Carlo simulations. Figure 6 shows mean and variance of the estimation error for two nodes. The figure suggests that the estimates of the node variables converge in the mean square sense. Note that the transition probability matrix is not known and the large state space makes it infeasible to compute the theoretical predictions of limiting mean and variances that are given in Lemma 6. One purpose of these simulations is therefore to test the performance of the algorithm when theoretical predictions are not available.

6 Summary

We analyzed a distributed algorithm for estimating clock skew and offset of the nodes of a mobile network and examined its convergence properties. Similar algorithms were previously proposed for skew and offset estimation but their convergence in mobile networks were not examined. The time variation of the network was modeled as a Markov chain, which makes the algorithm a jump linear system. Under the assumptions that the Markov chain is ergodic and the diagonal entries of its transition probability matrix are positive, the estimates were shown to be mean square convergent as long as the union of the graphs over time is connected. Expressions for the asymptotic mean and correlation are also provided by using results from jump linear systems from [19].

The estimation error dynamics turn out to be a leader-following consensus scheme, where the reference nodes (that know their global times) have their error states fixed at 0 at all times. We do not use existing consensus results since they are for the leaderless case and furthermore require additional assumptions than used here. In particular, we do not assume any kind of symmetry or balanced condition on the weighted Laplacians. Future work includes the analysis of the convergence rate. It is likely that the transition probabilities of the chain will play a role in the convergence rate. In some consensus papers, the effect of noise is counteracted by using a time-varying weights that satisfy a *persistence* condition. This is another area of future research, on using specifically designed time varying weights to obtain convergence of the variance to 0 instead of a finite number. The challenge here is that when nodes do not have synchronized clocks, it is not clear how to ensure the required persistence conditions on the weights. Prliminary work with promising results are reported in [29].

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