Energy-efficient control of an air handling unit for a single-zone VAV system

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***Abstract*— A large fraction of energy consumed by the HVAC (heating ventilation and air-conditioning) system in a commercial building is consumed at the AHUs (air handling units) that condition a mixture of outside and return air to specific temperature and humidity levels. Traditionally, the return air ratio and temperature of the air leaving the cooling coils in the AHU (conditioned air temperature) are maintained at pre-determined set points instead of being based on real-time measurements of occupancy, zone humidity, and outside weather. In this paper, we investigate the potential of energy savings as a function of the complexity of control algorithm. The inputs that can be commanded by the controllers are: air flow rate, return air ratio, conditioned air temperature, and the temperature of air leaving the heating coils in the AHU. Simulation results show that the controllers that use the measurements of occupancy, zone humidity, and outside weather result in significant savings over conventional controllers that do not use such measurements, without sacrificing thermal comfort or indoor air quality (IAQ). Surprisingly, a simple feedback control scheme is found to perform almost as well as a much more complex MPC (model predictive control) controller.**

I. INTRODUCTION

In the United States, buildings are responsible for more than 40% to the total energy consumption while HVAC systems account for 50% of the energy consumed in the buildings [1]. Poor design and inefficient operation of the HVAC system lead to a significant amount of energy wastage. Though it is possible to retrofit buildings with high efficiency HVAC equipment, doing so requires a substantial amount of investment [2]. In contrast, improving the control algorithms (that operate the HVAC system) to reduce energy usage is far more cost effective. Therefore, many researchers have focused on developing control algorithms to reduce energy usage in the buildings; see [3], [4], [5] and references therein.

We limit ourselves to variable-air-volume (VAV) HVAC systems for commercial buildings. In a VAV system, a building is divided into a number of zones, where a zone can be a room or a collection of rooms. The air leaving the zones is mixed with outside air based on the value of return air ratio, and the mixed air is sent to one or more AHUs. The air leaving the cooling coil in an AHU is called conditioned air, which is cooled down to conditioned air temperature to reduce the humidity ratio. The conditioned air goes to the VAV box of each zone, where the conditioned air may be heated up by using the heating coils before being supplied to the zone. The air supplied to the zone is called supply air. There are four control inputs that need to be decided for these multi-zone VAV systems. Two of the control inputs (the return air ratio and conditioned air temperature) are decided at the AHU while the other two control inputs are decided at the VAV box (the supply air temperature and flow rate).

Conventional control strategies used in the buildings use only zone temperature measurements but do not use any measurements of occupancy, zone humidity, and outside weather. Occupancy here means number of people in a zone. The control inputs at the VAV box are determined by the conventional controllers in such a way that the zone temperature is maintained in specific ranges based on predetermined occupancy schedules. While the control inputs at the AHU are usually kept constant at predetermined values irrespective of whether the building is occupied or not. This is inefficient in terms of energy usage since the indoor climate is maintained even during unoccupied times.

It has been shown in our recent work [3] that a significant amount of energy can be saved by using real-time occupancy measurements (instead of using predefined occupancy schedules) to decide zone-level control commands at the VAV boxes, while the control inputs at the AHU are kept constant. It is possible to improve the energy efficiency further by varying the AHU inputs. Our conjecture is that *a substantial amount of energy can be saved—while maintaining thermal comfort and IAQ—by not only controlling the inputs at the VAV box but also controlling the inputs at AHU and using the measurements of occupancy, zone temperature and humidity, and outside weather*. However, how to design a controller to achieve this is not obvious.

In this preliminary study, we only focus on a single-zone VAV system. In a single-zone VAV system, one AHU serves only one zone and heating coils are inside the AHU. The four control inputs that need to be determined are return air ratio, conditioned air temperature, supply air temperature and flow rate. In this paper, we address the following three questions:

1. How much savings can be obtained if a system model and measurements of zone temperature and humidity, outside temperature and humidity, and occupancy are available to a controller?

2. How do the savings depend on the fidelity of information and complexity of the controller?

3. Among the controllable variables, which is (are) the key one(s) that mostly affect the energy use and thermal comfort?

There have been many recent papers that have developed control algorithms to reduce energy consumption in buildings. Some of the papers [5], [6], [7] use optimal control based methods that are complex and computationally

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expensive, while others use feedback controllers [8], [9]. All of the papers show significant energy savings over the conventional baseline controllers. The strategy in [5] controls the return air ratio, supply air flow rate and temperature to minimize Predicted Mean Vote and energy consumption. The control strategies in [6], [7] reset the supply air temperature, ventilation rate, and chilled water temperature. The controller in [9] only resets the minimum supply air flow rate.

Contribution 1 : All of the above mentioned papers either compare the complex optimal control methods with the conventional controllers, or compare the simple feedback-based methods with the conventional controllers. However, they did not compare all three, i.e., the conventional, simple feedback, and complex MPC controllers. It is important to know how a complex MPC controller benefits over a simple feedback controller. Therefore, we compare a feedback, optimization-based MPC, and conventional controller. The outcome of our study may appear somewhat surprising: that a simple rule-based feedback controller performs just as well as a much more complex MPC controller.

Contribution 2 : None of the papers mentioned above show the effect of the type of measurements on the controllers performance. It is useful to know this from a implementation point of view as additional sensors imply extra investment. However, additional measurements may not always result in significant energy savings. Therefore, we study the value of measurements and control inputs in the performance of the controllers. Our study shows that occupancy is the most crucial measurement required to minimize energy use.

Contribution 3 : The controllers in above mentioned papers control a maximum of three variables, though there are four possible variables (supply air temperature and flow rate, conditioned air temperature, and return air ratio) that can be controlled. Controlling all the four variables may result in high savings/comfort. Also, these papers do not study the effect of an individual control input on the controllers performance. We show that the effect of the control inputs on the energy consumption decreases in the order 1) SA flow rate and temperature 2) RA ratio 3) CA temperature.

Furthermore, all the above-mentioned papers either do not include the conditioned air humidity ratio or assume constant humidity ratio. However, the conditioned air humidity ratio depends on the conditioned air temperature, which we consider here.

The rest of the paper is organized as follows. A description of single-zone VAV HVAC system and the models of a zone hygro-thermal dynamics and energy consumption are described in Section II. The control strategies are described in Section III. Section IV describes performance metrics related to thermal comfort and energy savings. Section V provides a description of the parameters chosen for the simulation study. Simulation results are discussed in Section VI. Section VII concludes the paper with possible future work.

II. System Description And Model

A schematic of a typical single-zone VAV-based HVAC system for commercial buildings, along with a schematic of a controller's implementation, is shown in Figure 1. In this type of system, a part of the air exhausted from the zone, which is called return air (RA), is mixed with the outside air (OA) before being sent to the AHU. The air sent to the AHU is called mixed air (MA). The mixed air is passed through the cooling coils inside the AHU, which condition the mixed air to temperature T^{CA} and humidity ratio W^{CA}. The air leaving the cooling coils is called conditioned air (CA), which is passed through the heating coils in the VAV box. The air leaving the heating coils at temperature (T^{SA}) and humidity ratio (W^{SA}), which is called supply air (SA), is supplied to the zone. The humidity ratio of the supply air (W^{SA}) is same as that of the conditioned air, i.e., ($W^{SA} = W^{CA}$), since reheating does not change the humidity ratio.

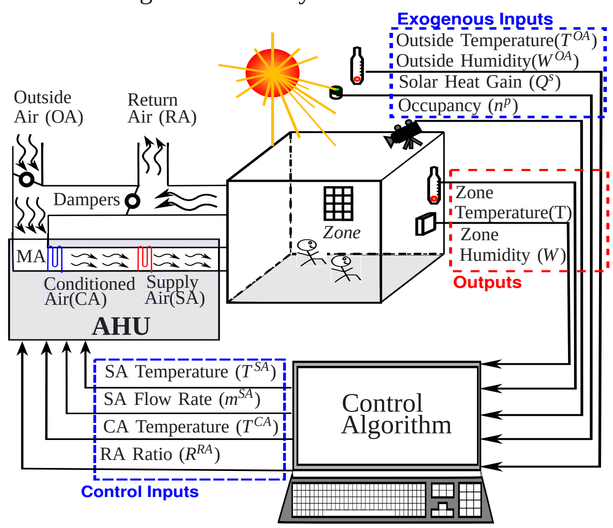

Fig. 1. Generic scheme of a single-zone VAV-based HVAC system.

The task of a control algorithm is to determine the control inputs in such a way that thermal comfort and IAQ are maintained in that zone. To study the performance of a control strategy through simulations, a hygro-thermal (humidity and temperature) dynamics model and a energy consumption model are required. A discrete-time hygro-thermal dynamics model with k being the discrete time index can be written as

$$X(k+1) = f(X(k), u(k), v(k)), \;\; X = [\mathbf{T}^T \;\; W]^T, \qquad (1)$$

where $u(k) = [m^{SA}(k), T^{SA}(k), T^{CA}(k), R^{RA}(k)]^T$ is the control input vector (command), while the exogenous inputs vector $v(k)$ consists of the outside temperature, outside humidity ratio, solar heat gain, and occupancy, i.e., $v(k) = [T^{OA}(k), W^{OA}(k), Q^s(k), n^p(k)]^T$. The total energy consumption during the time Δt between time indexes $k-1$ and k is denoted by $E(k)$. The energy consumption is dependent on the enthalpies of MA, RA, CA, OA, and SA, which are represented by h^{MA}, h^{RA}, h^{CA}, h^{OA}, and h^{SA}, respectively. We refer the interested reader to [10] for the details of the enthalpies, energy, and hygro-thermal dynamics model.

III. Control Algorithms

The four control algorithms (*BL*, *Z-FC*, *A-FC*, and *A-MPC*) are described in this section. The *BL* is a baseline controller and *Z-FC* controller was presented in our recent work [3], while the *A-FC* and *A-MPC* controllers are novel control algorithms. Information requirements and complexity of the controllers are summarized in Table I.

A. BL *(*Baseline*)*

We choose the dual maximum [11, Chapter 47] as the *baseline controller*, which determines the SA temperature and flow rate based on the zone temperature measurements. The RA ratio and CA temperature are kept constant. In this scheme, the control logic is divided into four modes: (i) Re-heating (ii) Heating (iii) Dead-Band and (iv) Cooling, which are shown in Figure 2. The mode of operation depends on the “Re-heating Set-Point (RTG)”, ‘Cooling Set-Point (CLG)”, and “Heating Set-Point (HTG)”. In the re-heating mode, the SA temperature is set to maximum possible value (T^{SA}_{high}), and the SA flow rate is varied. In the heating mode, the SA flow rate is set to the minimum allowed value, and the SA temperature is varied. The minimum allowed value for the flow rate is determined using ASHARE standard 62.1 [12]:

$$\text{Minimum Allowed Flow Rate} = m^{SA}_p n^p_d + m^{SA}_{low},$$
$$m^{SA}_p = m^{OA}_p/(1-R^{RA}),\ m^{SA}_{low} = m^A_z A_z/(1-R^{RA}), \qquad (2)$$

where m^{OA}_p is the amount of outside air required per person, m^A_z is the amount of outside air required per unit area, m^{SA}_{low} is the minimum amount of supply air during unoccupied time, A_z is the zone floor area, and n^p_d is designed occupancy. In the dead-band mode, $T^{SA} = T^{CA}$ and SA flow rate is set to the minimum allowed value. In the cooling mode, $T^{SA} = T^{CA}$, but the SA flow rate is varied to maintain a desired set-point T^{set} in the zone; see [3] for the details of the controller.

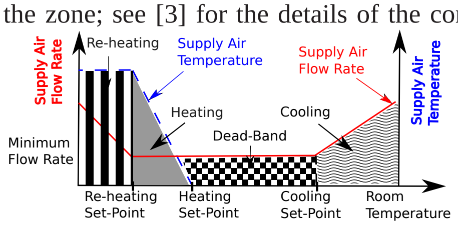

Fig. 2. Schematic representation of the baseline controller.

B. Z-FC *(*Zone-Level Feedback Control*): MOBS in [3]*

The *Z-FC* controller requires the measurements of occupancy and zone temperature. It is very similar to the *BL* controller described in Section III-A, except for two key differences. First, the minimum allowed flow mentioned in (2) is calculated based on the measured occupancy instead of the design occupancy as follows:

$$\text{Minimum Allowed Flow Rate } (K) = m^{SA}_p n^p(k) + m^{SA}_{low}, \quad (3)$$

where $n^p(k)$ is the occupancy measured at time index k, and m^{SA}_p, m^{SA}_{low} are computed using (2). Second, the temperature set-points are determined based on the zone occupancy:

$$\left.\begin{array}{l} RTG(t) = T^{unocc}_{RTG} \\ HTG(k) = T^{unocc}_{low} \\ CTG(k) = T^{unocc}_{high} \end{array}\right\} \text{if } n^p(k) = 0, \quad \left.\begin{array}{l} RTG(t) = T^{occ}_{RTG} \\ HTG(k) = T^{occ}_{low} \\ CTG(k) = T^{occ}_{high} \end{array}\right\} \text{if } n^p(k) \neq 0. \qquad (4)$$

The choice of design variables T^{unocc}_{RTG}, $T^{occ}_{RTG}, T^{unocc}_{low}$, $T^{occ}_{low}, T^{unocc}_{high}, T^{occ}_{high}$ involves a trade-off between energy savings and thermal comfort; see [3] for the details of this controller.

C. A-FC *(*AHU-Level Feedback Control*)*

The *A-FC* controller is a feedback strategy to determine all four inputs: the SA temperature, SA flow rate, CA temperature, and RA ratio. A flow chart that describes the *A-FC* control algorithm in detail is shown in Figure 3. The algorithm can be summarized in four steps: at every time index k, (1) obtain measurements, (2) determine the RA ratio by doing exhaustive search, (3) determine the CA temperature based on the MA enthalpy, OA enthalpy, and zone humidity, and (4) recalculate RA ratio to ensure zone humidity constraints are satisfied.

In step 2, the RA ratio is searched in the range $[max(R^{RA}(k) - R^{RA}_{rate}\Delta t, R^{RA}_{min}), min(R^{RA}(k) + R^{RA}_{rate}\Delta t, R^{RA}_{max})]$ due to the actuator constraints. The damper position can not change quickly, and so does the RA ratio. We assume that the maximum allowable rate at which RA ratio can change is R^{RA}_{rate}. The maximum and minimum allowable values of RA ratio are represented by R^{RA}_{max} and R^{RA}_{min}, respectively.

In step 3, the CA temperature is increased to reduce the energy consumption, and the CA temperature is decreased when the zone humidity ratio goes farther from the allowable range. The allowable ranges of the zone humidity ratio during unoccupied and occupied times are $[W^{unocc}_{low}, W^{unocc}_{high}]$ and $[W^{occ}_{low}, W^{occ}_{high}]$, respectively. As in the case of the RA ratio, there is a maximum allowable rate T^{CA}_{rate} at which the CA temperature can change. Also, the CA temperature should always be in the allowable range $[T^{CA}_{min}, T^{CA}_{max}]$.

Step 4 makes sure that the SA flow rate with minimum CA humidity is high enough to maintain the zone humidity within the allowable range. Otherwise the RA ratio is decreased, which increases the total flow rate.

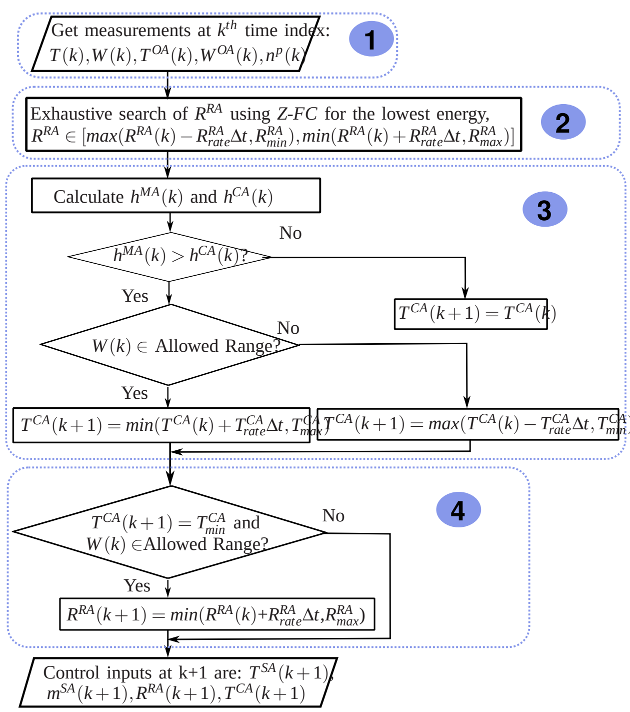

Fig. 3. Flow-chart of the *Z-FC* controller to determine the control inputs.

TABLE I

OVERVIEW OF THE CONTROL ALGORITHMS IN TERMS OF THE AMOUNT OF INFORMATION REQUIRED AND COMPLEXITY.

	Control algorithms	Controllable Inputs	Fixed Inputs	Measurements required	Predictions required	Model required	Computation requirements	Overall complexity
1	*BL*	T^{SA}, m^{SA}	T^{CA}, R^{RA}	T	-	No	Very Low	Very Low
3	*Z-FC*	T^{SA}, m^{SA}	T^{CA}, R^{RA}	T, n^p	-	No	Very Low	Low
4	*A-FC*	T^{SA}, T^{CA}, m^{SA}, R^{RA}	-	T, W, n^p, T^{OA}, W^{OA}	-	No	Medium	Medium
5	*A-FC* special case 1	T^{SA}, m^{SA}, R^{RA}	T^{CA}	T, W, n^p, T^{OA}, W^{OA}	-	No	Medium	Medium
6	*A-FC* special case 2	T^{SA}, T^{CA}, m^{SA}	R^{RA}	T, W, n^p, T^{OA}, W^{OA}	-	No	Medium	Medium
7	*A-MPC*	T^{SA}, T^{CA}, m^{SA}, R^{RA}	-	T, W, n^p, T^{SA}, m^{SA}	T^{OA}, W^{OA}, Q^s	Yes	High	Very High
8	*A-MPC* special case 1	T^{SA}, m^{SA}, R^{RA}	T^{CA}	T, W, n^p, T^{SA}, m^{SA}	T^{OA}, W^{OA}, Q^s	Yes	High	Very High
9	*A-MPC* special case 2	T^{SA}, T^{CA}, m^{SA}	R^{RA}	T, W, n^p, T^{SA}, m^{SA}	T^{OA}, W^{OA}, Q^s	Yes	High	Very High

While evaluating the performance of the *A-FC* controller, we consider two special cases to study the effect of each control input individually on the performance of energy savings and thermal comfort. The two special cases are:

Special Case 1: The CA temperature is kept constant at the minimum value T^{CA}_{min}, i.e., $T^{CA}_{max} = T^{CA}_{min}$, and $T^{CA}_{rate} = 0$.

Special Case 2: The RA ratio is kept constant, i.e., $R^{RA}_{rate} = 0$.

Note that *Z-FC* is a special case of the *A-FC* when both the RA ratio and CA temperature are kept constant.

D. A-MPC *(*AHU-Level Model Predictive Control*)*

The *A-MPC* controller also determines the all four inputs as in the *A-FC* controller, but by using an MPC-based strategy. The control inputs over K time indices are obtained by solving a constrained optimization problem: minimize total energy consumption over that period while maintaining thermal comfort and IAQ. The control inputs are applied at the current time index k. The optimization problem is solved again at time index $k+1$ to compute the control inputs for the next K time instants. The whole process is repeated again.

To solve the underlying optimization problem, the controller needs (i) a model of the zone hygro-thermal dynamics, which is the one described in Section II, (ii) initial state of the model, which is estimated using an Extended Kalman Filter-based state observer, and (iii) predictions of the exogenous input $v(k)$. Predictions of T^{OA}, W^{OA}, and Q^s (part of $v(k)$) are assumed available from weather forecasts. It is assumed that the instantaneous occupancy measurements are available at the time index k. The predicted occupancy over the prediction horizon K is assumed to be the same as the measured occupancy at the k-th time period: $n^p(i) = n^p(k), i \geq k$.

The control logic is divided into two modes: (i) Occupied and (ii) Unoccupied, which are explained below in detail.

Occupied Mode: The controller operates in the occupied mode if the measured occupancy at the time index k is at least 1. The optimal control inputs for the next K time indices are obtained by solving the following optimization problem:

$$U^{\star} := arg\min_{U} G(U), \tag{5}$$

where $U = [u^T(k), \cdots, u^T(k+K)]^T \in \mathbb{R}^{4(K+1)}$ and $G(U) = \sum_{i=k}^{k+K} E(i)$, subject to the following constraints:

$$\left.\begin{array}{c} T^{occ}_{low} \leq T(i) \leq T^{occ}_{high}, \\ W^{occ}_{low} \leq W(i) \leq W^{occ}_{high}, \\ T^{CA}(i) \leq T^{SA}(i) \leq T^{SA}_{high} \\ m^{SA}_{p} n^p(i) + m^{SA}_{low} \leq m^{SA}(i) \leq m^{SA}_{high} \\ R^{RA}(i) \leq min(R^{RA}(i-1) + R^{RA}_{rate}\Delta t, R^{RA}_{max}) \\ R^{RA}(i) \geq max(R^{RA}(i-1) - R^{RA}_{rate}\Delta t, R^{RA}_{min}) \\ T^{CA}(i) \leq min(T^{CA}(i-1) + T^{CA}_{rate}\Delta t, T^{CA}_{max}) \\ T^{CA}(i) \geq max(T^{CA}(i-1) - T^{CA}_{rate}\Delta t, T^{CA}_{min}) \end{array}\right\} \forall i = k, \ldots, k+K.$$

The first two constraints specify the range in which the zone temperature and humidity ratio are allowed to vary. The third constraint is simply to take into account actuator capabilities. The fourth constraint means that there is a lower and upper bound on the flow rate entering the zone (m^{SA}). The lower bound on the flow rate is same as (3), while the upper bound m^{SA}_{high} reflects the maximum flow rate possible. The last four constraints are on the rate of RA ratio and CA temperature, which are the same constraints as in Section III-B.

Unoccupied Mode: If the measured occupancy at the time index k is observed to be 0, then the controller operates in the unoccupied mode. At time k, the optimal control inputs for the next K time indices are obtained by solving the following optimization problem:

$$U^{\star} := arg\min_{U} G(U), \tag{6}$$

subject to the following constraints:

$$\left.\begin{array}{c} T^{unocc}_{low} \leq T(i) \leq T^{unocc}_{high} \\ W^{unocc}_{low} \leq W(i) \leq W^{unocc}_{high} \\ m^{SA}_{low} \leq m^{SA}(i) \leq m^{SA}_{high} \\ T^{CA}(i) \leq T^{SA}(i) \leq T^{SA}_{high} \\ R^{RA}(i) \leq min(R^{RA}(i-1) + R^{RA}_{rate}\Delta t, R^{RA}_{max}) \\ R^{RA}(i) \geq max(R^{RA}(i-1) - R^{RA}_{rate}\Delta t, R^{RA}_{min}) \\ T^{CA}(i) \leq min(T^{CA}(i-1) + T^{CA}_{rate}\Delta t, T^{CA}_{max}) \\ T^{CA}(i) \geq max(T^{CA}(i-1) - T^{CA}_{rate}\Delta t, T^{CA}_{min}) \end{array}\right\} \forall i = k, \ldots, k+K.$$

The reason for these constraints is the same as that explained previously. The constraints on the zone temperature and humidity ratio in the unoccupied mode, however, are chosen to be such that $[T^{unocc}_{low}, T^{unocc}_{high}] \supseteq [T^{occ}_{low}, T^{occ}_{high}]$, and $[W^{unocc}_{low}, W^{unocc}_{high}] \supseteq [W^{occ}_{low}, W^{occ}_{high}]$.

As in the *Z-FC* controller, the choice of the design variables during occupied/unoccupied times (T^{occ}_{low}, T^{occ}_{high}, W^{occ}_{low}, W^{unocc}_{high}, T^{unocc}_{low}, T^{unocc}_{high}, W^{unocc}_{low}, W^{unocc}_{high}) involves a trade-off between energy savings and occupant discomfort. We also consider two special cases for the *A-MPC* controller as

Special Case 1: The CA temperature is kept constant at T^{CA}_{min}.

Special Case 2: The RA ratio is kept constant.

Remark 1: All the controllers supply the minimum flow rate prescribed by ASHRAE ventilation standard 62.1-2010 [12], which ensures that IAQ is maintained by all the controllers.

IV. PERFORMANCE METRICS

An energy related performance metric is the % *savings over the baseline controller*, which is defined as

$$\% \text{ Savings} = (E_{BC} - E_C)/E_{BC}, \tag{7}$$

where E_C and E_{BC} are the energy consumed by the controller C and the baseline controller, respectively. Two metrics are chosen for analyzing the thermal comfort related performance of the controllers: (i) Temperature Violation D_T, and (ii) Humidity Violation D_H, which are defined as

$$D_T = \left\{ \begin{array}{c} -T(k) + T^{occ}_{low}, \text{ if } T(k) < T^{occ}_{low} \text{ and } n^p(k) \neq 0 \\ T(k) - T^{occ}_{high}, \text{ if } T(k) > T^{occ}_{high} \text{ and } n^p(k) \neq 0 \\ 0, \text{ otherwise} \end{array} \right\},$$

$$D_H = \left\{ \begin{array}{c} -W(k) + W^{occ}_{low}, \text{ if } W(k) < W^{occ}_{low} \text{ and } n^p(k) \neq 0 \\ W(k) - W^{occ}_{high}, \text{ if } W(k) > W^{occ}_{high} \text{ and } n^p(k) \neq 0 \\ 0, \text{ otherwise} \end{array} \right\}.$$

The *average temperature violation* ($\overline{D}_T$) and the *average humidity violation* ($\overline{D}_H$) over time ΔT are defined as

$$\overline{D}_T = \frac{1}{L}\sum_{k=1}^{L} D_T(k), \quad \overline{D}_H = \frac{1}{L}\sum_{k=1}^{L} D_H(k), \tag{8}$$

where $L = \Delta T/\Delta t$; see the details in [3].

V. Simulation Parameters

Simulations are carried out for a model of an auditorium in a building (Pugh Hall) at the University of Florida campus as shown in Figure 4. Parameters of the dynamic model for this zone are calibrated using the measured data in a manner done in [3]; we don't present the details due to lack of space.

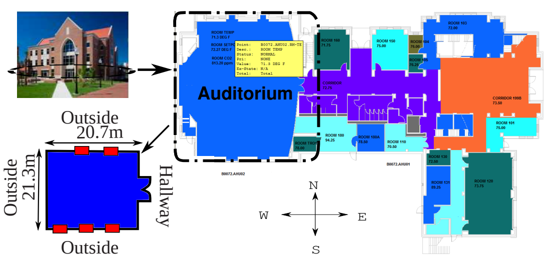

Fig. 4. Layout of auditorium on the first floor of Pugh Hall at the University of Florida, Gainesville, Fl. This zone is served by a dedicated AHU.

The maximum flow rate for all the controllers is chosen as 4.6 kg/s. For the *BL* and *Z-FC* controllers, the RA ratio and CA temperature are assumed to have constant values of 0.6 and 12.8°C, respectively. For the *BL* controller, the temperatures: RTG, HTG, and CLG are set to 21.8°C, 21.9°C, and 23.6°C, respectively, from $6:30$ a.m. to $10:30$ p.m. During the time $10:30$ p.m.–$6:30$ a.m., the temperatures: RTG, HTG, and CLG for the *BL* controller are chosen as 20.9°C, 21.1°C, and 24.4°C, respectively. This nighttime setback is currently used in the Pugh Hall. The relative humidity of the conditioned air is assumed constant at 90%. Other design parameters used by the controllers are shown in Table II.

VI. Results

We now compare the performance of *BL*, *Z-FC*, *A-FC*, and *A-MPC* control algorithms through simulations. Simulations are performed using MATLAB; while IPOPT [13] is used to solve the optimization problems in *A-MPC* controller.

The hallway shown in Figure 4 is assumed to have a constant temperature of 22.2°C. Three types of outside weather conditions: 1) cold (Jan 14, 2011), hot (Jul 31, 2011), and pleasant (Mar 16, 2011), are considered in Gainesville, FL. Weather data for this location is obtained from [14]. The zone is occupied by 200 people from $8:30$ a.m. to $4:30$ p.m. This is the current occupancy profile in the auditorium.

The total daily energy consumption, average temperature violation, average humidity violation, and % savings over the baseline controller are shown in Table III. It is clear from the table that all the controllers result in significant savings over the conventional baseline controller. The temperature and humidity violations are very close to zero for all the controllers, which means that the thermal comfort is maintained by all the controllers throughout the day.

There are three reasons for high energy savings by the *Z-FC*, *A-FC* and *A-MPC* controllers over the *BL* controller. The first reason is the reduction of the flow rate and increase in the allowable temperature range during unoccupied times. Reduction in the flow rate decreases fan-, conditioning-, and reheating-energy consumption. Increasing the allowable temperature range results in less reheating energy consumption since the zone temperature is allowed to be lower during unoccupied times than what the baseline controller allows. The second reason is the change of the RA ratio based on the enthalpies of OA, RA, CA, SA in such a way that the total energy is reduced. During pleasant weather when the OA enthalpy lies between the CA enthalpy and RA enthalpy, the RA ratio is low as lower energy is required by AHU to condition the outside air than to condition the return air. The third reason is the resetting of the CA temperature based on the zone humidity and enthalpies of MA and CA. When the MA enthalpy is less than the CA enthalpy and the zone humidity is within the allowable range, the CA temperature increases. Increase in the CA temperature increases humidity ratio, which reduces the energy consumed by the cooling coils. Some of these are not applicable to the *Z-FC* controller as it can not command the AHU inputs.

The *Z-FC* controller, which is a special case of the *A-FC* controller when both the SA temperature and RA ratio are kept constant while the SA temperature and SA flow rate are varied, results in $56-69\%$ energy savings. If the *A-FC* controller is allowed to vary the RA ratio as in the special case 1, the additional energy savings over the *Z-FC* controller are $5-26\%$. If the *A-FC* controller is allowed to vary only the CA temperature instead of the RA ratio as in the special case 2, the additional energy savings are $4-15\%$. When the *A-FC* controller is allowed to vary both the CA temperature and RA ratio, the additional savings over the *Z-FC* controller are $4-27\%$. A similar trend is observed for the *A-MPC* controller. These results suggest that varying the CA temperature with RA ratio does not offer any advantage in terms of energy savings over varying the RA ratio alone. Also, the effect of the SA flow rate and temperature on the energy savings is maximum among all the control inputs. Therefore, the effect of control inputs on the energy savings decreases in the order: 1) SA flow rate and temperature 2) RA ratio 3) CA temperature.

The *Z-FC* controller, which only uses the additional measurements of occupancy, results in $56-69\%$ energy savings

TABLE II

THE DESIGN PARAMETERS USED IN THE VARIOUS CONTROLLERS.

Temperature and time related parameters											
T^{set} $(^{\circ}C)$	T^{SA}_{high} $(^{\circ}C)$	T^{unocc}_{RTG} $(^{\circ}C)$	T^{occ}_{RTG} $(^{\circ}C)$	T^{occ}_{low} $(^{\circ}C)$	T^{occ}_{high} $(^{\circ}C)$	T^{unocc}_{low} $(^{\circ}C)$	T^{unocc}_{high} $(^{\circ}C)$	T^{CA}_{min} $(^{\circ}C)$	T^{CA}_{max} $(^{\circ}C)$	T^{CA}_{rate} $(\frac{^{\circ}C}{min})$	$K,\Delta t,\Delta T$ (no.,min,hr)
22.8	30.0	20.9	21.8	21.9	23.6	21.1	24.4	12.8	15.6	0.1	30,1,24
Humidity and other parameters											
W^{unocc}_{low} $(\frac{g}{kg})$	W^{occ}_{low} $(\frac{g}{kg})$	W^{unocc}_{high} $(\frac{g}{kg})$	W^{occ}_{high} $(\frac{g}{kg})$	m^{OA}_{p} $(\frac{kg}{sec})$	m^{A}_{z} $(\frac{kg}{m^2})$	m^{SA}_{high} $(\frac{kg}{sec})$	R^{RA}_{min} (%)	R^{RA}_{max} (%)	R^{RA}_{rate} $(\frac{\%}{min})$	n^{p}_{d}	A_z (m^2)
7.4	7.4	11	11	0.0042	$3.05*10^{-4}$	4.6	0	80	5	210	238

TABLE III

ENERGY CONSUMPTION, AVERAGE TEMPERATURE VIOLATION, AVERAGE HUMIDITY VIOLATION, AND % SAVINGS OVER A 24-HOUR PERIOD WITH VARIOUS CONTROLLERS. THE THREE WEATHER CONDITIONS ARE CHOSEN FOR GAINESVILLE, FL, USA.

	Cold				Hot				Pleasant			
Control Scheme	E MJ	Savings %	$\overline{D}_T$ $^{\circ}C$	$\overline{D}_H$ $\frac{g}{kg}$	E MJ	Savings %	$\overline{D}_T$ $^{\circ}C$	$\overline{D}_H$ $\frac{g}{kg}$	E MJ	Savings %	$\overline{D}_T$ $^{\circ}C$	$\overline{D}_H$ $\frac{g}{kg}$
BL	3142	-	0.008	0	7598	-	0.006	0	3877	-	0.007	0
Z-FC	980	68.8	0.015	0	3187	58.1	0.013	0	1687	56.5	0.014	0
A-FC special case 1	826	73.7	0.015	0	2280	70.0	0.001	0.001	659	83.0	0.011	0.002
A-FC special case 2	797	74.6	0.012	0	2595	65.8	0.005	0.003	1109	71.4	0.010	0.002
A-FC	851	72.9	0.013	0	2170	71.4	0.000	0.066	635	83.6	0.010	0.043
A-MPC special case 1	732	76.7	0.000	0	2152	71.7	0.000	0	615	84.1	0.000	0
A-MPC special case 2	815	74.1	0.000	0	2580	66.0	0.000	0	1103	71.5	0.000	0
A-MPC	703	77.6	0.000	0	2091	72.5	0.000	0	607	84.4	0.000	0

over the baseline controller that does not use occupancy measurements. If a controller uses the measurements of the zone humidity and outside weather along with the occupancy measurements as in the *A-FC* and *A-MPC* controllers, the energy savings are huge almost $71-85\%$. Hence, occupancy measurement is a key factor in reducing the energy usage.

VII. DISCUSSION AND FUTURE WORK

We examine how the performance of a controller is affected by its complexity; the goal of the controller is to minimize energy consumption while maintaining comfort and IAQ for a single-zone variable-air-volume HVAC system. To compare the performance vs complexity, we choose various controllers that require varying amount of information, computation, and implementation effort. Simulation results show occupancy measurement is a key factor to reduce the energy usage in buildings. Controllers that use only occupancy measurements result in $56-69\%$ energy savings. Another key finding is that a feedback-based controller that is simple and easy to implement, performs as well as a complex and computationally expensive MPC-based controller, if same measurements are provided to both the controllers. This is significant in light of the much higher effort required to implement the MPC-based controller due to the need for model identification [15] and on-line optimization.

The study shows that the effect of control inputs on the energy savings decreases in the order: 1) supply air flow rate and temperature 2) return air ratio 3) conditioned air temperature, and the conditioned air temperature has almost negligible impact on energy savings when the return air ratio is varied. Therefore, a feedback controller, with supply air temperature, return air ratio, and supply air flow rate as the control variables, is the most appropriate control algorithm to be used for single-zone VAV HVAC systems due to its simplicity, low computation, and similar performance to that of more complex control algorithms.

In this paper, we have assumed that one AHU serves only single zone. A detailed study for multi-zone buildings is a part of future work. Implementing the controllers in a real building is required to verify the simulation results. Work on the implementation of the *Z-FC* controller in each zone of the Pugh Hall is ongoing. Also, the effect of measurements error on the controllers performance is a part of future work.

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