

An autonomous MPC scheme for energy-efficient control of building HVAC systems

Tingting Zeng and Prabir Barooah
University of Florida, Gainesville, Florida USA

Abstract—Model Predictive Control (MPC) is a promising technique for energy efficient control of Heating, Ventilation, and Air Conditioning (HVAC) systems. However, the need for human involvement limits current MPC strategies from widespread deployment, since (i) model identification algorithms require re-tuning of hyper-parameters, and (ii) optimizers may fail to converge within the available control computation time, or get stuck in a local minimum. In this work we propose an autonomous MPC scheme to overcome these issues. Two major features are embedded in this architecture to enable autonomy: (i) a convex identification algorithm with adaptation to time-varying building dynamics, and (ii) a convex optimizer. The model identification algorithm re-runs periodically so as to handle changes in the building’s dynamics. The estimated model is guaranteed to be stable and has desirable physical properties. The optimizer uses a descent and convergent algorithm, with the underlying optimization problem being feasible and convex. Numerical results show that the proposed convex formulation is more reliable in control computation compared to the non-convex one, and the proposed autonomous MPC architecture reduces energy consumption significantly over a conventional controller.

I. INTRODUCTION

Heating, ventilation, and air conditioning (HVAC) systems are responsible for approximately 40% of the total energy consumption of buildings [1]. Model Predictive Control (MPC) is one of the most promising control strategies towards reducing the energy consumptions [2].

Although MPC has been extensively studied for control of HVAC systems, MPC architectures proposed in the literature cannot be used in an autonomous manner. By *autonomous MPC* we mean an MPC architecture capable of reliably computing high-quality control decisions at all times without the need for human involvement. A building’s and its equipment’s behavior changes with time, and consequently models used by the optimizer in MPC need to be updated over time. Control-oriented models that are simple enough for MPC need to be learned from data since first principles based models are quite complex. The overall architecture thus needs to be of an “adaptive MPC” type; see Figure 1. The system identification component of such a control system must be able to fit a model without needing human intervention and obtain a model of sufficient accuracy. A second requirement for autonomous operation is that the underlying optimization problem must be feasible and convex at every decision time. A non-convex controller may fail to converge

within the time available for control computation, or get stuck in a local minimum; see [2] and references therein. Infeasibility has the same effect. In either case, a rule-based controller must be used as back up when the non-convex optimizer cannot provide a control command. Switching between controllers induces concerns of instability and poor transient performance [3].

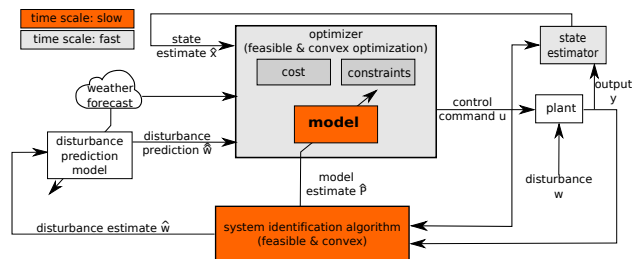


Fig. 1: Structure of the proposed building autonomous MPC architecture.

Identification of HVAC system models from data has a long history, but most of these algorithms are non-convex optimization problems with few guarantees on the quality of the model fit [4], [5]. Depending on the type and quality of data used, they require re-tuning of hyper-parameters by a human expert. A similar problem exists for control computation. Dynamic models of building HVAC systems are typically nonlinear, which lead to non-convex optimization problems in control action computation. An approach used to address this issue is linearization of models. Among works adopting this approach, some make limiting assumptions to obtain a linear thermal dynamic model [5]–[7]; others linearize the dynamics around a set-point or state trajectory, which requires an optimal, or at least near optimal trajectory first [8], [9]. The quality of a linearized model is thus sensitive to the choice of the optimal trajectory, and a-priori determination of such an optimal trajectory is challenging. After all, if it were easy there were no need for an MPC controller.

In this paper we propose an autonomous MPC architecture for HVAC systems. The system identification block is taken from our prior work [10]. The proposed identification algorithm is well-suited for autonomous operation: it involves solving an optimization problem that is always feasible and convex, and the model it returns is guaranteed to be stable and possess properties that are consistent with properties of a building HVAC system, such as an increase in outdoor temperature will lead to an increase in indoor temperature.

For the control computation block, we adopt a convexification approach in order to handle the bilinear terms in the nominal non-convex problem. Bilinear terms are commonly encountered in models of building thermal dynamics, and is one of the common sources of non-convexity of the optimization problem in HVAC MPC. We prove in this paper that among the list of many convex approximation methods to handle bilinearities, Convex-Concave Procedure (CCP) is the only applicable method for our problem structure. Our optimizer is therefore incorporated with the CCP approach, and we prove that the proposed algorithm is feasible, descent, and convergent. The proposed convex optimizer, and the analysis showing the inapplicability of the alternate convexification methods, is the first novel contribution of the paper. We put together components of the adaptive building model, the convex optimizer, and some other essential elements in a complete closed-loop autonomous MPC architecture. The second contribution is the performance assessment of the closed-loop adaptive MPC scheme in conjunction with a time-varying plant. Numerical results show the proposed convex MPC controller is more efficient and reliable in control computation compared to a nominal non-convex MPC controller, and the proposed autonomous MPC scheme consumes less energy than a conventional controller.

The rest of this paper is organized as follows. Section II describes the HVAC system. Section III describes structure of the proposed autonomous MPC scheme, together with the mathematical models we use for simulation. Section IV introduces our proposed convex controller and a nominal non-convex comparison controller. Simulation results are shown in Section V and Section VI concludes this work.

II. BUILDING HVAC SYSTEM

Figure 2 depicts the schematic of a typical single-zone variable air volume HVAC system used in a commercial building. In such system, part of the return air (at zone

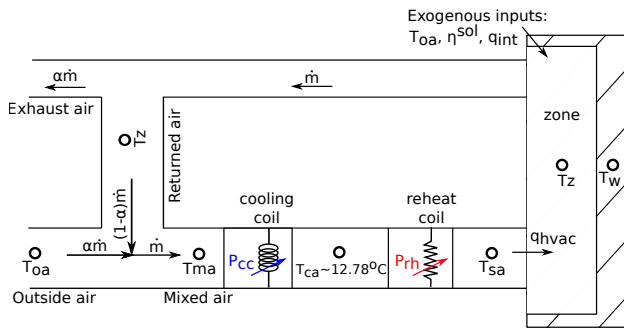


Fig. 2: A schematic of a typical single-zone variable air volume HVAC system used in a commercial building.

temperature T_z) is recirculated. This air is mixed with outside air (with temperature T_{oa}) at a constant ratio (outside air ratio α). However, this mixed air is usually hot and humid, especially for hot-humid climates. This not only makes occupants uncomfortable, but also and more importantly causes health concerns such as mold growth [11]. To avoid this problem, the mixed air (with temperature T_{ma}) is passed through the

cooling coil, where it is cooled and dehumidified. To ensure dehumidification, the conditioned air is usually maintained at a low value, typically 12.8°C [12]. This conditioned air is usually too cold for indoor climate. It is reheated by the reheat coil up to discharge air temperature T_{sa} , and then delivered into the zone. *The role of the energy-efficient control system is to vary the control commands, including the supply air flow rate and discharge air temperature, to maintain thermal comfort in the zone. That is, the controller has to compute \dot{m} and T_{sa} .*

III. PROPOSED CONTROL ARCHITECTURE AND MATHEMATICAL MODELS

In this paper we propose an autonomous MPC architecture as shown in Figure 1, of which the main features are:

- Adaptation of building model and disturbance to handle time-varying system dynamics, where the underlying optimization problem is feasible and convex.
- Computation of the global optimal control decisions from a convex control computation algorithm, which is proved to be feasible, convex, descent, and convergent.

At a discrete sampling time, sensor measurements from the plant and weather are collected, and are then fed into an algorithm to estimate the disturbance (\hat{w}) and building thermal model (\hat{P}). The building dynamics are unlikely to be static: they are a time-varying system depending on different operating conditions. Therefore we use an “adaptive MPC” procedure: the system identification algorithm re-runs periodically in order to cope with changes in the model and disturbance. In this paper, this self-learning process is done every week on Sunday at midnight with the identification algorithm from our previous work [10]. This algorithm involves solving a feasible and convex optimization problem, so that the global optimum can always be achieved. The obtained model is guaranteed to be stable and possess properties that are consistent with properties of a building HVAC system, such as an increase in outdoor temperature will lead to an increase in indoor temperature.

The future control decisions (u) are calculated from the proposed optimizer. This optimizer is formulated to be feasible and convex so that the global solution is always achieved.

The first several control decisions (over the control horizon N_c) are applied to the plant (P). The prediction time horizon is shifted by N_c and the above process is repeated until covering the total time span of interest (possibly endless).

Mathematical models that are used in simulations are described in the following subsections.

A. Plant model

The indoor zone temperature T_z ($^\circ\text{C}$) is affected by two types of inputs. The first one is input signals that can be commanded (u_c), which refers to cooling/heating added to the zone by the HVAC system, q_{hvac} (kW). The second is uncontrollable inputs (u_e), including the outside air temperature T_{oa} ($^\circ\text{C}$), the solar irradiance η^{sol} (kW/m^2), and the unknown disturbance q_{int} (kW), which is the internal heat gain due to occupants, lights, and equipments used by the

occupants. The wall temperature T_w ($^{\circ}\text{C}$) is unmeasurable, and the only measurable output is the zone temperature T_z .

The thermal dynamic behavior of building HVAC system is therefore represented by:

$$\begin{aligned} C_z \dot{T}_z &= \frac{T_w - T_z}{R_z} + q_{\text{hvac}} + A_e \eta^{\text{sol}} + q_{\text{int}} \\ C_w \dot{T}_w &= \frac{T_{\text{oa}} - T_w}{R_w} + \frac{T_z - T_w}{R_z}, \end{aligned} \quad (1)$$

where C_z, C_w, R_z, R_w are the thermal capacitances and resistances of the zone and wall, respectively, and A_e is the effective area of the building for incident solar radiation.

The cooling/heating added by HVAC system (q_{hvac}), is a bilinear function of the supply air flow rate \dot{m} (kg/s), and the deviation of discharge air temperature $T_{\text{sa}}(^{\circ}\text{C})$ from the zone temperature T_z :

$$q_{\text{hvac}}[k] = C_{\text{pa}} \dot{m}[k] (T_{\text{sa}}[k] - T_z[k]), \quad (2)$$

where C_{pa} is the specific heat of air at constant pressure.

B. Energy consumption

The power consumption of the HVAC system consists of three components: fan power, cooling power and reheat power. The fan power is modeled as:

$$P_{\text{fan}} = \alpha_f \dot{m}[k]^2. \quad (3)$$

The power consumed by cooling coil is modeled as the electric power used by a chiller to remove heat from the mixed air:

$$P_{\text{cc}}[k] = \begin{cases} \frac{C_{\text{pa}} \dot{m}[k] (T_{\text{ma}}[k] - T_{\text{ca}})}{COP_c} & T_{\text{ma}}[k] > T_{\text{ca}} \\ 0 & o.w. \end{cases}, \quad (4)$$

where $T_{\text{ca}}(^{\circ}\text{C})$ is the conditioned air temperature, COP_c is the chiller performance coefficient, and the mixed air temperature $T_{\text{ma}}(^{\circ}\text{C})$ is given by:

$$T_{\text{ma}}[k] = \alpha T_{\text{oa}}[k] + (1 - \alpha) T_z[k - 1]. \quad (5)$$

Note the cooling coil does not operate when $T_{\text{ma}}[k] \leq T_{\text{ca}}$, which only happens when the weather is very cold, and outside air ratio α is high. The power consumed by reheat coil is modeled as the heat it adds to the conditioned air:

$$P_{\text{rh}}[k] = C_{\text{pa}} \dot{m}[k] (T_{\text{sa}}[k] - T_{\text{ca}}). \quad (6)$$

C. Model used by the optimizer

In this paper, we use the following second-order model with $u_c := q_{\text{hvac}} \in \mathbb{R}$, $u_e := [T_{\text{oa}}, \eta^{\text{sol}}, q_{\text{int}}]^T \in \mathbb{R}^3$, $x := [T_z, T_w]^T \in \mathbb{R}^2$, $y := T_z \in \mathbb{R}$, and a sampling period Δt , to capture the building thermal dynamic:

$$\begin{aligned} x[k+1] &= Ax[k] + Bu_c[k] + Fu_e[k] \\ y[k] &= Cx[k] + Du_c[k] + Gu_e[k], \end{aligned} \quad (7)$$

where $A \in \mathbb{R}^{2 \times 2}$, $B \in \mathbb{R}^{2 \times 1}$, $C \in \mathbb{R}^{1 \times 2}$, $D \in \mathbb{R}$, $F \in \mathbb{R}^{2 \times 3}$ and $G \in \mathbb{R}^{1 \times 3}$ are appropriate matrices. Note q_{hvac} cannot be commanded by the controller directly, and only \dot{m} , T_{sa} can be controlled directly. Therefore in our work \dot{m} and T_{sa} are considered as the control commands. We treat q_{hvac} as

the input to the model in order to make the model (7) linear, and q_{hvac} is computed using measurements of \dot{m} and T_{sa} . The system identification algorithm learns the model (7) with q_{hvac} repeatedly, and then the control algorithm decides \dot{m} and T_{sa} accordingly.

IV. CONTROLLER DESIGN

The goal of the HVAC MPC controller is to minimize energy use subject to constraints such as occupants' comfort and actuator limitations. A direct translation of this goal into an optimization problem can be infeasible, but can be made into feasible using slack variables [13]. The later one is the nominal non-convex MPC controller formulated as follows.

A. Non-convex MPC controller

Define $z[k] := [\dot{m}, T_{\text{sa}}, T_{\text{ma}}, T_z, q_{\text{hvac}}, x_1, x_2, \epsilon^L, \epsilon^U]^T \in \mathbb{R}^9$ as the decision variables, in discrete time indices $k = 1, \dots, N$, where N is the prediction horizon. Let the cost be

$$\begin{aligned} J &= \sum_{k=1}^N (\Delta t (P_{\text{rh}}[k] + P_{\text{cc}}[k] + P_{\text{fan}}[k]) + \rho (\epsilon^L[k] + \epsilon^U[k])) \\ &= \sum_{k=1}^N \left(\frac{1}{2} z[k]^T P z[k] + q^T z[k] \right), \end{aligned} \quad (8)$$

where

$$\begin{aligned} P &= \begin{bmatrix} 2\alpha_f & C_{\text{pa}} \frac{C_{\text{pa}}}{COP_c} & 0_{1 \times 6} \\ \frac{C_{\text{pa}}}{COP_c} & & \\ 0_{6 \times 1} & 0_{8 \times 8} & \end{bmatrix}, \\ q^T &= \begin{bmatrix} -C_{\text{pa}} T_{\text{ca}} \frac{1+COP_c}{COP_c} & 0_{1 \times 6} & \rho & \rho \end{bmatrix}, \end{aligned}$$

are obtained using models (3)-(4) and (6). The optimization problem is:

$$\begin{aligned} \min_{z[k]_{k=1}^N} \quad & J \quad (9) \\ \text{s. t.} \quad & -q_{\text{hvac}}[k] + \frac{1}{2} z[k]^T P_c z[k] = 0 \quad (h_{1,k}) \\ & \text{equality constraints (5), (7)} \quad (h_{2,k}-h_{5,k}) \\ & |\dot{m}[k] - \dot{m}[k-1]| \leq \dot{m}^{\text{rate}} \Delta t \quad (f_{1,k}-f_{2,k}) \\ & |T_{\text{sa}}[k] - T_{\text{sa}}[k-1]| \leq T_{\text{sa}}^{\text{rate}} \Delta t \quad (f_{3,k}-f_{4,k}) \\ & (\dot{m}, T_{\text{sa}}) \in [\dot{m}^L, \dot{m}^U] \times [T_{\text{sa}}^L, T_{\text{sa}}^U] \quad (f_{5,k}-f_{8,k}) \\ & T_z \in [T_z^L - \epsilon^L, T_z^U + \epsilon^U] \quad (f_{9,k}-f_{10,k}) \\ & -\epsilon^L[k] \leq 0, \quad -\epsilon^U[k] \leq 0, \quad (f_{11,k}-f_{12,k}) \end{aligned}$$

where \times denotes the Cartesian product, and

$$P_c = \begin{bmatrix} 0 & C_{\text{pa}} & 0 & -C_{\text{pa}} & 0_{1 \times 5} \\ \frac{C_{\text{pa}}}{COP_c} & & & & \\ 0 & & 0_{8 \times 8} & & \\ -C_{\text{pa}} & & & & \\ 0_{5 \times 1} & & & & \end{bmatrix},$$

is from rewriting Eq. (2). Rate constraints ($f_{1,k}-f_{2,k}$) and ($f_{3,k}-f_{4,k}$) are for $k = 2, \dots, N$. The remaining constraints are for $k = 1, \dots, N$. As discussed earlier, the conditioned air is fixed at the set-point, $T_{\text{ca}} = 12.8^{\circ}\text{C}$. The

minimum allowed value for the supply air flow rate, \dot{m}^L , is computed based on the ventilation requirements specified in ASHRAE 62.1 [12], where $\dot{m}^L > 0$ in general. To ensure reheat coil can only add heat, we require $T_{sa}^L = T_{ca}$. Thermal comfort bounds are $[T_z^L, T_z^U]$. Slack variables ϵ^L, ϵ^U are added to guarantee feasibility, and ρ is a penalty parameter.

The nominal non-convex MPC problem (9) is feasible, but is non-convex since both the cost function and dynamic constraint ($h_{1,k}$) are non-convex. More specifically, coefficients of the quadratic terms are P and P_c , which are indefinite. It is possible that this non-convex controller fails to converge within the allowed time or get stuck in a local minimum.

B. Proposed convex MPC controller

The goal of this section is to solve problem (9) using an appropriate convexification method, so that the underlying optimization problem is convex, and the obtained solution provides good approximation to the original one.

The proposed control computation algorithm is designed using CCP approach. We show this algorithm is descent and convergent, and the underlying optimization problem is convex and feasible. Additionally, analysis showing the inapplicability of the alternate convexification methods is provided at the end of this section.

The basic idea of CCP is to use a convex model of the problem and repeatedly minimize it. Specifically, convex portions of the problem are handled exactly and efficiently, while the non-convex portions of the problem are modeled by convex functions that are (at least locally) accurate.

We propose the following convex control computation algorithm incorporated with CCP approach.

Algorithm 1. At each iteration near current solution z_0 , approximate \hat{J} of cost J (8) and $\hat{h}_{1,k}$ of constraint ($h_{1,k}$) as

$$\hat{J} = \sum_{k=1}^N \left(\frac{1}{2} z_0^T P z_0 + q^T z_0 + (P z_0 + q)^T (z[k] - z_0) + \frac{1}{2} (z[k] - z_0)^T P^+ (z[k] - z_0) \right), \quad (10)$$

where P^+ is the positive semi-definite part of P , and can be computed by keeping the non-negative eigenvalues from eigen-decomposition of P , and

$$\hat{h}_{1,k} : q_{\text{hvac}} = \frac{1}{2} z_0^T P_c z_0 + z_0^T P_c (z[k] - z_0). \quad (11)$$

Then we solve the following convex problem

$$\begin{aligned} \min_{z[k]_{k=1}^N} \quad & \hat{J} \\ \text{s. t.} \quad & \hat{h}_{1,k}, h_{2,k} - h_{5,k}, f_{1,k} - f_{12,k}, \end{aligned} \quad (12)$$

and repeat steps (10)-(12) until convergence condition is met.

Proposition 1. Problem (12) is feasible and convex, and Algorithm 1 is a descent and convergent method.

Proof of Proposition 1. Problem (9) is feasible due to the presence of the slack variables, and the same for Problem (12). Problem (12) is convex as both quadratic cost and linear constraints are convex over the feasible region.

The proof of CCP being a descent method can be found in [14]. It can be shown that cost (8) is lower bounded by 0 over the feasible region. Therefore Algorithm 1 is convergent, because it has a lower-bounded, monotonically non-increasing cost. \square

We remark that properties from Proposition 1 are the key features to guarantee reliable performance of our controller. The global optimal is always obtained at each iteration as Problem (12) is formulated to be feasible and convex.

Even though convergence may not be achieved during the time available for control computation, the outcome obtained from Algorithm 1 within the allowed time is still the best solution that can be achieved over solutions from previous iterates. In practice, this leads to a suboptimal controller. Moreover to alleviate computation, we can always initialize z_0 using old decisions to help in convergence.

1) *Choice of convexification methods:* The following propositions show important properties of problems (9), which help us to explain why we choose CCP over some other commonly-used approximation methods such as BnB. It is due to the inapplicability of the other method to our problem structure.

Proposition 2. The dual of Problem (9) is unbounded from below.

Proof of Proposition 2. The Lagrangian of (9) is:

$$\mathcal{L}(z, \lambda, v) = \sum_{k=1}^N \left(\frac{1}{2} z[k]^T P z[k] + q^T z[k] + \sum_{i=1}^5 \lambda_{i,k} h_{i,k} + \sum_{j=1}^{12} v_{j,k} f_{j,k} \right). \quad (13)$$

To find the dual function of (13), we minimize over z . According to the general formula [13]:

$$\inf_{z \in \mathbb{R}^n} z^T P z + q^T z + r = \begin{cases} r - \frac{1}{4} q^T P^\dagger q, & P \geq 0 \\ -\infty, & \text{o.w.} \end{cases},$$

Lagrangian (13) for our problem is unbounded from below as the coefficient of the quadratic term is $\sum_k \frac{1}{2} (P + \lambda_{1,k} P_c)$, which is indefinite. \square

Branch-and-Bound (BnB) [15] is one the the most commonly-used methods in literatures, it is inapplicable to for Problem (9) based on Propositions 2. BnB requires construction of a tight convex under-estimator of the NLP within any given region of the space of the variables. The most widely-used under-estimators are Lagrangian relaxation [16] or convex relaxation. However Proposition 2 shows the dual of our problem is unbounded from below, therefore a lower bound through duality cannot be found and Lagrangian relaxation cannot be applied here. Common convex relaxation options are McCormick envelope [15] and Reformulation Linearization Technique (RLT) [17]. Both of them reformulate a problem via the addition of certain nonlinear constraints that are generated by using the products of the bounding constraints. However, constructing such

products require knowledge of bounds on variables that are involved. In our problem, we do not know thermal comfort limits because of the introduction of slack variables. Hence convex relaxation is also not applicable for our case.

C. Baseline controller

The baseline controller – against which the performance of the proposed MPC controller is compared – is chosen as the single maximum controller which is widely used in practice [18]. The controller operates the HVAC system in three modes depending on the zone temperature. When the zone temperature exceeds an upper bound, the supply air flow rate varies within the allowed range to maintain thermal comfort, and the discharge air temperature is kept at minimum allowed value. When the zone temperature is below a threshold, the discharge air temperature is varied within an allowed range, and the flow rate is kept at a minimum. When the zone temperature is within the upper and lower bounds, the discharge air temperature and the flow rate are both kept at the minimum allowed values.

V. SIMULATION RESULTS AND DISCUSSIONS

A number of comparisons are provided. One is comparison of the proposed control architecture with a conventional rule based controller, called single-max controller, that is widely used in buildings. The other is a comparison between the proposed architecture with another one that uses the original non-convex formulation of the optimization problem. The purpose of the second comparison is to check if autonomous operation is still possible even without convexification.

A. Simulation setup

The simulation parameters are chosen based on an auditorium ($\sim 470m^2$) in Pugh Hall located in the University of Florida campus. The values for plant parameters are listed in Table I. The plant is a time-varying system, for which the resistance and capacitance parameters have 3% of increment per week, and the effective area for incident solar radiation decreases 3% every week.

TABLE I: Simulation parameters for plant

C_z	9.4 ~ 10.7	kWh/K	C_w	20.0 ~ 21.9	kWh/K
R_z	0.70 ~ 0.76	K/kW	R_w	0.70 ~ 0.76	K/kW
A_e	6.3 ~ 7.0	m^2			

TABLE II: Parameters for MPC

\dot{m}^L	0.42	kg/s	T_z^L	21.9	$^\circ\text{C}$
\dot{m}^U	4.72	kg/s	T_z^U	23.6	$^\circ\text{C}$
\dot{m}^{rate}	0.2	kg/s/min	T_{ca}	12.8	$^\circ\text{C}$
T_{sa}^L	12.8	$^\circ\text{C}$	α	0.5	N/A
T_{sa}^U	30.0	$^\circ\text{C}$	α_f	417.5	W/(kg/s) ²
T_{sa}^{rate}	0.56	$^\circ\text{C}/\text{min}$	COP_c	3.5	N/A

The prediction horizon for MPC is $N = 24$ hours with sampling time $\Delta t = 5$ minutes, and the control horizon is $N_c = 15$ minutes. The number of decision variables for problems (9) and (12) is $2592 (= 9N)$. Estimates of model

and disturbance are obtained using the algorithm in [10], and the algorithm re-runs periodically at every Sunday midnight, using measurements from the previous week. The remaining parameters are listed in Table II.

The exogenous inputs are chosen as follows: ambient temperature is taken from weatherunderground.com, and solar irradiance data is taken from NSRDB: <https://nsrdb.nrel.gov/>, both for Gainesville, FL. The disturbance is chosen by scaling CO₂ data from Pugh Hall.

All numerical results presented in this work are obtained through MATLAB[®]. Specifically, the plant is simulated in SIMULINK[®]. Nonlinear programming solver Ipopt[®] [19] is used for solving the non-convex problem (9), and CVX[®] [20] package is used for estimating model and disturbance, and for solving the proposed convex problem (12). We used a Desktop computer running Linux with a 3.60GHz \times 8 CPU and 16.2 GB RAM.

B. Results and discussions

1) Comparison with the non-convex MPC controller:

We compare the control performance when the proposed convex controller (12) and the non-convex controller (9) are implemented in the adaptive MPC scheme. Both controllers have similar performance in maintaining thermal comfort; see top plot from Figure 3. On average it takes about 2.3 seconds for the non-convex controller to find a local optimum successfully. Ipopt fails to converge to a local minimum for 0.3% of the time during the tested period. When this happens, decision from a single maximum controller is used and as a result, control decisions and power consumption increase abruptly; see the middle of week 2 from Figure 3. The proposed controller is always able to find a global optimum, and its takes on average 1.5 seconds to do so. During the three weeks of experiments, the convex controller consumes approximately 1% more energy compared to the non-convex one; see Figure 5.

The above results indicates that the proposed convex controller is more reliable in control computation than the non-convex MPC controller, as the later one may occasionally fail to converge within the available computation time. As a trade-off, the proposed controller consumes slightly more energy.

2) Comparison with the baseline controller:

The proposed controller outperforms the baseline controller both in maintaining temperature (see Figure 4, top) and energy use (see Figure 5).

3) Comparison with non-adaptive MPC scheme:

We also compare the energy consumption of the proposed adaptive MPC scheme against an MPC scheme without model adaptation. Figure 5 shows that during the three weeks of the study, the adaptive scheme reduces energy use compared to the non-adaptive scheme, in both convex and non-convex optimization formulations.

VI. CONCLUSION

An autonomous MPC architecture for HVAC control is presented, which involves self-adaptation of the time-varying

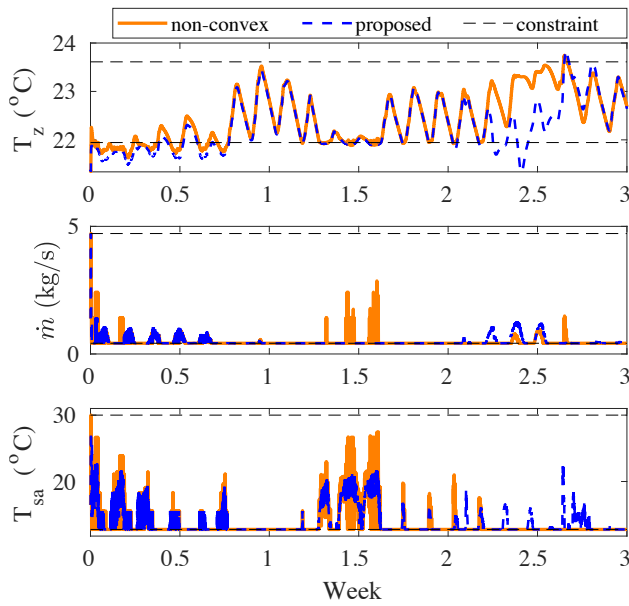


Fig. 3: Decisions made by the convex and non-convex MPC controllers.

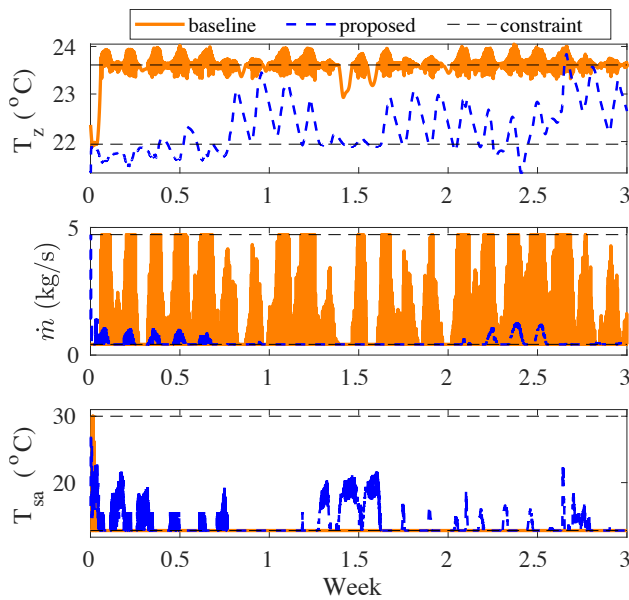


Fig. 4: Decisions made by the convex MPC and baseline controllers.

disturbance and building thermal dynamics model, and control computation using a convex optimizer. The proposed MPC scheme reduces energy use significantly over a baseline rule-based controller. Simulation studies show the proposed controller is more reliable in control computation compared to a nominal non-convex MPC controller, and that model adaptation is beneficial. Conducting additional simulation studies for a wide variety of weather conditions and experiments in real buildings are topics of future work.

REFERENCES

[1] United States Energy Information Administration, “Annual energy outlook,” April 2018. [Online]. Available: <https://www.eia.gov/>

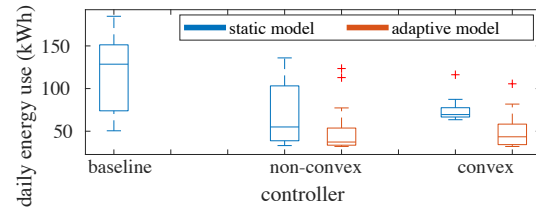


Fig. 5: Daily energy consumption used by the baseline, non-convex and proposed convex controllers.

- outlooks/aeo
- [2] A. Afram and F. Janabi-Sharifi, “Theory and applications of HVAC control systems—a review of model predictive control (mpc),” *Building and Environment*, vol. 72, pp. 343–355, Feb 2014.
 - [3] D. Liberzon, *Switching in systems and control*. Springer Science & Business Media, 2003.
 - [4] A. Kelman and F. Borrelli, “Bilinear model predictive control of a HVAC system using sequential quadratic programming,” *IFAC Proceedings Volumes*, vol. 44, no. 1, pp. 9869–9874, 2011.
 - [5] M. Razmara, M. Maasoumy, M. Shahbakhti, and R. Robinett III, “Optimal exergy control of building HVAC system,” *Applied energy*, vol. 156, pp. 555–565, Oct 2015.
 - [6] A. Aswani, N. Master, J. Taneja, D. Culler, and C. Tomlin, “Reducing transient and steady state electricity consumption in HVAC using learning-based model-predictive control,” *Proceedings of the IEEE*, vol. 100, no. 1, pp. 240–253, 2011.
 - [7] A. Parisio, L. Fabietti, M. Molinari, D. Varagnolo, and K. H. Johansson, “Control of HVAC systems via scenario-based explicit mpc,” in *53rd IEEE conference on decision and control*. IEEE, 2014, pp. 5201–5207.
 - [8] M. Maasoumy and A. Sangiovanni-Vincentelli, “Total and peak energy consumption minimization of building HVAC systems using model predictive control,” *IEEE Design & Test of Computers*, vol. 29, no. 4, pp. 26–35, 2012.
 - [9] Y. Ma, J. Matuško, and F. Borrelli, “Stochastic model predictive control for building HVAC systems: Complexity and conservatism,” *IEEE Transactions on Control Systems Technology*, vol. 23, no. 1, pp. 101–116, 2015.
 - [10] T. Zeng, J. Brooks, and P. Barooah, “Simultaneous identification of linear building dynamic model and disturbance using sparsity-promoting optimization,” *Automatica*, 2018, under review; available at ArXiv.org, arXiv:1711.06386.
 - [11] A. Baughman and E. A. Arens, “Indoor humidity and human health—Part I: Literature review of health effects of humidity-influenced indoor pollutants,” *ASHRAE Transactions*, vol. 102 Part 1, pp. 192–211, 1996.
 - [12] ASHRAE, “ANSI/ASHRAE standard 62.1-2016, ventilation for acceptable air quality,” 2016.
 - [13] S. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge university press, 2004.
 - [14] T. Lipp and S. Boyd, “Variations and extension of the convex–concave procedure,” *Optimization and Engineering*, vol. 17, no. 2, pp. 263–287, 2016.
 - [15] G. P. McCormick, “Computability of global solutions to factorable nonconvex programs: Part i—convex underestimating problems,” *Mathematical programming*, vol. 10, no. 1, pp. 147–175, 1976.
 - [16] A. d’Aspremont and S. Boyd, “Relaxations and randomized methods for nonconvex qcqps,” *EE392o Class Notes, Stanford University*, vol. 1, pp. 1–16, 2003.
 - [17] H. D. Sherali and A. Alameddine, “A new reformulation-linearization technique for bilinear programming problems,” *Journal of Global optimization*, vol. 2, no. 4, pp. 379–410, 1992.
 - [18] M. Hydeman, S. Taylor, J. Stein, E. Kolderup, and T. Hong, “Advanced variable air volume system design guide,” *California Energy Commission*, 2003.
 - [19] A. Wächter and L. T. Biegler, “On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming,” *Mathematical programming*, vol. 106, no. 1, pp. 25–57, 2006.
 - [20] M. Grant and S. Boyd, “CVX: Matlab software for disciplined convex programming, version 1.21,” <http://cvxr.com/cvx>, Feb. 2011.