

Issues in identification of control-oriented thermal models of zones in multi-zone buildings

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Abstract—There is significant recent interest in applying model-based control techniques to improve the energy efficiency of buildings. This requires a predictive model of the building’s thermal dynamics. Due to the complexity of the underlying physical processes, usually system identification techniques are used to identify parameters of a physics-based model. We investigate the effect of various model structures and identification techniques on the parameter estimates through a combination of analysis and experiments conducted in a commercial building. We observe that a second order model can reproduce the input-output behavior of a full-scale model (with 13 states). Even a single state model has enough predictive ability that it may be sufficient for control purposes. We also show that the application of conventional techniques to closed-loop data from buildings (that are collected during usual operation) leads to poor estimates; their inaccuracy becomes apparent only when forced-response data is used for validation where there is sufficient difference among various inputs and outputs. The results of this investigation are expected to provide guidelines on do’s and don’ts in modeling and identification of buildings for control.

I. INTRODUCTION

Model-based control of buildings, in particular, of the HVAC (heating, ventilation, and air conditioning) system, has generated excitement in the community of control researchers in recent years. This problem is of great societal importance since buildings account for 34% of total energy use in the United States and HVAC account for roughly half of that [1]. If novel control techniques can increase the efficiency of HVAC systems, it will lead to substantial savings in total energy use.

The most popular candidate for control of building HVAC systems is MPC (Model Predictive Control); a slew of recent papers have been published on this topic [2], [3], [4], [5], [6], [7], [8], [9], [10]. MPC requires a model of the process that can predict the evolution of temperature and humidity (and possibly other environmental variables such as CO_2 concentration) given the inputs (mass flow and temperature of the supply air, temperatures of the surroundings, etc.) In this paper, we focus our attention to the control of a single zone in a multi-zone commercial building with VAV (variable air volume) system.

The underlying processes that govern the dynamics of temperature evolution are complex and uncertain, so simplified lumped parameter models are usually used, among which RC network model is a popular one. It captures inter-zone heat

transfer through solid surfaces, which is linear in structure, and a non-linear term that captures the effect of enthalpy exchange with the outside due to the supply and extract air. We restrict ourselves to this class of models, which is denoted by $\mathcal{M}_{RC}(n, q, p)$, where n refers to state dimension, q to the number of inputs, and p to the number of uncertain parameters. Two questions are relevant. (i) Q1: What is the minimum model complexity (measured by n, q, p) that is required to predict the temperature dynamics of a single zone with an acceptable degree of accuracy so that it can be used in MPC? (ii) Q2: How to identify the values of the uncertain parameters from measured data, and what kind of measured data are required, to achieve this level of accuracy?

ASHRAE (American Society of Heating, Refrigeration and Air conditioning and Engineers) handbook [11] describes how to determine the R,C (resistance/capacitance) values for a solid surface given its material and construction type. This information is now available in software such as HAP [12]. However, there is uncertainty in these parameter values. First, information on wall and window construction and material are not always easy to obtain for existing buildings due to poor record keeping. Second, due to cracks in windows and walls, the effective resistance of an window and wall is likely to be lower than what is inferred from construction data. Therefore there is a need to *identify/estimate* R,C parameters from measured data, a process referred to as model calibration and/or system identification.

The identification problem is not trivial. Powerful state-space identification methods (such as subspace methods [13]) cannot be directly employed as they do not lead to an identification of R,C parameters. Rather, they identify the system matrices in an arbitrary state space. Frequency domain and adaptive techniques [14] suffer similar problems due to the complex relationship between the R,C parameters and the coefficients in the polynomials that describe the transfer functions. The need for obtaining values of R,C parameters (as opposed to obtaining a system realization) is manifold. First, R,C values have intuitive, physical meaning. Second, one can check if the identified parameters values are reasonable - and therefore if the model is - by comparing against ASHRAE values. This provides a sanity check and can help unearth potentially grossly inaccurate system identification. Third, since R,C modelling paradigm is prevalent in the HVAC/buildings community, it is useful simply as a common language.

In this paper, we attempt to answer Q1 by a comparison of response between a model of a high state dimension and various low order models of same class $\mathcal{M}_{RC}(n, q, p)$. Both

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time- and frequency-response comparison are performed. Comparisons show that a second order model with 2 parameters is capable of reproducing the input-output behavior of a 13 state model (with 30 parameters) with a high degree of accuracy. Even a first order model has acceptable degree of accuracy.

To answer Q2, we apply two parameter estimation techniques to identify the R,C values of low order models from data collected from a building in the University of Florida campus. A few authors have proposed methods for estimating R,C parameter values of building thermal models, e.g., [15], [16], [17], [18], [19]. Such papers show the effectiveness of their proposed methods by comparing the prediction of the identified model with measured data. Our analysis shows that for almost all closed-loop data, such comparison is meaningless; grossly “wrong” models can reproduce such data quite accurately. That the model is wrong becomes evident only when it is asked to predict measurements with very specific features, namely when there is large differences between the temperature of the room and the temperatures of the surroundings. This is in fact the situation that the model needs to be able to predict if it is going to be useful for control algorithms that seek to reduce energy use, because the controller may let the temperature float up or down when the zone is unoccupied [2], [3]. Thus, unless specific forced response test are conducted to gather certain types of data, one is better off with ASHRAE values of parameter than to estimate parameters from closed-loop data.

II. MODEL

In this section, we will discuss the states, inputs, and parameters of three different models of a zone in the class $\mathcal{M}_{RC}(n, q, p)$. Consider a typical zone in a multi-zone building with a VAV system, which is separated from the outside through an external wall and a window and from five internal spaces (floor, ceiling, one room on each side, and a hallway) through internal walls and a door to the hallway. The major heat transfer mechanisms include the following: (1) heat conduction through external and internal walls, windows, roof, and ceiling; (2) heat convection with outside air due to the air supplied to and extracted from the room by the HVAC system; (3) solar radiation through the window and external wall; (4) casual heat gain from occupants and equipments; and (5) infiltration and exfiltration. A schematic figure of the room is shown in Figure 1.

The following assumptions are made throughout the paper. (i) The air inside the room is well mixed, so that we have one uniform temperature in the room. (ii) We ignore in/ex-filtration. (iii) We confine our study to night time data when there is no solar radiation and occupant heat gain, which often have large uncertainty and not easy to measure accurately. More importantly, since the occupants are the main source of latent heat gain in an office, this restriction eliminates the need to consider nonlinear humidity dynamics [20].

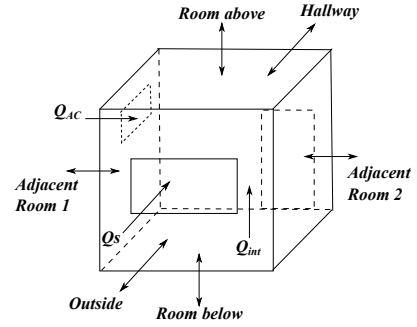


Fig. 1. Schematic figure of the room

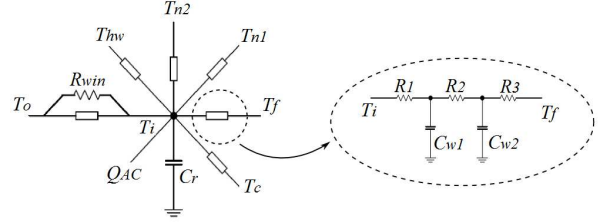


Fig. 2. Full-scale model structure

The main variable of interest is the temperature of the space inside the zone, denoted as T_i . This is the first state, and depending on the model dimension, there may be other states. We now describe the various models.

A. Full-scale model of a zone

The inputs that affect the room temperature are outside temperature, temperatures of the surrounding spaces, and heat gain inside the zone. We define the input vector to be $\bar{u} = [T_o, T_f, T_c, T_{n1}, T_{n2}, T_{hw}, T_s, m, Q]^T$, where T_o is the outside temperature, T_f is the temperature of the space below the floor, T_c is the temperature of the space above the ceiling, T_{n1} and T_{n2} are the temperature of the adjacent rooms to the side, T_{hw} is the temperature of the hallway, Q is additional casual heat gain from appliances etc., m is the flow rate of supply air and T_s is its temperature. The net heat gain due to the supply air, and equals to the enthalpy of incoming air minus the enthalpy of exhaust air:

$$Q_{AC} = mC_p T_s - mC_p T_i \quad (1)$$

where m is the supply air mass flow rate, C_p is the specific heat of air, and T_s is the supply air temperature. We employ the commonly used choice of 3R-2C (3 resistors and 2 capacitors) [21] in modelling the heat transfer through a solid surface separating two rooms. Since windows have very low heat capacitance, it is modelled as a single resistor. Each surface element is then connected to the room “node” to form a RC network model. An additional capacitor is included to model the heat stored by the air and other objects in zone. The structure of the full-scale model is shown in Figure 2. Since each wall is a 3R-2C component, there are two states for each wall. Together with T_i , we have a 13-state vector: $T = [T_i, T_{fw1}, T_{fw2}, \dots]^T$, where T_{*w1}, T_{*w2} are the temperatures of two nodes associated with the surface $*$.

The dynamics of T_i can be then expressed as:

$$C_r \dot{T}_i = \left(-\frac{1}{R_{win}} - \frac{1}{R_{ow_1}} - \frac{1}{R_{fw_1}} - \frac{1}{R_{cw_1}} - \frac{1}{R_{n_1w_1}} - \frac{1}{R_{n_2w_1}} - \frac{1}{R_{hw_1}} \right) T_i + \frac{T_o}{R_{win}} + \frac{T_{ow_1}}{R_{ow_1}} + \frac{T_{fw_1}}{R_{fw_1}} + \frac{T_{cw_1}}{R_{cw_1}} + \frac{T_{n_1w_1}}{R_{n_1w_1}} + \frac{T_{n_2w_1}}{R_{n_2w_1}} + \frac{T_{hw_1}}{R_{hw_1}} + Q_{AC} + Q \quad (2)$$

The dynamics of the wall nodes of each wall have similar structure:

$$C_{*w_1} \dot{T}_{*w_1} = \left(-\frac{1}{R_{*w_1}} - \frac{1}{R_{*w_2}} \right) T_{*w_1} + \frac{T_i}{R_{*w_1}} + \frac{T_{*w_2}}{R_{*w_2}}$$

$$C_{*w_2} \dot{T}_{*w_2} = \left(-\frac{1}{R_{*w_2}} - \frac{1}{R_{*w_3}} \right) T_{*w_2} + \frac{T_{*w_1}}{R_{*w_2}} + \frac{T_{*w_3}}{R_{*w_3}}$$

The full-scale model has a total of 32 R and C parameters, thus it is of the class $\mathcal{M}_{RC}(13, 8, 32)$.

B. Low order models

We now consider two low order models which still employs the RC network analogy of conductive heat transfer as the full-scale model described above, but with fewer states and parameters.

1) *First-order model:* We start with a model of lowest possible state dimension, namely, one. Here all the capacitive elements of a zone (walls, air, furniture) are aggregated into a single capacitor, so that the dynamics of T_i become

$$C_r \dot{T}_i = \frac{T_o - T_i}{R_{win}} + \frac{T_{sd} - T_i}{R_w} + Q_{AC} + Q \quad (3)$$

where T_{sd} is an average of all the surrounding space temperatures. This model has a single state T_i , five inputs $\bar{u} = [T_o, T_{sd}, T_s, m, Q]^T$, and three parameters: $\theta = [C_r, R_{win}, R_w]^T$. Thus, it is of the class $\mathcal{M}(1, 5, 3)$.

2) *Second-order model:* The response of the room temperature T_i to changes in mass flow rate and temperature of the supply air is usually faster than its response to changes in the surrounding temperatures. A natural idea is to use two capacitors to reproduce the two-time scales of the process. One capacitor (room capacitance C_r) is used for the low thermal mass of the air and other objects in the room, and the other (C_w) is used for the heat capacity of all the walls combined. We choose to model the integrated wall as a 2R-1C element, which leads to:

$$C_r \dot{T}_i = \frac{T_o - T_i}{R_{win}} + \frac{T_{w1} - T_i}{R_1} + Q_{AC} + Q$$

$$C_w \dot{T}_w = \frac{T_i - T_w}{R_1} + \frac{T_{sd} - T_w}{R_2} \quad (4)$$

where T_w is the temperature of the wall node. This model has 2 states $T = [T_i, T_w]^T$, the same five inputs as the single-state model, and five parameters $\theta = [C_r, C_w, R_{win}, R_1, R_2]^T$. Thus, it is of class $\mathcal{M}(2, 5, 5)$.

3) *The LTI case:* Note that the Q_{AC} term in (2), (3), and (4) is the only nonlinear term in each of the three models described above. When the supply air flow rate, m , is constant, the Q_{AC} term becomes linear in the state T_i and input T_s . The system then becomes a LTI system:

$$\dot{T} = FT + Gu, \quad z = HT = T_i, \quad (5)$$

where the state T and input u varies depending on which of the three models is under consideration.

III. Q1: MODEL STRUCTURE COMPARISON

Various models within the class $\mathcal{M}_{RC}(n, q, p)$ can be compared among themselves. In the LTI case, it is possible to compare their frequency response as well. A comparison between their frequency responses is useful to determine whether a low-order model is powerful enough to characterize the temperature dynamics within a range of input frequencies as well as a high order model.

We do not deal with calibration in this section. Instead, we choose the parameters of the full-scale model as their ASHRAE values. In the low-order models, we consider the surfaces that are aggregated to form the integrated wall to be parallel components, so that the total capacitance is the sum of capacitance of each surface, and the total conductance is the sum of conductance (inverse of resistance) of each surface.

A. Field Data

For model calibration and validation, we choose a typical office, Room 241, in Pugh Hall, University of Florida. The room is connected to two adjacent offices and the hallway through internal walls; and to the outside through an external wall and window. Three data sets are studied. All data are collected from 6pm in the evening to 6am next morning. The first data set, set A, was collected on a summer day from Aug. 2nd to Aug. 3rd, 2011, under normal closed loop control strategy. The second data set B, was collected on a winter day from Dec. 5th to Dec. 6th, 2011 under normal closed loop control strategy. These two data sets are chosen among data collected over several months to meet the criteria that the room and supply air temperatures should have as much variation as possible (to ensure persistency of excitation), while the mass flow rate should be constant for long periods of time (so that the models become LTI). The third data set, set C, was collected from Oct. 24th to Oct. 25th, 2011, under a forced response test where the inputs are determined by given commands. The room temperature, supply air temperature, and supply air flow rate of the three data sets are plotted in Figure 3.

B. Time domain comparison

The time domain comparisons of the full-scale and reduced model (along with measured temperatures from the three data sets described in Section III-A) are shown in

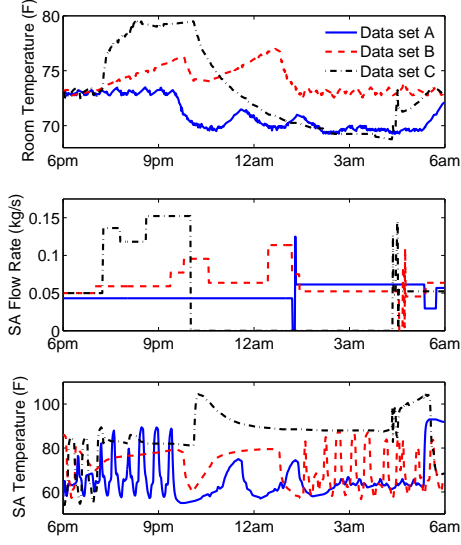


Fig. 3. Three data sets collected at Pugh Hall.

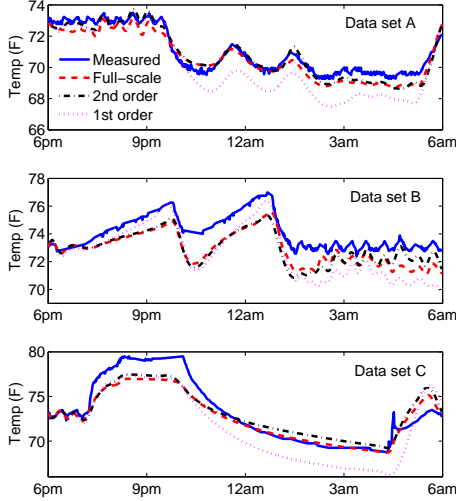


Fig. 4. Time domain comparison of different model structures.

Figure 4. We see that all the three models have similar prediction.

Note that, though the three data sets was collected at night with no occupants, we conjecture that an additional heat gain due to the appliance in the room was present. An inactive desktop PC & monitor can produce 20-30 W of heat, and an idle desktop printer can produce 10-35 W [11]. So we used a constant load of $Q(t) \equiv 50W$ in all time domain simulation in this paper.

C. Frequency domain comparison

For comparison, we need common inputs in different models. Let $H_\ell(s)$ be the transfer function from the ℓ -th input to the output T_i . In full scale model, we define a transfer function from a single surrounding temperature T_{sd} to output in the full scale model as the sum of transfer functions of

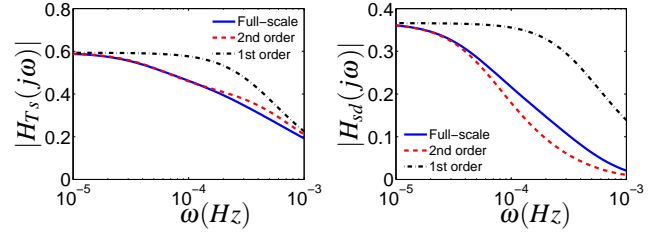


Fig. 5. Comparison of gains of the three models.

all five surrounding temperatures, i.e.,

$$H_{sd}(s) := \sum_{i=1}^5 H_{sd_i}(s) \quad (6)$$

where H_{sd_i} are the transfer functions from each surrounding temperature to output.

Figure 5 shows the frequency response comparison (only the magnitude, not phase) of the three models: full-scale (13-th order), first order, and second order. The parameters are chosen as described earlier. The frequency of interest is chosen to be $\frac{1}{5}$ hours ($10^{-5}Hz$) to $\frac{1}{5}$ mins ($10^{-3}Hz$). The value of the mass flow rate is set to be the maximum possible value, $m = 0.12(Kg/s)$, as we have observed that the maximum error occurs at the maximum values of m . We also observed that the outside temperature T_o has small impact on the output compared to the other inputs (in the sense of small gain), which is consistent with the fact that commercial buildings are usually well insulated. Therefore, we only show the result of the two inputs T_s and T_{sd} here.

The effect of an input on the output depends not only on the transfer function but also on the magnitude of input. The variation in different inputs can be significantly different from each other. The error in the prediction of T_i may be different due to these inputs even though they might have the same gain. Therefore we define the *error with respect to i -th input* as:

$$e_i(\omega) = \left| |H_{fs_i}(j\omega)| - |H_{d_i}(j\omega)| \right| \delta u_i, \quad (7)$$

where $H_{fs_i}(\cdot)$ and $H_{d_i}(\cdot)$ correspond to full-scale and low-order models, and δu_i is the maximum variation in the i -th input. By examining the data from Pugh Hall, we observe that the largest variations in supply air temperature, surrounding room temperatures, and outside temperature are $45^{\circ}F$, $10^{\circ}F$, and $45^{\circ}F$. The largest estimation error again appears in the inputs T_s and T_{sd} . The error e_i as a function of frequency is shown in Figure 6.

The following conclusions are drawn from the comparison. *First*, for the range of frequencies deemed of interest, the second order model is almost as accurate as the 13-th order model, with a maximum prediction error of less than $1^{\circ}F$. *Second*, the 1st order model is also quite accurate in terms of predicting the response due to supply air temperature T_s , but less so in case of the surrounding temperature. The maximum prediction error is about $3^{\circ}F$ in this case.

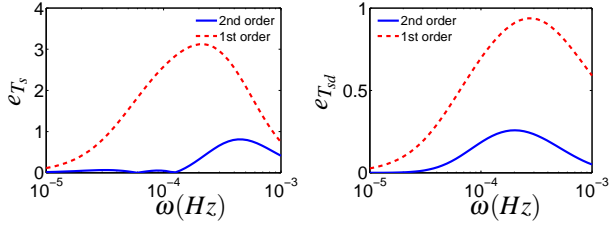


Fig. 6. The error defined in (7) for the two inputs T_s and T_{sd} .

IV. Q2: MODEL CALIBRATION AND VALIDATION

Model calibration refers to estimating the R,C parameters. As we saw in the previous section that the second order model provides similar prediction power as full-scale model, we limit our attention to second order model for the purpose of identification. The following assumptions are made to further reduce the number of uncertain parameters in the second order model: (i) The resistance of the wall is symmetric, i.e., $R_1 = R_2 = \frac{1}{2}R_w$. (ii) The wall capacitance C_w and window resistance R_{win} are assumed known (ASHRAE values). (iii) The surrounding temperature T_{sd} is taken to be the average of all the surrounding room temperatures (above the ceiling, below the floor, two adjacent rooms and the hallway).

A. Identification methods

We pick two methods for parameter estimation, with different cost functions described below.

1) *Least-squares*: For a given model structure with fixed parameters, we define a *prediction error cost* J_{LS} as:

$$J_{LS} = \int_0^{\tau} (T_{im}(t) - T_{ip}(t))^2 dt \quad (8)$$

where $T_{im}(t)$ is the measured room temperature at time t , $T_{ip}(t)$ is the room temperature at time t predicted by the model with a given set of parameter values, and τ is a user-specified time interval.

2) *Maximum likelihood (ML) method*: This method is applicable to LTI models in discrete time. The likelihood function of the parameter vector θ is given by the joint density function of all observations. With some Gaussian and independent assumptions, it can be calculated by using a Kalman filter. The reader is referred to [17] for details. We define the cost function J_{ML} as:

$$J_{ML} = -\log L \quad (9)$$

where L is the likelihood function.

In minimizing the cost functions, we pick the direct search method. The reason is that the cost functions are non-convex, optimization algorithms like gradient descent often got stuck in local minimum. This situation is likely to be exacerbated with a model with higher number of parameters. By using direct search method, we avoid this problem.

B. Attempt 1: apparent success but really a failure

We use data set A for model calibration and data set B for model verification. The resulting best-fit parameter values are shown in Table I.

parameter	least-squares	ML
$C_r(J/K)$	5×10^4	5×10^4
$R_w(K/W)$	6.44×10^{-4}	5.6×10^{-4}

TABLE I

BEST FIT PARAMETERS WHEN DATA SET A IS USED FOR MODEL CALIBRATION.

The room temperature predicted by the model with the best-fit parameter values and the measured room temperature are shown in Figure 7. As shown in the figure, the estimated model predicts well for both data set A and B, although they have quite different inputs profile (one during summer time, the other during winter time). However, prediction of the zone temperature of data set C with this calibrated model is extremely poor.

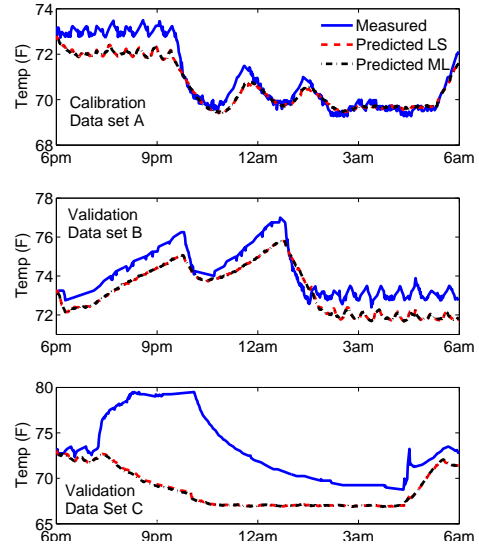


Fig. 7. Calibration/validation when data set A is used for calibration.

The reason for the failure was found to be the data. In the calibration data (data set A), the surrounding room temperatures are almost the same as the room we study. So the best fit resistance values are those that are so small that the room temperature essentially follows the temperature of the surroundings, leading to small prediction error. In validation data set B, though the room temperature profile is quite different from that in set A, the surrounding room temperatures are still close to the room temperature, so the model predicts well. However, in validation data set C, the surrounding room temperatures are significantly different from the room temperature, so it shows that the calibrated parameter values are incorrect.

C. Attempt 2: a more reliable calibration

Now we use data set C for calibration, which leads to best-fit parameter values shown in Table II. While in the previous case both methods yield a much lower value of the resistance compared to their ASHRAE values, this time the estimated resistance value is much closer to its ASHRAE value. The time-domain simulations for calibration and validation data sets are shown in Figure 8. From the figures, we can see that though the maximum prediction error is large (3^0F), this set

parameter	least-squares	ML
$C_r(J/K)$	7.8×10^5	4.7×10^5
$R_w(K/W)$	0.01	0.005

TABLE II

BEST FIT PARAMETERS WHEN DATA SET C IS USED FOR MODEL CALIBRATION.

of parameters predict the trend of the temperature well in all three data sets.

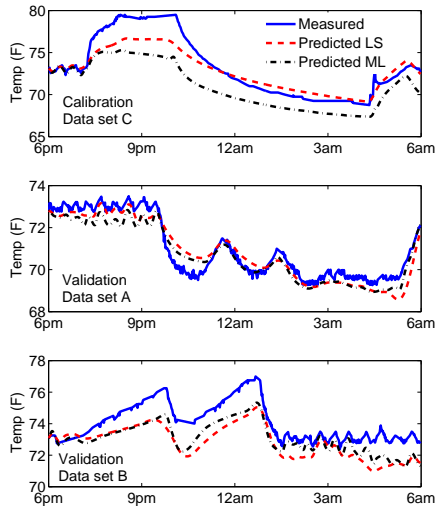


Fig. 8. Calibration/validation when data set C is used for calibration.

From the results, we see that calibration accuracy crucially depends on the data set chosen. This by itself is not surprising. What is surprising is that having large variation in the measured inputs (to have persistency of excitation) and outputs is not enough. The data should be chosen so that the output (room temperature) does not have a strong correlation with any of the surrounding temperatures. The features required in the data to ensure identification of parameters seem to be possible only through forced response experiments.

V. SUMMARY AND FUTURE WORK

We examined two questions for models of HVAC zones that can be used for predictive control: required model complexity and parameter identification. We examined models of varying complexity within the popular class of non-linear RC network models. By comparing low order models with high order ones, we conclude that a second order model reproduces the input-output behavior of the full-scale, 13th order model quite accurately. Even a first order model is accurate enough that it may very well suffice for the purpose of predictive control. Thus, complex models with high state dimension and large number of resistance/capacitance models are not needed.

The work reported here on parameter identification of low-order models from experimental data has revealed that calibrating the parameters of the R,C network model to closed-loop data from a building is likely to lead to grossly inaccurate parameter estimates, so that the resulting model is unlikely to be useful in predictive control. Even data with

sufficiently exciting input is not enough. The results reveal the features that the data should have to enable correct identification. It seems that these features can only be ensured through forced response tests. We have not incorporated effect of an open door so far. Future work will address this question, as well as adding other heat sources such as solar radiation and occupancy.

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