Effect of Various Uncertainties on the Performance of Occupancy-Based Optimal Control of HVAC Zones

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***Abstract*— Model Predictive Control (MPC) has emerged as a potential control architecture for operating buildings in a more energy efficient manner. We study through simulations the effect of several sources of uncertainty that arise in the implementation of MPC on the energy consumption, thermal comfort, and indoor air quality (IAQ). These include occupancy profile, measurement errors and mismatch between the plant and its model that the control algorithm uses. Simulations are carried out for two extreme cases: a winter day with no solar load and a summer day with high solar load. The study shows that increasing fluctuations in occupancy, errors in measuring occupancy, and model mismatch have the strongest impact on the energy consumption. However, measurement errors in outside temperature and solar load does not have significant impact. Therefore, it is possible to improve the controller performance by using more accurate occupancy sensors. Furthermore, implementation cost can also be reduced by eliminating the sensors and prediction algorithms for predicting outside temperature and thermal loads without compromising the controller performance. Even with these uncertainties, MPC delivers $12-37\%$ reduction of energy use over conventional control methods without affecting thermal comfort and IAQ.**

I. INTRODUCTION

Buildings are one of the primary energy consumers worldwide. Inefficiencies in the operation of HVAC (Heating Ventilation and Air-Conditioning) system are major causes for the large energy consumption of buildings. A possible way to improve the efficiency of HVAC systems is through the use of novel control algorithms. Improving efficiency through change in the control algorithm alone has a strong economic incentive: retrofitting existing buildings with new control software is far cheaper than redesigning the buildings to be more efficient.

Several recent studies have explored the use of model-based control to reduce energy use, particularly Model Predictive Control (MPC) based techniques [1], [2], [3], [4], [5], [6]. These papers show that MPC can reduce energy use significantly compared to conventional control schemes that are currently used to operate buildings, such as rule based control or single-maximum control [7].

Conventional control methods are easy to implement since they are pure feedback strategies based on measured temperature. MPC, in contrast, requires additional information such as model of the plant and predictions of exogenous inputs. While the studies mentioned above indicate that MPC does have potential to increase energy efficiency, they have examined the performance of MPC under idealized assumptions, such as perfect knowledge of exogenous inputs and accurate knowledge of plant dynamics. It is therefore important to examine the sensitivity of MPC performance to various sources of uncertainty, since the thermal dynamics of buildings are complex and highly uncertain.

The effect of plant-model mismatch and occupancy prediction on the controller performance is studied in [1], [5], [6]. Plant-model mismatch refers to the difference between the process dynamics, and model of the plant that the MPC controller uses for optimization and/or state estimation. These papers only consider temperature control, whereas humidity and IAQ are also important factors in ensuring the comfort and health of occupants [8]. The paper [1] examines the effect of occupancy information available on the energy consumption. It also discusses the effect of total occupancy period on the energy consumption, but not the effect of occupancy fluctuations. In [5], parameters of the model are perturbed from their nominal values to determine the critical parameters that have the most effect on energy consumption. The paper [6] studies the effect of a fixed increase/decrease in window and wall resistances.

The purpose of this paper is to examine the sensitivity of MPC performance to uncertainties introduced by plant-model mismatch and errors in predictions of exogenous inputs through extensive simulations. The MPC-based control scheme proposed in our earlier work [2], [9] is chosen for the sensitivity study undertaken in this paper. Perfect knowledge of plant was assumed in [2], [9], so that there was no plant-model mismatch. In [2], full state information was assumed available to the MPC controller. These are common assumptions in the recent papers on MPC for green buildings [1], [4]. A key exogenous variable whose prediction is required is occupancy (number of occupants). Significant amount of energy savings can be obtained by relaxing the constraints on space temperature and/or humidity when the zone is unoccupied [1], [2]. It is shown in [2], [9] that a significant amount of energy can be saved by using only measurements.

In this study, we introduce various uncertainties in the prediction of exogenous inputs and plant-model mismatch. Since occupancy is a key exogenous input, it is likely that error in the prediction of occupancy will have a significant effect on the performance of MPC controller. In the context of the prediction method used in [2], [9], even if occupancy measurements are noise free, such a prediction will be frequently erroneous if occupancy in the zone changes frequently. This raises the question of sensitivity to *occupancy profile*. Namely, if a zone A is occupied for 8 hours

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continuously, does it consume the same amount of energy as when it is occupied sporadically but with a total occupied duration of 8 hours? In addition, occupancy measurements will be corrupted with noise, which may lead the controller to believe that a zone is occupied when it is not and vice versa. We therefore study how MPC performance varies with occupancy profile and measurement noise. We study plant-model mismatch by changing the resistance of the window and the door in the process dynamics (which is likely in practice when the door and/or the window is opened), while the controller uses the nominal model with fixed parameters.

The overall conclusion of the study is that the MPC scheme of [2], [9] is quite robust to these uncertainties, though the degree of robustness varies. It is seen to be most sensitive to occupancy profile (for the same total occupied duration), occupancy measurement errors, and plant-model mismatch. These factors can lead to energy use increase of up to 35% over the baseline MPC case (with no uncertainty) with no significant increase in occupant discomfort. In the worst case, the MPC controller yields a 12% energy savings compared to the conventional (single-maximum) control strategy. MPC is observed to be highly robust to uncertainties in measurements of outside temperature and solar load. The overall robustness of MPC to uncertainty indicates that MPC is a good candidate for building control, in spite of high degree of uncertainty in building dynamics.

The rest of paper is organized as follows. The model used to simulate the zone, and the models used in MPC/Kalman filter are described in Section II. Section III briefly describes the control algorithm. Section IV explains how the uncertainty in occupancy measurements, inputs predictions and model mismatch are chosen. Simulation results with the effect of occupancy type, model and measurements uncertainty are shown in Section V. Section VI concludes the paper and discusses ways to extend this work in future.

II. Control Architecture

A common configuration of HVAC systems used in modern buildings is the so-called variable-air-volume (VAV) system with a reheat box. In this configuration, cold and dry air is supplied by the Air Handling Unit (AHU) to the VAV box, which may heat up the air before supplying it to the zone. The VAV box also controls the flow rate of the air supplied to the zone. For the sake of simplicity, we assume that one AHU supplies air to only one zone.

The vector u of controllable input signals and v of measured exogenous inputs to a zone are

$$u = [m^{in}, T^{in}],\ v = [Q^p, Q^s, T_0, W^{in}]^T, \qquad (1)$$

where m^{in}, T^{in} and W^{in} are the flow rate, temperature and humidity ratio of supply air, respectively, Q^p is the heat gain due to occupants, Q^s is the solar load gain, and T_0 is the outside temperature.

Figure 1 shows a generic scheme for implementation of MPC in a zone. Noisy measurements of outside temperature, solar radiation, occupancy, zone temperature and humidity are used for state estimation and prediction of exogenous inputs for a fixed time horizon into the future. These predictions, together with a model of the zone's hygro-thermal dynamics are used in the computation of the optimal values of the control inputs.

The total power consumption (P_T), which includes fan power (P_F), reheating power (P_R) and conditioning power (P_U) can be written in the following form

$$P_T \triangleq P_F + P_U + P_R = g(u, T^{AHU}, W^{AHU}), \qquad (2)$$

where T^{AHU} and W^{AHU} are the temperature and humidity ratio of air being supplied by the AHU respectively. Details of the power model can be found in [2], [9].

MPC requires a model of the dynamics to perform optimization. A model is also needed for state estimation since MPC needs full state information. For state estimation and control computation, approximations of a *nominal* process model is used, which is denoted by M_0, and is referred to as "belief model". In studying the effect of plant-model mismatch, a process model M_δ is used for simulating the process, which may differ from the nominal model. The scalar δ is used to indicate how different the process model is from the belief model. These models are described below.

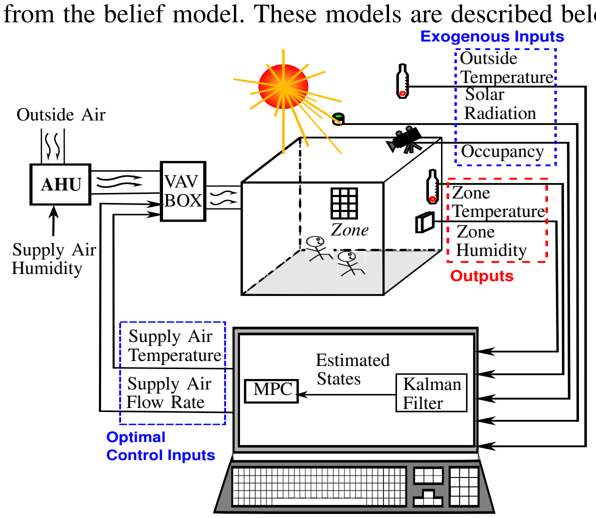


Fig. 1. Implementation of model predictive control for zone-level control.

A. Process dynamics

A model of a building's thermal dynamics can be constructed by combining elemental models of conductive interaction (RC networks) between two spaces separated by a solid surface such as a wall, as well as heat exchange due to supply and extract air. Humidity dynamics can be derived from mass balance. The resulting model of the hygro-thermal dynamics is a set of coupled nonlinear ODEs

$$\dot{T} = AT + Bv + f(T_1, W, u, v), \quad \dot{W} = g(T_1, W, u, v), \qquad (3)$$

where the vector $T \in \mathbb{R}^n$ consists of zone temperature (T_1) and the temperatures of the nodes interior to the walls, $W \in \mathbb{R}$ is the humidity ratio of the air in the zone; see [10] for details.

The plant model (3) with nominal parameter values is represented by M_0, while a model in which these parameters are different from their nominal values is denoted by M_δ. The scalar δ represents the degree of variation of the parameters from their nominal values.

B. "Belief Models" for state estimation and optimization

The belief model M_{0r} is an approximation of the nominal process model M_0 that is obtained by applying the model reduction technique of [10] to M_0. The interested reader is referred to [10] for details. The reason for using a lower order approximation is that not only does it reduce computation complexity but that the full order model M_0 is found to be unobservable for certain resistance/capacitance values, while the reduced model M_{0r} is observable by construction.

All of the models M_δ, M_0, M_{0r} are continuous time models. However, to solve the optimization problem numerically, some form of discretization is needed. We use a discrete-time version of the model M_{0r} for this purpose, which is denoted by $M_{0r}^{(d)}$. The model $M_{0r}^{(d)}$ is obtained by discretizing M_{0r} using Euler Forward method [11]. The resulting discrete time model is of the form $x_{k+1} = f_k(x_k, u_k, v_k)$, where $k = 0, 1, \ldots$ corresponds to the discrete time index, Δt is the time period between index k and $k+1$, and the state x_k is the value of temperature vector and zone humidity at time k, while u_k, v_k are the controllable and exogenous inputs, respectively, as described in (1), at time k.

The block diagram for the MPC implementation is shown in Figure 2. As shown in the Figure 2, the process model uses the true values for exogenous inputs, while the noisy occupancy measurements and uncertain predictions of outside temperature and solar load are used by MPC/Kalman filter. In the Figure 2, n_ε^p represents the noisy occupancy measurements from sensor. The subscript ε represents the variation of occupancy measurements from their true values, e.g. n_0^p represents the true occupancy value.

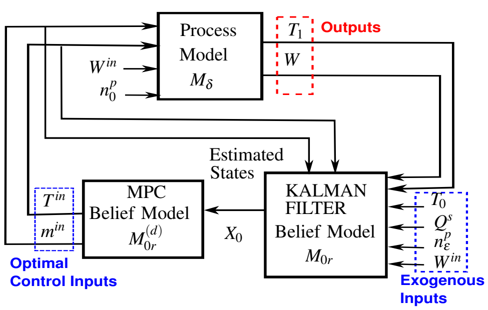


Fig. 2. Block diagram of MPC implementation for a zone-level control.

III. Control Algorithm

A. MPC Algorithm

The MPC based controller in [2], [9] seeks to reduce energy consumption by maintaining temperature, humidity and IAQ. The optimal control inputs are computed for a discrete time horizon of length K, but only the first of these K samples is executed. It is assumed that control inputs are constant for m time indices, so that only $z = K/m$ control inputs have to be computed for a horizon of length K. After m time steps, the whole process is repeated.

The control logic is divided into two modes: (i) Unoccupied, and (ii) Occupied. If the measured occupancy at time index k, i.e., at the beginning of the k-th time interval of length Δt, is observed to be 0, then the prediction of the occupancy for the future is set at 0, and the controller turns on the unoccupied mode. The occupied mode is turned on if the measured occupancy of the zone is at least 1 at the k-th time index. In this mode, the occupancy prediction for the future is set to be the value measured at the k-th time index.

1) Unoccupied Mode: The optimal control inputs for the next K time indices are obtained by solving the following optimization problem:

$$arg \min_{m^{in}, T^{in} \in \mathbb{R}^z} \sum_{i=k}^{k+z} P_T(im) \tag{4}$$

subject to the following constraints:

$$\left.\begin{array}{r} T_{low}^{unocc} \leq T(im) \leq T_{high}^{unocc} \\ m_{low}^{in} \leq m^{in}(im) \leq m_{high}^{in} \\ T^{AHU} \leq T^{in}(im) \leq T_{high}^{in} \\ W_{low}^{unocc} \leq W(im) \leq W_{high}^{unocc} \end{array}\right\} \forall i = k, \ldots, k+z,$$

where T_{low}^{unocc}, T_{high}^{unocc}, W_{low}^{unocc}, W_{high}^{unocc} and m_{low}^{in} are design variables, whereas m_{high}^{in}, and T_{high}^{in} are actuator constraints.

2) Occupied Mode: In this mode, the optimal control inputs for the next K time indices are obtained by solving the following optimization problem.

$$arg \min_{m^{in}, T^{in} \in \mathbb{R}^z} \sum_{i=k}^{k+z} P_T(im) \tag{5}$$

subject to the following constraints:

$$\left.\begin{array}{r} T_{low}^{occ} \leq T(im) \leq T_{high}^{occ} \\ T^{AHU} \leq T^{in}(im) \leq T_{high}^{in} \\ m_{low}^{in} + m_p^{in} n^p(im) \leq m^{in}(i) \leq m_{high}^{in} \\ W_{low}^{occ} \leq W(im) \leq W_{high}^{occ} \end{array}\right\} \forall i = k, \ldots, k+z,$$

where T_{low}^{occ}, T_{high}^{occ}, m_p^{in}, W_{low}^{occ} and W_{high}^{occ} are design variables. Note that IAQ is always maintained by the design of minimum flow rate, which is provided as a constraint to the control algorithm; see [2], [9] for details.

To perform the optimization in both occupied and unoccupied modes, the controller needs predictions of the exogenous input vector $v = [Q^p, Q^s, T_0, W^{in}]^T$ over the time horizon of optimization and the initial state of the belief model $M_{0r}^{(d)}$ (i.e., the state at the current time index). Prediction of the Q^p depends only on the occupancy, i.e., $Q^p = \alpha n_\varepsilon^p$, where α is specified by ASHRAE handbook [8]. Prediction of T_0, Q^s is assumed available from weather forecasts, while $W^{in} = W^{AHU}$ is assumed constant. The state is estimated using a continuous-discrete extended Kalman filter that uses the belief model M_{0r}. We use output measurements sampled at the end of the interval to estimate the state.

B. Performance metrics

The energy used over a period T is $E := \int_0^T P_T(t)dt$. We refer to the predicted energy use when the model predictive controller is used in the ideal conditions (no plant-model mismatch, full state information available without need for state estimation, and perfect measurement of occupancy is available) as the *baseline MPC energy use*, and denote it by E_0^{MPC}. An energy related performance metric is the

$$\Delta E^{MPC} := (E^{\mathrm{MPC}} - E_0^{\mathrm{MPC}})/E_0^{\mathrm{MPC}}. \qquad (6)$$

where E^{MPC} is the predicted energy use with MPC, when any uncertainty is present there.

We divide the discomfort level into two parts 1) Temperature Discomfort (D_T), and 2) Humidity Discomfort (D_H). The instantaneous discomfort is deviation of the temperature/humidity from the allowed range during the occupied time. During the unoccupied mode, D_T is considered 0 since there is no one in the zone. The expressions for D_T and D_H are written below

$$D_T = \left\{ \begin{array}{c} T_1(t) - T_{low}^{occ}, \text{ if } T_1(t) < T_{low}^{occ} \text{ and } n_0^p(t) \neq 0 \\ T_1(t) - T_{high}^{occ}, \text{ if } T_1(t) > T_{high}^{occ} \text{ and } n_0^p(t) \neq 0 \\ 0, \text{ if } n_0^p(t) = 0 \end{array} \right\},$$

$$D_H = \left\{ \begin{array}{c} W(t) - W_{low}^{occ}, \text{ if } W(t) < W_{low}^{occ} \text{ and } n_0^p(t) \neq 0 \\ W(t) - W_{high}^{occ}, \text{ if } W(t) > T_{high}^{occ} \text{ and } n_0^p(t) \neq 0 \\ 0, \text{ if } n_0^p(t) = 0 \end{array} \right\}.$$

Total temperature discomfort ($D_T^\star$) and total humidity discomfort ($D_H^\star$) over a T time period can be written as

$$D_T^\star = \int_0^T D_T dt, \; D_H^\star = \int_0^T D_H dt. \qquad (7)$$

IV. Types of uncertainties studied

In this section, we describe the various uncertainties whose effect on MPC performance metrics we wish to study.

A. Occupancy profile

We assume that the total time occupied by the occupants is constant. However, number of times occupants leave or enter the zone is varied. Consider a typical schedule of a person in an office, which is shown in Figure IV-A a). In Figure IV-A a), a person enters the office at 8 AM and leaves the office at 12 PM for lunch, the person comes back to the office at 1 PM and leaves it at 5 PM. In this case, the person enters or leaves the office 4 times in a day, which we call "No. of changes" (N^c). Similarly, in Figure IV-A b), the no. of changes is 8: $N^c = 8$.

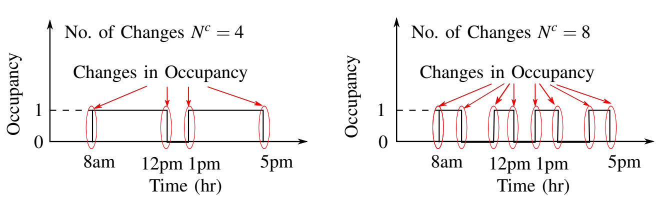


Fig. 3. Multiple occupancy profiles for fixed occupied time and different no. of changes (N^c).

B. Occupancy measurement error

The controller uses occupancy measurements (n_ε^p), which are invariably corrupted by sensor noise. To study the effect of this noise, we let the state estimator and optimization routine in the controller use the noisy occupancy measurements n_ε^p while the true occupancy value n_0^p is used in computing predictions of the process model M_δ.

When the zone is not occupied, $Pr(n_\varepsilon^p = 0) = Pr(n_\varepsilon^p = 1) = 0.5$, where $Pr(.)$ denotes probability. During the occupied time, $Pr(n_\varepsilon^p = 0) = Pr(n_\varepsilon^p = 1) = Pr(n_\varepsilon^p = 2) = 1/3$. This model produces occupancy measurements (n_ε^p) as 0 or 1 when no one is there, while it produces occupancy measurements as 0, 1 or 2 when someone is there.

C. Plant-model mismatch and input uncertainty

1) Plant-Model Mismatch: The plant-model mismatch we study arises due to the uncertainty in the thermal resistance and capacitance values of wall, window, etc. The uncertainty in these parameters are mainly due to two reasons: a) infiltration/exfiltration due to the gap or leakage through window or opening door of the zone, and b) aging of the wall material. Leakage through the window or opened door causes the thermal resistance values to change by a large factor but not the thermal capacitances. However, aging causes comparatively small change in the thermal resistances and capacitances. We only study the change in thermal resistance values due to opened door and/or window, which is comparatively much larger than the thermal resistance change due to aging. It is also assumed that thermal capacitances do not change when door or window is opened.

The perturbed resistance values of door (R_δ^d) and window (R_δ^w) used by the process model, M_δ, can expressed as

$$R_\delta^d = R_0^d(1 + W_\delta), \; R_\delta^w = R_0^w(1 + D_\delta), \qquad (8)$$

where $-1 < W_\delta < 1$ and $-1 < D_\delta \leq 1$ are the % change in the thermal resistance values of the door and window, respectively. R_0^d and R_0^w are the thermal resistances of the door and window, respectively, when there is no uncertainty in the model.

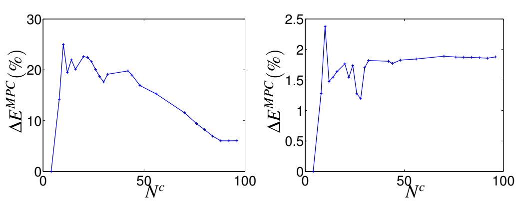
2) Input uncertainty: We also study two other types of uncertainty: errors in the prediction of outside temperature T_0 and solar load Q^s. Uncertainty in T_0 is studied by providing a constant value to the controller that is different from the true value that the process is subjected to. Similarly, uncertainty in the load is introduced by providing a load forecast of 0 in the controller, while using a non-zero load in the process model. This corresponds to a situation when there is no weather forecast available.

V. Results

Simulations are carried out for a model of a zone from the second floor in a building (Pugh Hall) at the University of Florida campus, Gainesville, FL. Numerical results presented here are obtained from simulations conducted in MATLAB©. Simulations are carried out for two extreme boundary conditions, in a a) winter and, b) summer day; see [12] for the details of inputs, building, model and design parameters.

A. Effect of occupancy profile

Figure 4(a) and 4(b) show the % change in the total energy consumption in a day over the nominal case as a function of number of fluctuations in the occupancy during the summer and winter day, respectively. It is clear from the both the figures that the energy consumption change increases first till $N^c = 10$, and then decreases and becomes constant later. The first increase in the energy consumption is due to the controller fluctuating between the occupied and unoccupied mode. The maximum energy consumption is at $N^c = 10$. This is because the time period between the fluctuations is large enough for the zone temperature to attain the lowest allowed value during the unoccupied mode. When a person enters a zone, maximum power is used to bring the zone temperature back in the comfortable range as quickly as possible. However, if the fluctuations are more frequent, the zone temperature is not able to drift very far away from the comfortable range. This leads to the increase in energy consumption becomes constant. The % change in the energy consumption during the summer is smaller than that during the winter. This is because AHU contributes a major portion of energy to the total energy consumption and the controller is mostly in the cooling mode. The total temperature and humidity discomfort are quite low (i.e. very close to the comfortable region) during the winter and summer day, which are not shown here due to the space limit.



(a) Winter (b) Summer

Fig. 4. Percentage change (ΔE^{MPC}) in energy consumption as a function of the number of the changes in the occupancy (N^c) during the a) Winter, and b) Summer day, where the total occupancy time period is 8hrs.

B. Effect of occupancy measurements error

Simulation results for the winter day are shown in Figures 5-6. In the figures, mean (μ) is represented by the solid line, while the dashed line represents a band of three times standard deviation ($\pm 3\sigma$) from the mean value, which we call variation, i.e. "variation" = $\pm 3\sigma$. Fifty (50) random numerical experiments are used to compute the mean and variation in studying the effect of occupancy measurement error. Figure 5 shows the mean (μ) and variation ($\pm 3\sigma$) of temperature and humidity discomfort during the winter, which are very close to zero. This guarantees good thermal comfort in the zone. Figure 6 shows the mean (μ) and variation ($\pm 3\sigma$) of the total power consumption and occupancy. The mean and variation of total energy consumed in a day are 34.56MJ and 0.1MJ, respectively. However, the energy consumed (E_0^{MPC}) during the nominal winter day is 29.13MJ when there is no error in the occupancy measurement. Therefore, there is an increase of 18% in the total energy consumption in a day.

Simulation results for the summer day are shown in Figures 7-8. It is explicit from Figure 7 that IAQ and comfort level is not compromised. Mean (μ) and variation ($\pm 3\sigma$)

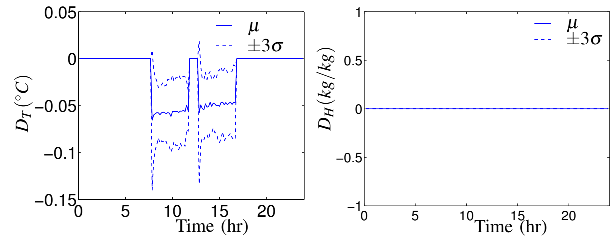


Fig. 5. Mean (μ) and variation ($\pm 3\sigma$) of the temperature discomfort (D_T) and humidity discomfort (D_H) during the winter, when the outside temperature is 15.56°C and no solar radiation is present.

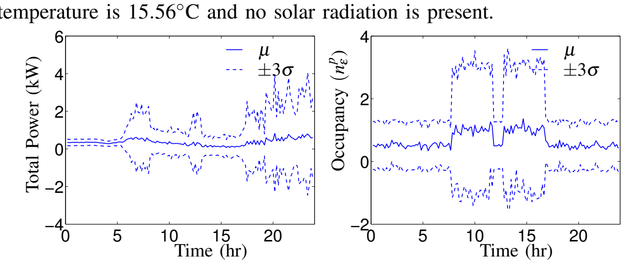


Fig. 6. Mean (μ) and variation ($\pm 3\sigma$) of the total power (kW) and occupancy during the winter, when the outside temperature is 15.56°C and no solar radiation is present.

of the power consumption and occupancy are shown in Figure 8. The power consumption is more during the summer because a lot of energy is consumed at the AHU to cool down the hot outside air from 29.44°C to 12.8°C.

The mean (μ) and variation ($\pm 3\sigma$) of the total energy consumption in day during the summer is 124.47MJ and 0.24MJ. However, the total energy consumption (E_0^{MPC}) during the nominal summer day is 106.84MJ when there is no measurement error in occupancy. Hence, there is an increase of 16.5% in the total energy consumption in a day. The change in energy consumption during the summer is close to the energy consumption increase during the winter.

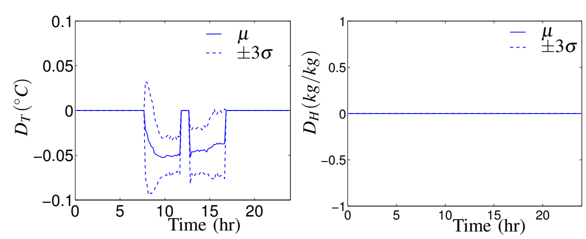


Fig. 7. Mean (μ) and variation ($\pm 3\sigma$) of the temperature discomfort (D_T) and humidity discomfort (D_H) during the summer, when the outside temperature is 29.44°C and solar load is present.

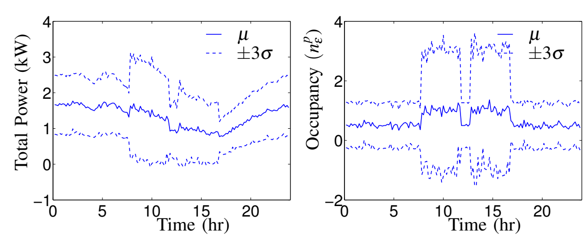
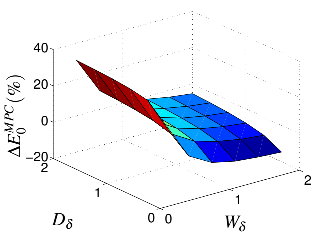


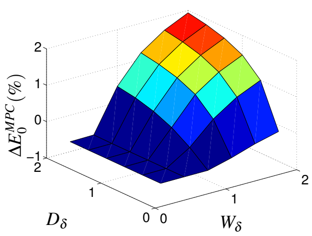
Fig. 8. Mean (μ) and variation ($\pm 3\sigma$) of the total power (kW) and occupancy during the summer, when the outside temperature is 29.44°C and solar load is present.

C. Model Mismatch

In this section, resistance values of the door and window are varied as mentioned in the Section IV-C.1. The parameters D_δ and W_δ are varied from -0.67 to 1, and surfaces for the % change in the total energy consumption (ΔE_0^{MPC}) are plotted as shown in Figures 9(a) and 9(b) during the winter and summer day, respectively. Figure 9(a) shows that there is a decrease in the total energy consumption during the winter, when W_δ is increased and D_δ is decreased. This is because the surroundings temperature is maintained at 22.22°C, and decreasing D_δ means easy transfer of energy through the walls. However, when W_δ is decreased there is more interaction with the cold outside weather. Hence, more heat is being supplied to maintain the comfortable temperature in the zone. Figure 9(b) shows that there is not much change in the energy consumption during the summer. This is because reheating mode is mostly not turned on and only outside fresh air is being circulated. The temperature and humidity discomfort values in a day are very low during the winter and summer, which ensures good thermal comfort in the zone. We don't see any significant change in the total energy consumption or discomfort level due to the uncertainty in solar load and outside temperature. This is because of the high resistance of the window; see [12] for the results in detail.



(a) Winter



(b) Summer

Fig. 9. Percentage change in the total energy consumption (ΔE_0^{MPC}) in a day as a function of parameter variation during the a)Winter, and b) Summer.

Note that we did not study the effect of any of these uncertainties on IAQ, since the bounds on the mass flow rate chosen in the MPC controller assures that good IAQ will be maintained at all times, occupied or not.

VI. Discussion and future work

We examine the effect of multiple occupancy profiles, error in the occupancy measurements, outside temperature and solar load predictions, as well as model uncertainty on the total energy consumption and thermal comfort with MPC. We conclude that the energy consumption of the MPC controller is most sensitive to occupancy profiles, occupancy measurements error and plant-model mismatch, although the controller is quite robust to errors in outside temperature and solar load predictions. The controller is able to maintain IAQ and thermal comfort in the zone in all the cases.

Plant-model mismatch due to the opened door or leakage through the window changes the energy consumption by 35% over the baseline MPC case (with no uncertainty). Increasing the occupancy fluctuations increases the total energy consumption by at-most 25% over the baseline case. While uncertainty in the occupancy measurements causes the total energy consumption to increase by 18% and 16.5% during the winter and summer day, respectively, over the baseline case. Uncertainty in the solar load and outside temperature predictions does not cause significant increase in the energy use over the baseline case (with no uncertainty). In the worst case, the MPC controller yields a 12% energy savings compared to the conventional (single-maximum) control strategy, while in the best case, the MPC controller results in 37% energy savings.

We conclude that the MPC controller's performance is sufficiently robust to uncertainties to make it a good candidate for building control. Even with uncertainties, the MPC controller is able to provide energy savings of $12-37\%$ over the conventional (single-maximum) controller while maintaing thermal comfort and IAQ. High accuracy occupancy sensors can make energy savings of MPC over convention control more consistent. Furthermore, it is possible to reduce the cost of MPC implementation by eliminating sensors and prediction algorithms for measuring and predicting outside temperature and solar load.

The avenues of future work are: i) include the multi-zone interactions while solving the optimization problem and studying sensitivity, ii) consider return air recirculation while minimizing energy consumption, iii) study the effect of other design parameters used in the control algorithm, and finally, iv) implement the control algorithm in a real building and analyze the controller performance.

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