Mean-Field Control for Energy Efficient Buildings

Kun Deng, Prabir Barooah, and Prashant G. Mehta

***Abstract*— In this paper, we consider the problem of distributed set-point temperature regulation in a large building. With a large number of zones, the problem becomes intractable with standard control approaches due to the large state space dimension of the dynamic model. To mitigate complexity, we develop here a mean-field control approach applicable to large-scale control problems in buildings. The mean-field here represents the *net effect* of the entire building envelope on any individual zone. Rather than solving the large-scale centralized problem, we explore distributed game-theoretic solution approaches that work by optimizing with respect to the mean-field. The methodology is illustrated with a numerical example in a simulation environment.**

I. Introduction

Buildings are one of the primary consumers of energy. In the United States, buildings are responsible for 30% of energy consumption, and 71% of electricity consumption, while accounting for 33% of CO_2 emissions [1]. A large amount of the energy consumed in buildings is wasted. A major reason for this wastage is *inefficiencies* in the building technologies, particularly in operating the HVAC (heating, ventilation and air conditioning) systems. These inefficiencies are in turn caused by the manner in which HVAC systems are currently operated. The temperature in each zone is controlled by a local controller, without regards to the effect that other zones may have on it or the effect it may have on others. Substantial improvement may be possible if inter-zone interactions are taken into account in designing control laws for individual zones.

In fact, there is a growing interest in optimal control methods to minimize building-wide energy consumption based on dynamic models [2]–[5]. Such control techniques require a model of the transient thermal dynamics of the building that relates the control signals to the space temperature of each zone. A challenge in developing such techniques is the *complexity* of the underlying models due to large dimension of state-space and a large number of control objectives. A model based on the first-principles will be a large set of coupled PDEs, which is intractable in general. Even the so-called reduced order models that rely on a lumped resistor-capacitor analogy of walls and windows lead to models with large state space. For example, a reduced order lumped paramater model for a medium-size commercial building with about 100 zones will have a state dimension close to 1000 [6].

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In this paper, we propose a *decentralized* optimal control strategy for the zones of a multi-zone building where model complexity is mitigated by using a two pronged approach. First, we use recently developed aggregation-based model reduction techniques [7] to construct a reduced-order model of the multi-zone building's thermal dynamics. Second, we use the mean-field intuition from statistical mechanics so that the effect of other zones on a particular zone is captured though a *mean-field* model [8]. Then the whole model (even the reduced model) does not have to be used in computing the controls over short time scales.

By using the mean-field idea, we cast the control problem as a game, whereby each zone has its own control objective modeled as set-point tracking of the local (zonal) temperature. In general, the control problem quickly becomes intractable for even a moderate number of competing objectives. In order to mitigate complexity, we employ the Nash Certainty Equivalence principle to obtain a mean-field description [9]. The mean-field here represents the *net effect* of the entire building envelope on any individual zone. A local optimal zonal control is designed based on the local model of thermal dynamics and its interaction with the building via the mean-field. A consistency relationship is used to enforce the mean-field in a self-consistent manner. The methodology is shown to yield distributed control laws that can easily be implemented on large-scale problems.

We compare the performance of the proposed controller with that of a PI controller. Controllers currently used in commercial buildings use a combination of discrete logic and PID type controllers. Simulations show that the proposed scheme achieves comparable temperature tracking performance while reducing energy consumption by reducing the mass-flow rates entering the zones.

II. Building Thermal Model

A. Configuration of HVAC system

A typical multi-zone HVAC system used in modern buildings is the Variable-Air-Volume (VAV) system. Such a system supplies air at a constant temperature. The airflow to the zones is controlled based on room thermal load requirements. Figure 1 depicts the configuration of a four-zone building equipped with a VAV system: Upstream, an air handling unit (AHU) conditions air by passing it across the cooling coil. A series of ducts is used to supply the cold and dry air to VAV boxes for each of the downstream zones. Each VAV box contains a local controller, tasked with maintaining

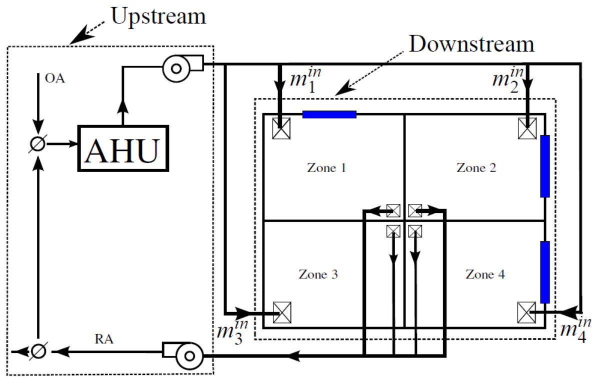

Fig. 1. The configuration of a four-zone building HVAC system.

the zone temperature at a specified value, by controlling the mass flow rate of air supplied to the zone using dampers. The total air flow rate of the entire system therefore varies with time. Zonal controllers at VAV boxes may also apply "reheat" (adding heat) or "pre-cooling" (removing heat) to the conditioned air before supplying it to the zone.

B. Baseline building thermal model

A building thermal model is constructed by combining elemental models of conductive interaction between two zones separated by a solid surface (e.g., walls, windows, ceilings, and floors). A lumped parameter model of conduction across a surface is assumed to be a RC-network, with current and voltage being analogs of heat flow and temperature [10].

The resulting model can be described by an *undirected graph* $G = (\mathcal{V}_0, \mathcal{E})$, where $\mathcal{V}_0 := \{0\} \cup \mathcal{V}$ denotes the set of *nodes* of the graph. The node $\{0\}$ denote the outside and the set $\mathcal{V} := \{1, \dots, n\}$ denotes the building nodes. The nodes are so indexed that the first N nodes of $\mathcal{V}$ correspond to the zones $1, \dots, N$, and these are called the *zone nodes*. The next $(n - N)$ nodes of $\mathcal{V}$ correspond to the internal points of the surfaces. These are called the *internal nodes*. An edge (i, j) exists between nodes i and j if there is a resistance connecting them directly. The set $\mathcal{E} \subset \mathcal{V}_0 \times \mathcal{V}_0$ is the set of all *edges*. Therefore each edge (i, j) has an associated *thermal resistance* $R_{ij} \in \mathbb{R}_+$. Since the graph is undirected, $R_{ji} = R_{ij}$ by convention. Each node $i \in \mathcal{V}$ has an associated thermal capacitance C_i.

The states and inputs of the building thermal model are summarized below:

$$\text{States}: T_1, \dots, T_N, T_{N+1}, \dots, T_n$$
$$\text{Inputs}: T_0, T^s; \dot{m}_i^{in}, \dot{Q}_i^r, \dot{Q}_i^{int}, \dot{Q}_i^{ext}, i = 1, \dots, N$$

where $T_1, \dots, T_N$ denote the space temperature of the zones, $T_{N+1}, \dots, T_n$ denote the temperature of the points internal to the surface elements, T_0 denotes the outside temperature, T^s denotes the temperature of the air supplied by the AHU, $\dot{m}_i^{in}$ denotes the mass flow rate of the supply air entering the ith zone, $\dot{Q}_i^r$ denotes the rate of heat due to reheat, $\dot{Q}_i^{int}$ denotes the rate of heat generated by occupants, equipments, and lights in the ith zone, and $\dot{Q}_i^{ext}$ denotes the rate of solar radiation entering the ith zone.

The supplied air temperature T^s is usually constant for a VAV system, at least over short intervals of time. All other inputs are time varying. In this paper we assume that (estimates of) the outside temperature T_0 and the heat gains $\dot{Q}_i^r, \dot{Q}_i^{int}, \dot{Q}_i^{ext}$ are available based on historical data, weather predictions, and various sensors.

The thermal dynamics of a multi-zone building, described by a graph G, is represented by the following coupled differential equations: For each $i \in \mathcal{V}$,

$$C_i \dot{T}_i(t) = \dot{Q}_i(t) + \Delta H_i(t) + \sum_{j \in \mathcal{N}_i} (T_j(t) - T_i(t))/R_{ij}$$

where $\mathcal{N}_i := \{j \in \mathcal{V}_0 : j \neq i, (i, j) \in \mathcal{E}\}$ denotes the set of *neighbors* of the node i. The *heat gain* $\dot{Q}_i$ is the rate of thermal energy entering the node i from external sources, other than ventilation air and conduction from neighboring nodes:

$$\begin{aligned} &\dot{Q}_i(t) = \dot{Q}_i^r(t) + \dot{Q}_i^{int}(t) + \dot{Q}_i^{ext}(t), && i = 1, \dots, N \\ &\dot{Q}_i(t) = 0, && i = N+1, \dots, n. \end{aligned}$$

The *ventilation heat exchange* ΔH_i is the rate of thermal energy entering the node i due to ventilation:

$$\begin{aligned} &\Delta H_i(t) = C_{pa} \dot{m}_i^{in}(t)(T^s - T_i(t)), && i = 1, \dots, N \\ &\Delta H_i(t) = 0, && i = N+1, \dots, n \end{aligned}$$

where C_{pa} is the specific heat capacitance of the supplied air at constant pressure. In the following, the mass flow rate $\dot{m}^{in}$ is the control input u, i.e., $u_i = \dot{m}_i^{in}$ for $i = 1, \dots, N$.

C. Reduced building thermal model

In this section, we describe a reduced-order building thermal model by using an aggregation technique [7]. The reduced models will be used in Section III to develop the mean field control strategies. To obtain the reduced model, we aggregate a subset of nodes into *super-nodes*. Mathematically, suppose we want to reduce the state space dimension from n to m, where $m \leq n$ is the (user-specified) number of super-nodes. The first step is to determine a *partition function* $\phi : \mathcal{V} \to \bar{\mathcal{V}}$, where $\bar{\mathcal{V}} := \{1, \dots, m\}$ such that ϕ is onto but possibly many-to-one. The elements of $\bar{\mathcal{V}}$ are the super-nodes, and for every $k \in \bar{\mathcal{V}}$, the node set $\phi^{-1}(k) \subset \mathcal{V}$ includes the nodes in the baseline model that are aggregated into the kth super-node. Similar to the baseline model, we let $\{0\}$ denote the outside node and define the set $\bar{\mathcal{V}}_0 := \{0\} \cup \bar{\mathcal{V}}$.

Given a fixed m-partition function ϕ, we introduce the following quantities for the reduced model:

- The *super-capacitance* of the kth partition is the combination of all capacitances of the nodes in kth partition:
$$\bar{C}_k^{(\phi)} := \sum_{i \in \phi^{-1}(k)} C_i, \quad k \in \bar{\mathcal{V}}.$$
- The *super-resistance* between kth and lth partitions is the parallel-equivalence of all resistances connecting the nodes between two partitions:
$$\bar{R}_{kl}^{(\phi)} := \frac{1}{\sum_{i \in \phi^{-1}(k)} \sum_{j \in \phi^{-1}(l)} 1/R_{ij}}, \quad k \neq l \in \bar{\mathcal{V}}.$$

- The *super-load* of the kth partition is the combination of all thermal loads for the zones in the kth partition:

$$\dot{\bar{Q}}_k^{(\phi)}(t) := \sum_{i\in\phi^{-1}(k)} \dot{Q}_i(t), \quad k \in \bar{\mathcal{V}}.$$

The reduced-order model is also a RC-network defined on super-nodes with super-edges connecting these super-nodes. Its thermal dynamics is represented by the following coupled differential equations: For each $k \in \bar{\mathcal{V}}$,

$$\begin{aligned}\bar{C}_k^{(\phi)}\dot{\bar{T}}_k(t) = \dot{\bar{Q}}_k^{(\phi)}(t) + \Delta\bar{H}_k^{(\phi)}(t) \\ + \sum_{l\in\bar{\mathcal{N}}_k}(\bar{T}_l(t) - \bar{T}_k(t))/\bar{R}_{kl}^{(\phi)}\end{aligned} \tag{1}$$

where $\bar{T}_k$ is the temperature of the kth super-node, $\bar{\mathcal{N}}_k \subset \bar{\mathcal{V}}_0$ denotes the set of neighbors of the kth super-node, and the ventilation heat exchange for the kth super-node is given by

$$\begin{aligned}\Delta\bar{H}_k^{(\phi)}(t) &:= \sum_{i\in\phi^{-1}(k)\cap\{1,\dots,N\}} \Delta H_i(t) \\ &= \sum_{i\in\phi^{-1}(k)\cap\{1,\dots,N\}} C_{pa}\dot{m}_i^{in}(t)(T^s - T_i(t)).\end{aligned}$$

The initial condition of the reduced model (1) at initial time t_0 is defined as

$$\bar{T}_k(t_0) = \sum_{i\in\phi^{-1}(k)} (C_i/\bar{C}_k^{(\phi)})T_i(t_0), \quad k \in \bar{\mathcal{V}}.$$

Note that the reduced model (1) requires for its inputs: the mass flow rate $\dot{m}_i^{in}$ and the zone temperature T_i for $i = 1, \dots, N$. These are assumed to be available, or are measured.

The reduced model described so far depends on the choice of the partition function ϕ. We should note that any m-partition function ϕ induces a reduced model with m super-states. In [7], we proposed a recursive bi-partition algorithm to search for the sub-optimal m-partition function ϕ^*. However, one can also directly choose a sub-optimal ϕ^* based on physical intuition (e.g., floors in a multi-zone building), or some kind of expert-based heuristics. The goodness of the reduced model (1) with ϕ^* can be verified in practice. In this paper, we will not discuss the algorithms for choosing the optimal partition function ϕ^*. In the following, we assume that ϕ^* has already already properly specified, and we mainly focus on how to design optimal control laws by taking advantage of the reduced building model.

III. Mean-Field Control

We consider N zones, each with its local set-point tracking control objective. The dynamics of the ith zone is given by

$$\dot{T}_i = l_i^\circ(T_i; T_{-i}) + b_i(T_i)u_i + d_i \tag{2}$$

where

$$\begin{aligned}l_i^\circ(T_i; T_{-i}) &:= -\sum_{j\in\mathcal{N}_i}(T_i - T_j)/(C_iR_{ij}), \\ b_i(T_i) &:= C_{pa}(T^s - T_i)/C_i, \quad d_i := \dot{Q}_i/C_i,\end{aligned}$$

with $T_{-i} := (T_j)_{j\neq i}$. The control problem for the ith zone is to minimize the finite-horizon cost function

$$J_i^\circ(u_i; u_{-i}) = \int_{t_0}^{t_1} c(T_i, u_i)dt \tag{3}$$

where

$$c(T_i, u_i) := \frac{1}{2}\Delta T_i^2 + \frac{1}{2}ru_i^2, \tag{4}$$

with the tracking error $\Delta T_i := T_i - T_i^{set}$, and a given scalar $r > 0$ as control penalty. A *Nash equilibrium* in control policies is given by $\{u_i^*\}_{i=1}^N$ such that u_i^* minimizes $J_i^\circ(u_i; u_{-i}^*)$ for $i = 1, \dots, N$.

We denote $R_i := (\sum_{j\in\mathcal{N}_i} 1/R_{ij})^{-1}$ and define the time constant for the ith zone as $\tau_i := C_iR_i$. The individual zones are distinguished by their initial conditions $T_i(0)$, set-point T_i^{set}, loads d_i and the time-constants τ_i. We introduce a parameter $\omega := (T(0), T^{set}, \tau, d)$, and consider a large number N of zones, where ω is sampled from a given distribution $\rho(\omega)$. For each zone i, the parameter ω_i is assumed to be i.i.d., with common distribution $\omega_i \sim \rho(\omega)$.

We seek a control solution that is decentralized and of the following form: For each $i \in \mathcal{V}$ and $t_0 \le t \le t_1$, the control input $u_i(t)$ depends only on *local information* $\{T_i(s) : t_0 \le s \le t\}$, and perhaps some *aggregate information*. This amounts to a dynamic game, whose exact solution is infeasible for large N.

Instead we construct an approximation of the form described in [11]. This approximation is based on the aggregated models described in Sec. II-C with the following steps:

(i) We identify a small number of super-nodes that describe the slow evolution of the thermal dynamics of the building. To simplify the introduction of the mean-field control method, we consider here only the simplest case: we use a single super-node to represent the entire building.

(iii) We consider an approximation of the interaction between a single zone and the entire building. Motivated by the consideration of the physics of thermal interactions (large time constants for interactions) and the separable nature of the control objectives (e.g., (3)), we consider an approximation based on replacing $T_{-i}(t)$ by $F(t)$, a known function of time. In particular, $l_i^\circ(T_i(t); T_{-i}(t))$ in (2) is replaced by

$$\bar{l}_i(T_i(t); F(t)) := -\frac{T_i(t) - F(t)}{\tau_i}. \tag{5}$$

Comparison of l_i° and $\bar{l}_i$ suggests the following approximation

$$F(t) \approx \bar{T}(t). \tag{6}$$

(iii) For the local model (2) with $l_i^\circ(T_i; T_{-i})$ replaced by $\bar{l}_i(T_i; F)$, the game reduces to decentralized optimal control problems. The individual zones are "oblivious" to the state of the entire system and make their control decisions based only on local state variables.

(iv) A form of *self-consistency* is required: oblivious actions of individual zones reproduce the evolution of $\bar{T}$ as described by the aggregated model.

In the following subsection we develop the "oblivious" solution described in (iii). We then turn to the self-consistent aggregated model in (i) that defines the approximate interaction (6) in (ii). Mathematically, we obtain a fixed-point problem.

A. Local optimal control of a single zone

Suppose the interaction function $F(t)$ is given, possibly in a time-dependent form for $t \in [t_0, t_1]$. We consider the following dynamics for the single-zone:

$$\dot{T}_i = \bar{l}_i(T_i; F) + b_i(T_i)u_i + d_i$$

where $\bar{l}_i$ is given by (5).

The control problem for single zone model is to choose the control law u_i so as to minimize the finite-horizon cost function

$$J_i(u_i; F) = \int_{t_0}^{t_1} c(T_i, u_i)dt.$$

The solution of the optimal control problem with the cost function $J_i(u_i; F)$ is standard. It is given in terms of the *optimal cost-to-go function* or *value function*:

$$J_i^*(T_i, t) = \min_{u_i} \left\{ \int_t^{t_1} c_i(T_i, u_i)dt \right\}.$$

The value function J_i^* is known to satisfy the Hamilton-Jacobi-Bellman (HJB) equation

$$\frac{\partial J_i^*}{\partial t} + \min_{u_i} \left\{ H_i \left(T_i, u_i, \frac{\partial J_i^*}{\partial T_i} \right) \right\} = 0 \tag{7}$$

with the boundary condition $J_i^*(T_i, t_1) = 0$. The *Hamiltonian* in (7) is defined for $\lambda \in \mathbb{R}$

$$H_i(T_i, u_i, \lambda) := c_i(T_i, u_i) + \lambda \left(l_i(T_i; F) + b_i(T_i)u_i \right)$$

with

$$l_i(T_i; F) := \bar{l}_i(T_i; F) + d_i.$$

The optimal control in (7) is explicitly obtained as

$$u_i^*(T_i; F) = -\frac{b_i(T_i)}{r} \left(\frac{\partial J_i^*}{\partial T_i}(T_i; F) \right). \tag{8}$$

Substituting (8) into (7), we obtain the HJB equation for $J_i^*(T_i, t)$:

$$\frac{\partial J_i^*}{\partial t} = \frac{1}{2} \frac{b_i^2(T_i)}{r} \left(\frac{\partial J_i^*}{\partial T_i} \right)^2 - l_i(T_i; F) \left(\frac{\partial J_i^*}{\partial T_i} \right) - \frac{1}{2} \Delta T_i^2. \tag{9}$$

B. Coupled model

We now provide a complete description of the *coupled model* that is intended to approximate the game model for large N. This model is based on the interaction function $F(t)$ introduced in the preceding section. A value function function $J^*(T, t; \omega)$ for the large N model is defined by the following differential equation identical to the HJB equation (9) for the single-zone model.

$$\frac{\partial J^*}{\partial t} = \frac{1}{2} \frac{b^2(T)}{r} \left(\frac{\partial J^*}{\partial T} \right)^2 - l(T; F) \left(\frac{\partial J^*}{\partial T} \right) - \frac{1}{2} \Delta T^2.$$

The associated optimal feedback control law is then defined by

$$u^*(T; F) = -\frac{b(T)}{r} \left(\frac{\partial J^*}{\partial T}(T; F) \right). \tag{10}$$

Given the feedback control law (10), the differential equation that defines the evolution of the super-temperature $\bar{T}$ is given by

$$\bar{C}\dot{\bar{T}}(t) = (T_0(t) - \bar{T}(t))/\bar{R}_0 + \bar{U}(t)$$

where

$$\bar{U}(t) = N \int (\dot{Q}(t; \omega) + C_{pa} u^*(T(t; \omega); F)(T^s - T(t; \omega)))\rho(\omega)d\omega.$$

The only difference thus far is notational: $J_i^*(T, t)$ is the value function for a single zone with parameter ω_i, and $J^*(T, t; \omega)$ is the value function for a large number of zones, distinguished by their own ω. Such is the case because we have *assumed* $F(t)$ is a known deterministic function that is consistent across the population. All that remains is to specify $F(t)$ in a self-consistent manner. The consistency enforced here is inspired by the approximation given in (6). The two PDEs are coupled through this integral that defines the relationship between the interaction function F and the mean temperature $\bar{T}$:

$$F(t) = \bar{T}(t).$$

In summary, the coupled PDE model is given by: For $t \in [t_0, t_1]$,

$$\frac{\partial J^*}{\partial t} = \frac{1}{2} \frac{b^2(T)}{r} \left(\frac{\partial J^*}{\partial T} \right)^2 - l(T; F) \left(\frac{\partial J^*}{\partial T} \right) - \frac{1}{2} \Delta T^2 \tag{11}$$

$$\bar{C}\dot{\bar{T}}(t) = (T_0(t) - \bar{T}(t))/\bar{R}_0 + \bar{U}(t) \tag{12}$$

$$F(t) = \bar{T}(t) \tag{13}$$

with boundary conditions $J^*(T, t_1; \omega) = 0$ and $\bar{T}(t_0) = \sum_{i=1}^n (C_i/\bar{C})T_i(t_0)$.

Numerically, the optimal control may be obtained by iteratively solving the equations (11) and (12) over a sufficiently long time-horizon. A waveform relaxation algorithm for solving such equations appear in our earlier paper [12]. In the following, we propose an approximate solution based on the observation that the value function is known to approximately become a constant for large terminating times [13].

C. Approximate local optimal control

In this section, we propose an approximation approach to the solution of coupled PDE (11)–(13) by considering the the *equilibrium solutions*. By setting $\partial J^*/\partial t \approx 0$ and letting $F = \bar{T}$ in (11), we consider the equilibrium solution to (11):

$$k(T)\left(\frac{\partial J^*}{\partial T}\right)^2 - 2m(T,\bar{T})\left(\frac{\partial J^*}{\partial T}\right) - n(\Delta T) = 0 \quad (14)$$

where we define

$$k(T) := b^2(T),\ m(T,\bar{T}) := rl(T;\bar{T}),\ n(\Delta T) := r\Delta T^2.$$

Since T^s (the temperature of supplied air) is always strictly less/more than T (the temperature of zone) when cooling/heating, then we always have

$$k(T) = (C_{pa}(T^s - T)/C)^2 > 0.$$

Thus (14) is a second order equation, whose solutions are given by,

$$\left(\frac{\partial J^*}{\partial T}\right)_{\pm} = \frac{m(T,\bar{T})}{k(T)} \pm \frac{\sqrt{m^2(T,\bar{T}) + k(T)n(\Delta T)}}{k(T)}.$$

For any T, ΔT, and $\bar{T}$, we can check

$$\left(\frac{\partial J^*}{\partial T}\right)_{+} \geq 0, \quad \left(\frac{\partial J^*}{\partial T}\right)_{-} \leq 0.$$

We would like to construct a "value function" $\widehat{J}^*$ such that it is approximately convex with respect to T with minimum achieved at $\Delta T = T - T^{set} = 0$. One possible choice is

$$\frac{\partial \widehat{J}^*}{\partial T} = \begin{cases} (\partial J^*/\partial T)_+, & \text{if } \Delta T \geq 0 \\ (\partial J^*/\partial T)_-, & \text{if } \Delta T < 0. \end{cases} \quad (15)$$

However, such a choice of $(\partial \widehat{J}^*/\partial T)$ is not a smooth function of T, and neither is the associated control law (10). Note that the mass flow rate (the control) is usually varied continuously to regulate the zone temperature for a building. To obtain a smooth control law, here we consider a smooth approximation to the sign function,

$$\text{sgn}(x) \approx \tanh(cx) = \frac{1 - e^{-2cx}}{1 + e^{-2cx}}, \quad \text{for} \quad c \gg 1. \quad (16)$$

Then we modify (15) to obtain the following smooth approximation:

$$\frac{\partial \widehat{J}^*}{\partial T} = \frac{m(T,\bar{T})}{k(T)} + \tanh(c\Delta T)\frac{\sqrt{m^2(T,\bar{T}) + k(T)n(\Delta T)}}{k(T)}$$

The approximate local optimal control law is chosen as

$$\widehat{u}^*(T;\bar{T}) = -\frac{b(T)}{r}\left(\frac{\partial \widehat{J}^*}{\partial T}(T;\bar{T})\right). \quad (17)$$

By setting $\dot{\bar{T}} \approx 0$ in (12) and substituting (17) into (12), we can obtain the equilibrium solution $\bar{T}^s > 0$ by solving a second order equation (we omit the details here). Finally, the stationary local optimal control law is given by

$$\widehat{u}^{*,s}(T) = -\frac{b(T)}{r}\left(\frac{\partial \widehat{J}^*}{\partial T}(T;\bar{T}^s)\right). \quad (18)$$

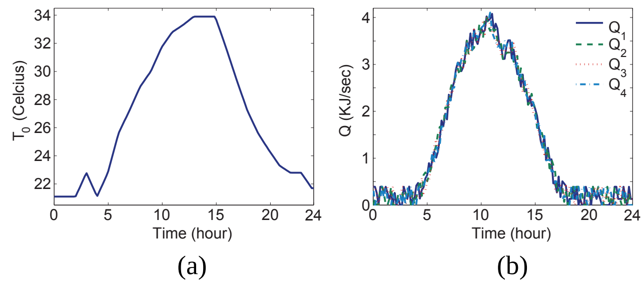

Fig. 2. Exogenous inputs for a 24 hour period in Gainesville, FL: (a) outside temperature and (b) heat gains of each zone.

IV. Simulation and Discussion

A. Basic setup

Simulations are carried out for the four-zone building shown in Fig. 1: All four zones/rooms have an equal floor area of $5m \times 5m$ and each wall is $3m$ tall, which provides a volumetric area of $75m^3$ for each room. The RC-network representation of the four-zone building has totally 36 building nodes plus 1 outside node [7]. Each building node is assigned with a thermal capacitance, two adjacent nodes are connected with a thermal resistance. The windows are modeled as single resistors since they have relatively little capacitance. The values of the capacitances and resistances are obtained from Carrier's Hourly Analysis Program [14].

The HVAC system used for simulation is designed to supply maximal mass flow rate of 0.25 kg/s per zone. The mass flow rate m_i^{in} for $i = 1, \ldots, 4$ for four zones can be adjusted based on designed control laws. The supplied air temperature is fixed at $T^s = 12.8°C$. Here we assume there is no return air and 100% of the outside air is sent to chiller. Number of people in each zone is uniformly generated as a random integer ranging between 0 and 4. Outside temperature and outside solar radiation data is obtained for a summer day (05/24/1996) of Gainesville, FL [15]. The outside temperature and the heat gains (due to solar radiation and people occupancy) of each zone are depicted in Fig. 2 (a) and Fig. 2 (b), respectively.

Numerical results presented in the following are obtained using ode45 function in Matlab for 24 hours with the time step size chosen as 10 minutes. All temperatures of the building nodes are initialized at $24°C$, respectively. The desired zone temperatures T_i^{set} for $i = 1, \ldots, 4$ are varying with time and are depicted as solid lines in Fig. 3.

B. Simulation results

To compare performance of the proposed controller with existing control algorithms commonly used in commercial buildings, we consider the following decentralized PI control law: For $i = 1, \ldots, 4$,

$$m_i^{in}(t) = K_p \Delta T_i(t) + K_i \int_0^t \Delta T_i(s) ds \quad (19)$$

where the tuned proportional gain $K_p = -0.00005$, the tuned integration gain $K_i = 0.0001$, and the temperature tracking error $\Delta T_i := T_i^{set} - T_i$.

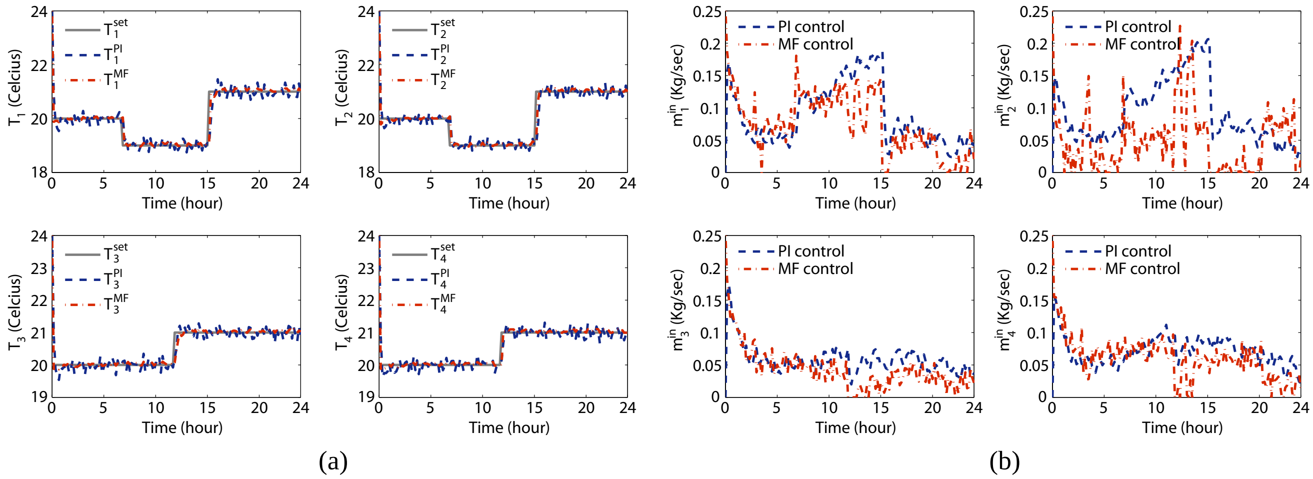

Fig. 3. Comparison of (a) zone temperatures and (b) zone mass-flow rates obtained by using PI control law and mean-field control law with $r = 10$.

The mean-field control is implemented based on the stationary nonlinear policy (18). For the four-room building, we only consider the reduced model with one super-node. The control performance becomes slightly better by adding more super-nodes into the reduced model. One may expect larger performance improvement by considering more super-nodes for more complex building topologies.

In this paper, we take $r = 10$ in the individual cost function (4), and we take $c = 5$ for smooth approximation of sign function in (16). We apply mean-field control law (18) and PI control law (19) to each zone, respectively. The comparison results of simulated zone temperatures are depicted in Fig. 3 (a). The comparison results of mass-flow rates associated with two control laws are depicted in Fig. 3 (b). We observe that the mean-field control has better temperature tracking performance than that for PI control (see Fig. 3 (a)).

The total energy consumption of each zone can be computed based on the mass-flow rate entering each zone [6]. Here the total energy consumption is the combination of fan power and the chiller power consumptions. For PI control law, the energy consumption (kWh) for each zone is 69.3, 72.5, 45.2, and 55.9, and the total energy consumption for all four zones is 242.9; For mean-field control law, the energy consumption for each zone is 59.6, 38.9, 34.0, and 47.7, and the total energy consumption for all four zones is 179.2. In this case, the mean-field control thus reduces total energy consumption by 25% over the PI control. One could expect more energy savings (but worse temperature tracking performance) by increasing the control penalty parameter r.

V. Conclusions

In this paper, we develop the mean-field methodology as a means to mitigate complexity associated with large-scale control problems in buildings. Rather than solving the large-scale centralized problem, we explore distributed game-theoretic solution approaches that work by optimizing with respect to the mean-field. Simulation results show that the proposed mean-field scheme achieves comparable temperature tracking performance while reducing energy consumption in the operation of HVAC systems. Moreover, the tradeoff between tracking performance and the energy savings can be made by adjusting the control penalty parameter in the mean-field scheme.

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