ASSIGNMENT-MA597

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QUASI-BIRTH-DEATH-M (QBD-M) PROCESSES AND EXISTING METHODS FOR THEIR STEADY STATE ANALYSIS

The search of numerical solutions for queueing models applied in performance analysis of computer systems and communication networks has practical importance and is always a hot research topic. In recent years, the extensive application of two- dimensional queueing systems which state is described by a phase and a level has been widely witnessed, particularly on the performance evaluation of ATM (Asynchronous Transfer Mode) systems . If the level transitions are only possible between adjacent ones, such queueing systems are called QBD (Quasi Birth-Death) processes. For the steady state solution of this class of two-dimensional Markov chains, several efficient methods have been developed and improved over recent years. However, there are only few works dealing with such two- dimensional queueing systems in which upper-bounded arrival and or departure batches occur, i.e. multiple jumps in level dimension are possible. The involvement of this kind of queueing systems is quite reasonable and useful during modelling and performance analysis of a variety of problems arising in telecommunication networks and computer systems field.

Consider a discrete-time two dimensional Markov process. Each state of the process is denoted by two integer valued random variables I_n and J_n. I_n is taking finite set of values $\{0, 1 \cdots\}$ and J_n is taking an infinite set of values $\{0, 1, \cdots\}$. The states with the same value of $J_n(J_n = j)$ define level (j). The transition probabilities of the underlying Markov process are given by the following transition probability matrixes:

- A_j: purely phase transitions-From state (i, j) to state (k, j) $(0 \leq i, k \leq N; i \neq k; j = 0, 1, 2....)$
- $B_{j,s}$: bounded s-step upward transitions-From state (i, j) to state $(k, j+s)$ $(0 \leq i, k \leq N; 1 \leq s \leq y_1; y_1 \geq 1; j = 0, 1, 2....)$

- $C_{j,s}$: bounded s-step downward transitions-From state (i, j) to state $(k, j-s)$ $(0 \leq i, k \leq N; s \leq j; 1 \leq s \leq y_2; y_2 \geq 1; j = 0, 1, 2....)$

we assume there is a threshold M, $M \geq y_1$, such that for $j \geq M$ the transition matrixes become level-independent.That is

$$A_j = A,\ j \geq M; \quad B_{j,s} = B_s,\ j \geq M - y_1, \quad C_{j,s} = C_s,\ j \geq M. \tag{1}$$

This is similar to the QBD process, except that in level dimension are allowed only between adjacent cells in QBD's case. Thus, from the aspect of regarding level changes, the process considered so far is the extension of QBD process. Therefore, in what follows this process is referred to as QBD-M process , where the letter M stands for multiple jumps in level dimensions.

Our task is to determine $p_{i,j}$: the steady state probability of the state (i, j)

$$p_{i,j} = \lim(I_n = i, J_n = j)\ i = 0, 1, 2, \cdots, N,\ j = 0, 1, \cdots \tag{2}$$

Our task is determining these probabilities in terms known parameter of the system.
v_j: the row vector defined as

$$v_j = (p_{0,j}, p_{1,j}, ..., p_{N,j}) \qquad j = 0, 1, \cdots \tag{3}$$

e: the column vector of $(N+1)$ elements each of which is equal to 1. For $j = 0, 1, ..., M-1$,the balance equations of the system are:

$$v_j = \sum_{s=1}^{y_1} v_{j-s} B_{j-s,s} + v_j A_j + \sum_{s=1}^{y_2} v_{j+s} C_{j+s} \tag{4}$$

(It is assumed that $v_{j-s} = 0, if j < S$ For $j \geq M$, the corresponding j-independent set becomes the set is

$$v_j = \sum_{s=1}^{y_1} v_{j-s} B_s + v_j A + \sum_{s=1}^{y_2} v_{j+s} C_s, j \geq M \tag{5}$$

In addition,since the sum of all probabilities must be one, we have:

$$\sum_{j=0}^{\infty} v_j \mathbf{e} = 1, \tag{6}$$

for the infinite case.

Available methods for steady state solution of QBD-M processes fall into two categories depending on whether the calculation of level probability vectors takes place in a direct or indirectway. The methods of Naumov et al. and the spectral expansion method are most appealing to use due to their fast and accurate performance. We will refer to these two methods as NA after BL and SE after BL method,respectively. The direct solution way is represented by the spectral expansion method,that is capable to solve QBD-M processes without re-blocking. However numerical experiments do not encourage use of this method, because it is sometimes inclined to fail in ill-conditioned troubles.

A new method has been proposed.This method makes use of re-blocking technique and special relations between level probability vectors to reduce calculation efforts. An iterative procedure has been provided for calculation of the reduced part of the so called rate matrix. This method is referred to as ITE method. Theoretically, it has been proved that regarding each iterative step, the complexity of the ITE method is less than the complexity of matrix geometry based methods(one of them is Naoumovs method itself)applied after re-blocking. Moreover,its gain is also offered by a favourable storage requirement that is definitely less than the case of matrix geometry based methods such as the NA after BL method.

Bibliography

[1] HungT.TRAN and TienV.DO, *Genralised Invariant Subspace Based Method for Steady State Analysis of QBD-M Processes*, 2000