# **MA 402 Term paper**

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## APPLICATION OF QUEUEING THEORY WITH MONTE CARLO SIMULATION TO THE STUDY OF THE INTAKE AND ADVERSE EFFECTS OF ETHANOL

Abstract: Due to a phenomenological analogy between a queueing system and the systems in the body dealing with ethanol,. When using queueing theory, a stochastic approach was used for modelling the study of ethanol. Consumption of ethanol and removal of the adverse effects associated with its consumption from the body can be random processes; and thus requirement for detailed information on the anatomical structure etc. can be minimized. Using queueing theory, estimations can be made regarding
1) To the accumulated adverse effects of ethanol in the body and
2) The time needed to remove the adverse effects etc.

Introduction: Consumption of ethanol is a random process in most real-life cases can be considered as a reasonable assumption. The reason being, ethanol is taken more frequently on unfixed occasions, rather than on fixed ones and also it is also reasonable to assume that the removal from the body of the adverse effects associated of its consumption is also a random process for the following reasons (i) the absorption, metabolism, and elimination of ethanol are affected by numerous factors of varying degrees (ii) a random consumption of ethanol would result in a random beginning of the removal of the adverse effects associated with this consumption; and (iii) the absorption, metabolism, and elimination systems in humans would not be expected to work at the same rate throughout life. Basing on these assumptions, the stochastic approach might be helpful in the modeling of these processes, and could provide useful information that cannot be obtained using the deterministic approach. The aim of this current approach is to apply queueing theory with Monte Carlo simulation to the study of ethanol.

## Queueing modeling:

Figure 1 represents the simplest comparison between the customer-server servicing system and the consumption of ethanol and removal of adverse effects of its consumption in humans in terms of queueing theory approach.

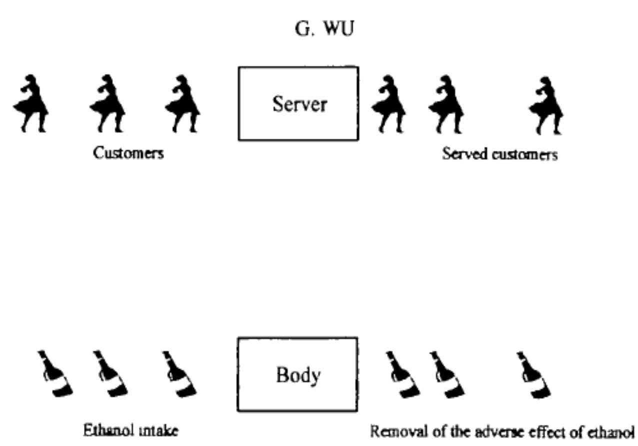


Fig. 1

In the ethanol study, a 'server' can be defined as the systems in humans, the 'service' is to remove from the body, the adverse effects associated with the consumption of ethanol, the accumulated adverse effects of ethanol in the body can be defined as the queue waiting to be 'served', and the 'served' adverse effects are the adverse effects removed from the body. In this manner, a servicing system is analogous to the systems in humans in dealing with the intake, metabolism, and elimination of ethanol. The target parameters for the estimation using this approach being (i) the accumulated amounts of adverse effects of ethanol in the body in terms of bottles of beer and (ii) the time needed to remove the adverse effects of ethanol from the body.

The following data are to be known for the estimation of the above said parameters: (1) the probability distributions of inter-arrival times, in other words, it is the probability distributions of lengths of time between any two equal successive amounts of ethanol consumption and characterizes the process of ethanol intake, i.e. random intake, fixed time intake etc. (2) the probability distributions of service times, which is the probability distributions of lengths of time in the removal of equal amount of adverse effects of ethanol from the body, and characterizes the removal of the adverse effects of ethanol, i.e. random removal, constant rate removal etc. (3) the queue discipline, being the regulation by which an equal amount of adverse effects of ethanol is selected to be removed, i.e. first-come-first-served, last-come first-served, priority service, random service etc.; and (4) the number of servers, i.e. the body can be defined as one server, or gastrointestinal absorption, gastrointestinal metabolism, and liver metabolism can be defined as three different servers etc.

Monte Carlo simulation: The assumption used in this is that a person consumes 3 bottles of beer during non-leeping and non-working time per day and that the body can also remove the adverse effects associated with the consumption of 3 bottles of beer per day. Another assumption is that the removal of the adverse effects associated with the consumption of 3 bottles of beer per day from the body is not at the same rate, i.e. the time needed to remove the adverse effects associated with the consumption of each bottle of beer is different. Figure 2 shows the simulation on drinking 21 bottles of beer and removal of the adverse effects associated with the consumption of these 21 bottles of beer in terms of the behavior predicted by queueing theory.

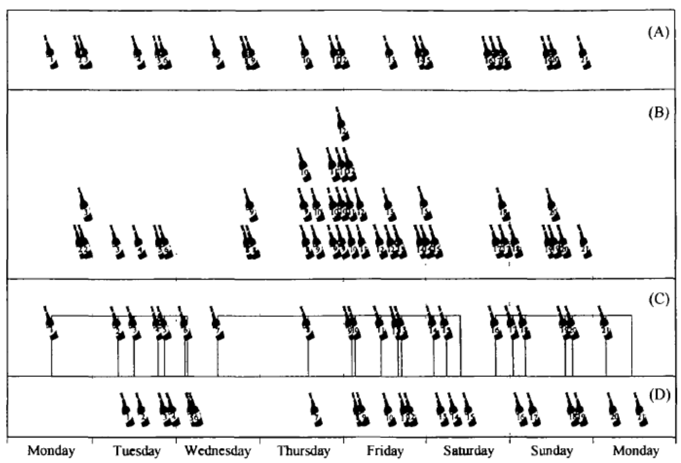


Fig. 2. Simulation of the drinking of 21 bottles of beer and the removal of the adverse effects associated with their intake. (A) The assumed drinking times from Monday to Sunday are 12:30, 21:00, 21:30; 13:06, 20:00, 21:00; 12:00, 20:30, 22:00; 13:30, 22:00, 22:48; 14:00, 23:00, 23:48; 20:00, 21:00, 22:00; 12:00, 13:00, and 21:00. (B) The adverse effects associated with the drinking of bottles of beer accumulated in the body during the second to the 21st bottle of beer. (C) The assumed random lengths of time needed to remove the adverse effects in terms of each bottle of beer are 18.833, 4.473, 7.110, 1.876, 5.841, 0.716, 25.685, 12.769, 1.125, 7.552, 4.872, 1.046, 9.123, 3.781, 3.885, 4.912, 3.527, 11.456, 2.113, 9.652, and 7.622 h. (D) The finishing time for the removal of the adverse effects in terms of each bottle of beer from Tuesday to the following Monday is 07:20, 11:48, 18:55, 20:48; 02:38, 03:21; 13:41; 02:27, 03:35, 11:08, 16:00, 17:03; 02:10, 05:57, 09:50; 00:55, 04:26, 15:54, 18:00; 03:40, and 11:17.

Figure 2A shows the assumed drinks during non-sleeping and non-working time from Monday to Sunday; this panel can be regarded as the arrival of customers to a servicing system. The position of the lower-left corner of each bottle represents the drinking time. Figure 2B shows the accumulated adverse effects in the body in terms of how many bottles of beer; this panel can be regarded as the queue waiting to be served in a servicing system. The position of each bottle is changed along the time course, and the panel constructs the dynamics of the queue. Figure 2C shows the removal of the adverse effects from the body; this panel can be regarded as the service provided by the servicing system. The lower-left corner of each bottle represents the start time to remove the adverse effects in terms of each bottle of beer and the rectangle (from the left line to the right line) is the time needed to remove the adverse effects in terms of each bottle of beer. Figure 2D shows the removal of the adverse effects from the body in terms of each bottle of beer; this panel can be regarded as the served customers who have left the servicing system. The upper-left corner of each bottle represents the finishing time for the removal of the adverse effects in terms of each bottle from the body (all the data are given in the caption to Fig. 2).

Queueing theory with Monte Carlo simulation further shows: (1) the adverse effects are accumulated to the maximum amount in terms of 4 bottles of beer in the body waiting for removal, in means ± SEM (1.52 ± 1.03 bottles, Fig. 2B); (2) the maximum length of time waiting is 28.45 h (7.87 ± 7.24 h/bottle); (3) the maximum length of time in which the body is not involved in the removal of the adverse effects is 10.16 h (0.89 ± 2.98 h/bottle); (4) the maximum length of time that the adverse effects are in the body is 29.95 h (14.91 ± 7.79 h/bottle); (5) the length of time from the beginning of drinking to the complete removal of the adverse effects associated with the drinking of 21 bottles of beer is 166.78 h (6.95 days); (6) there are about 18.81 h (about 11.28% of 166.78 h) between drinks in which the body is not involved in the removal of the adverse effects; and (7) there are two intervals between drinks in which the body is not involved in the removal of the adverse effects (before the seventh and 17th bottles in Fig. 2C). This simulated example suggests that the body can successfully remove the adverse effects of the consumption of 21 bottles of beer within 166.78 h, the maximum accumulated adverse effects in terms of 4 bottles of beer can result, which may lead to some adverse effects in humans.

Table 1. Equations deduced by queueing theory for determination of operating characteristics in the *M/M/*1 queueing model

| | | | |
|---|---|---|---|
| $L_s = \dfrac{\lambda}{\mu - \lambda}$ | $L_q = \dfrac{\rho\lambda}{\mu - \lambda}$ | $W_s = \dfrac{1}{\mu - \lambda}$ | $W_q = \dfrac{\rho}{\mu - \lambda}$ |

In the ethanol study, $\lambda$ is the parameter used in the Poisson input, i.e. the mean amount of intake of ethanol per time; $\mu$ is the parameter used in the negative exponential distributions of service times, i.e. the mean amount of the adverse effects of ethanol removed from the body per time; $\rho$ is the probability of a servicing system being busy and is equal to $\lambda/\mu$, i.e. the probability of the body being busy removing the adverse effects of ethanol from the body; $L_s$ is the mean number of customers in the queueing system being served and waiting, i.e. the mean amount of the adverse effects of ethanol being removed and waiting to be removed in the body; $L_q$ is the mean number of customers in the queue, i.e. the mean amount of the adverse effects of ethanol waiting to be removed from the body; $W_s$ is the mean length of time that a customer spends in the queueing system including service and waiting time, i.e. the mean length of time that the equal amount of the adverse effects of ethanol spend in the body including removing and waiting time; and $W_q$ is the mean length of time that a customer spends in the queue, i.e. the mean length of time that the amount of the adverse effects of ethanol spend waiting to be removed from the body.

The parameters in the queueing process can also be calculated using the deduced equations. The model of ethanol intake, metabolism, and elimination in Fig. 1 can correspond to the simplest model in the queueing theory, i.e. M/M/l model, the single-server model with Poisson input and exponential service defined by endell's notation, where *M* is a Markovian process, the first *M* is that the probability distributions of inter-arrival times are the Poisson input, the second *M* is that the probability distributions of service times are the negative exponential distributions, and 1 is the single server, the queue discipline is the first-come first-served.

## Calculation and estimation:

In the modeling of the consumption of ethanol and the removal of the adverse effects associated with such consumption, the *M/M/l* model denotes that: (1) the consumption of ethanol is a random process; (2) the removal from the body of the adverse effects associated with the consumption of ethanol would take an identical length of time for the same amount of adverse effects in terms of each bottle; and (3) the body is one 'server'. Table 1 shows several equations used in the *M/M/l* model, which are suited at the statistical equilibrium state. Drinking to be 3 bottles per day, we have A = 3/24 = 0.125 (bottle/h). The body can remove the adverse effects associated with the consumption of 3 bottles of beer per day; as the length of time of removal meets the negative exponential distributions of service times (Appendix), we have *H* = 3/24 = 0.125 (bottle/h). Here we have A = /z, i.e. the arrival rate is equal to the service rate, this is also the case in our second Monte Carlo simulation. In these cases, we cannot use the equations in Table 1. Similarly, when the body can remove the adverse effects associated with the consumption of 2.5 bottles of beer from the body per day, and we have /i = .5/24 = 0.1042 (bottle/h) the same is true. Thus we have A > */i,* i.e. the arrival rate is larger than the service rate and we still cannot use the deduced equations in Table 1. However, we would expect that the adverse effects of ethanol in the body would increase with drinking because the intake rate is larger than that of removal.

In the first Monte Carlo simulation, the person randomly drinks the average 3 bottles during non-sleeping and non-working time per day, A = 3/24 = 0.125 (bottle/h), and the body can remove the adverse effects associated with the consumption of 10 bottles of beer from the body per day, and we have *fx*= 10/24 = 0.4167 (bottle/h). Then we have *p = X/fj, = 0.125/* 0.4167=0.72, L5=A/(/x- A) = 0.125/(0.4167 - 0.125) = 0.43 (bottle), *Lq* = *p\i(n* - A) = 0.72 x 0.125/(0.4167-0.125) = 0.13 (bottle), *Ws* = l/(/t-A) =1/(0.4167-0.125) = 3.43 (h), and *Wq* = 0.72/(0.4167 -0.125)= 1.03 (h). Part of these results is similar to the simulated results, i.e. 0.13 bottles vs 0.20 ± 0.54 bottles, but other results are different from the simulated results. This difference is due to the fact that the drinks are randomly distributed throughout the day (00:00 to 24:00) in the calculation using the deduced equations, whereas the drinks are randomly distributed only during the non-sleeping and non-working time in the calculation using Monte Carlo simulation.