Queueing Theory Model for Insulin Level and Number of Insulin Receptors in Body

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***Abstract*- Blood glucose in our body is regulated by level of Insulin, a hormone. Its deficiency or over secretion results in several disorders. The level of Insulin in our body is related to the number of Insulin receptors in a cell. In this paper a queueing theory model is applied to estimate the optimum level of Insulin and the optimum number of Insulin receptors in our cells.**

***Index Terms*- Insulin level, number of Insulin receptors.**

I. Introduction

Insulin is one of the important hormones in our body. Any increase or decrease in its normal production rate causes several disorders such as polyurea, polydipsia and wight loss in metabolism. Hence it becomes necessary to determine the optimum level of Insulin in the body in order to have a check on these and other deadly diseases due to Insulin deficiency or over production. In literature we have a lot of mathematical models to understand the behavior of Insulin. One common shortcoming in all of the previous works is that none of the models is able to give the estimate of the several important parameters such as optimum energy use for metabolic energy balance. This paper provides a detailed modeling of the optimum level of Insulin in the body based on the number of Insulin receptors in the cells and is organized as follows. Section II discusses about the implied queueing model, definitions of the parameters of interest and application of that model in present case. The results and observations are explained in Section III. Finally, conclusions are drawn in Section IV.

II. Queueing model and problem formulation

The number of Insulin receptors in our cells is found to have an exponential distribution with a average rate of $1/\mu$. The Insulin level is also exponentially distributed with an average rate of $1/\lambda$. Since the number of Insulin receptors in the cells operating at a time is always ≥ 1, M/M/C queueing model is the optimum choice. The queue discipline is First come First serve (FCFS).

In context of the M/M/C model,
C = maximum number of Insulin receptors in cells (servers)
N_s = number of Insulin molecules in the system (customers in system)
N_q = number of Insulin molecules in the queue (customers in queue)
λ = average arrival rate of Insulin molecules in the system
μ = average service rate of Insulin receptors.

The above model is similar to the Birth-Death process as depicted in Fig .1.

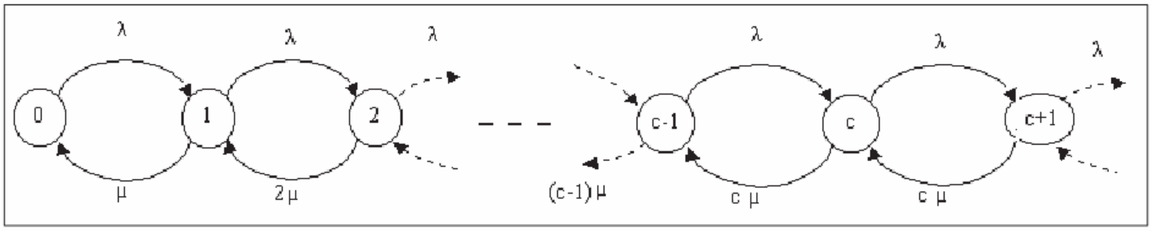

Fig.1. State transition diagram for M/M/C model

Let n denotes the number of Insulin molecules in the system. Then the total service rate of the receptors is given as

$$\mu_n = n\,\mu \qquad n < C$$
$$\mu_n = C\,\mu \qquad n \geq C$$

$\rho = \lambda / \mu$ (Insulin-Insulin receptor complex utilization factor)
p_n = probability that there are n insulin molecules in the system

$$p_n = \begin{cases} \dfrac{\lambda^n}{\mu(2\mu)(3\mu)\ldots(n\mu)} p_0 = \dfrac{\lambda^n}{n!\,\mu^n} p_0 & n \leq c \\[2ex] \dfrac{\lambda^n}{\mu(2\mu)\ldots(c-1)\mu(c\mu)^{n-c+1}} p_0 = \dfrac{\lambda^n}{c!\,c^{n-c}\mu^n} p_0 & n > c \end{cases}$$

Now from the probability equation

$$\boldsymbol{\sum p_n = 1 \quad \text{for } n = 0,1,2\ldots..\infty}$$

hence

$$p_0 = \left\{ \sum_{n=0}^{c-1} \frac{\rho^n}{n!} + \frac{\rho^c}{c!} \left(\frac{1}{1 - \frac{\rho}{c}} \right) \right\}^{-1}, \qquad \frac{\rho}{c} < 1$$

Now the average number of Insulin molecules waiting in the queue is given as

$N_q = p_0 * \rho$^ C+1 / (C-1)! (C- ρ)^2

The average waiting time of Insulin molecules in the queue is given as

$W_q = p_0 * \rho$^ C+1 / λ* (C-1)!* (C- ρ)^2

The average number of Insulin molecules in the system is given as

$N_s = p_0 * \rho$^ C+1 / (C-1)! (C- ρ)^2 + ρ

The average waiting time of Insulin molecules in the system is given as

$W_s = p_0 * \rho$^ C+1 / λ* (C-1)!* (C- ρ)^2 + 1/ μ

III. Results and Observations

For practical verification of the above queueing model **λ** = 12.3 μU/mL and **μ** = 6.6 μU/mL were taken. The simulation is done for **N_q** and **N_s** for different number of number of Insulin receptors in the system.

A tabulated data of the observation made is given below.

No of receptors	N_s (μU/mL)	N_q (μU/mL)	W_s (min)	W_q (min)
4	**1.99**	**0.124**	**0.161**	**0.01**
5	**1.89**	**0.027**	**0.153**	**0.002**
6	**1.87**	**0.005**	**0.152**	**0.000**
7	**1.86**	**0.001**	**0.151**	**0.000**
8	**1.86**	**0.00**	**0.151**	**0.000**
9	**1.86**	**0.00**	**0.151**	**0.000**

The figure below shows the graph of **N_s** and **N_q**

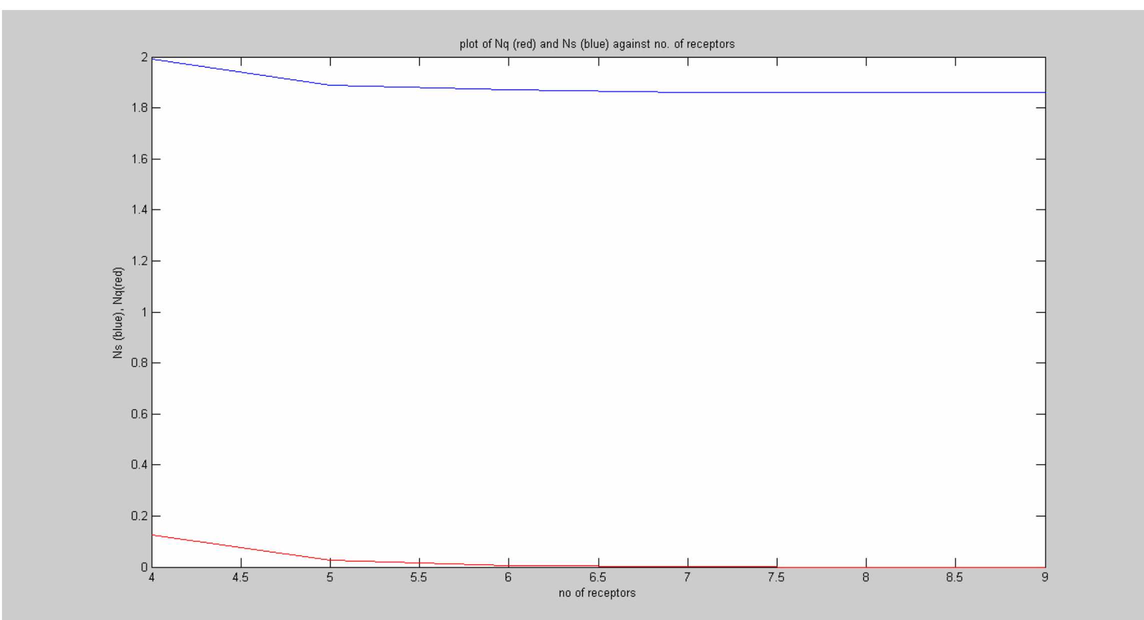

Fig.2.

The figure below shows the plot between $\mathbf{W_s}$, $\mathbf{W_q}$ and no of receptors

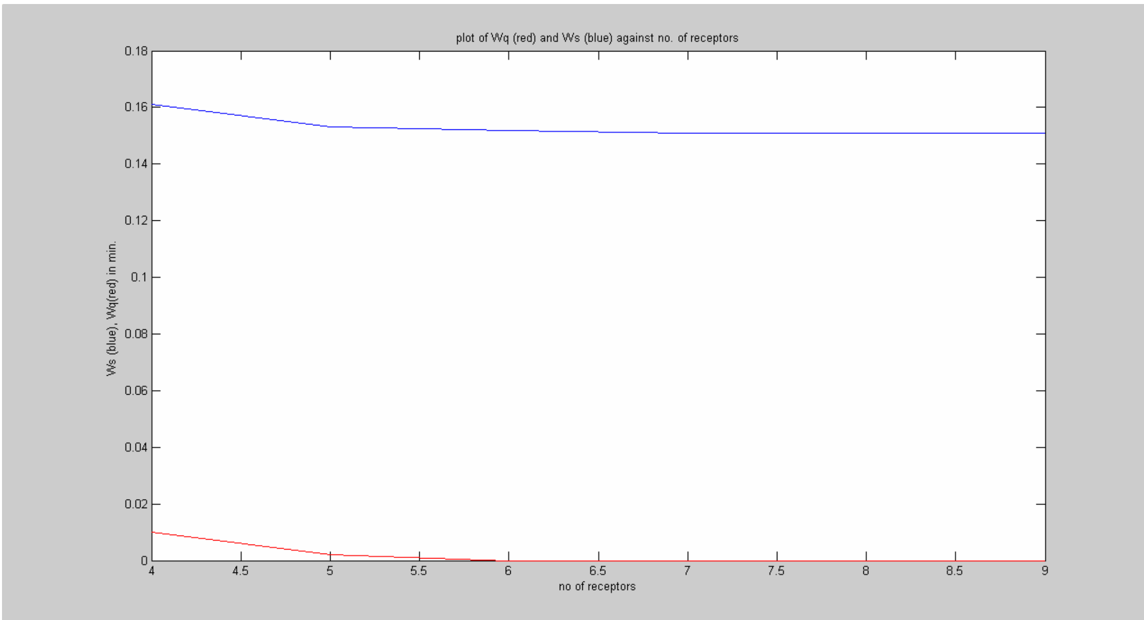

Fig.3.

It is obvious from the above two plots that as no of receptors increases from 5 to 6 the waiting time in queue becomes 0 which means no insulin molecules has to wait in the queue and system size becomes constant at 1.86 µU/mL. Hence 6 is the optimum number of Insulin receptors in the system and 1.86µU/mL is the optimum Insulin level in the body.

IV. Conclusion

This paper presents a novel method of determining the optimum level of Insulin in our body in order to study the diseases arising due to it's over and under regulated production. The M/M/C queueing model is applied and observation is made in order to find out the average level of insulin in our body and average number of Insulin receptors to keep the Insulin level at the optimum value.

REFERENCES

[1] C. Kandemir and L.Cavas , "*An Application of Queueing Theory to the Relationship Between Insulin level and Number of Insulin Receptors ,*" *Turkish Journal of Biochemistry*, vol. 32(1), p. 32-38, May 2007.