Topic: **Sub-Nyquist Sampling: Recent Advances and Applications**

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The Shannon-Nyquist sampling theorem is a widely used method for discrete representation of analog bandlimited signals. In this framework, the signal is represented by its samples taken at the Nyquist rate, which is greater than equal to twice the maximum frequency present in the signal. In 1977, Papoulis proposed a generalized multichannel sampling method for bandlimited signals where the signal is represented by samples of filtered signals taken at a sub-Nyquist rate. Hence, the overall sampling rate is equal to Nyquist rate. In many practical applications, signals are not bandlimited. To accommodate such signals within the Shannon-Nyquist framework, one passes the signal through an anti-aliasing filter, which retains most of the signal energy in a certain band.

With the advancement of the communication technologies such as cognitive radio, software defined radio, ultra wideband communication for infostations, wideband RADAR, etc. the requirement of higher sampling rate is continuously increasing. It is difficult to realize analog-to-digital converters with higher bits per sample at large sampling rates. Hence, one has to look for alternate ways to discretize such wideband analog signals.

In the past two decades, various sampling methods have been developed for sampling non-bandlimited signals by using prior information about the analog signals. In applications such as RADAR, SONAR, ultrasound imaging, optical coherence tomography (OCT), multiband signal communication, wideband spectrum sensing, etc., the signals to be sampled have a certain structure, which could be of in a form such as: (i) sparsity or parsimony in certain basis function; (ii) signals belonging to a shift-invariant space; (iii) multiband structure of the signal spectrum; (iv) a finite rate of innovation, etc. By using the structure to advantage, one can sample and reconstruct such signals at much lower rates than the Nyquist rate.

The goal of this tutorial is to introduce the audience to some recent sub-Nyquist sampling methods and give a review of major technical advancements and challenges. Starting from the Shannon sampling technique, we shall discuss different sampling and reconstruction methods for multiband signals. We will give a brief overview of sampling signals in shift-invariant spaces and analog compressive sensing. We conclude with a detailed discussion on the topic of finite-rate-of-innovation signals sampling, which has been widely used for low-rate sampling in many practical applications. The major topics we intend to cover are as follows:

Contents:

1. Shannon-Nyquist sampling framework:
    - 1.1 Sampling of bandlimited and essentially bandlimited signals
    - 1.2 Papoulis generalized sampling
2. Sub-Nyquist sampling:
    - 2.1 Sampling of signals in shift-invariant space:
        - I. Introduction to shift invariant spaces
        - II. Basis splines, exponential splines, and wavelets
        - III. Kernel-based sampling and reconstruction
    - 2.2 Multiband signal sampling:
        - I. Introduction to multiband signals
        - II. Landau's theorem for minimal sampling rate
        - III. Multicoset sampling
        - IV. Xampling: Random demodulated wideband converters
    - 2.3 Analog compressive sensing (CS)
        - I. Parsimonious representation
        - II. Conventional CS: sparse vector recovery problem
        - III. Random demodulators-based analog CS
        - IV. A generalized analog CS
    - 2.4 Finite-rate-of-innovation signal sampling:
        - I. Introduction
        - II. Sampling kernels
        - III. Reconstruction mechanism
        - IV. Applications: Sub-Nyquist RADAR, sub-Nyquist ultrasound imaging, and high-resolution frequency-domain optical-coherence tomography.

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Speakers' biographies:

Chandra Sekhar Seelamantula obtained a Bachelor of Engineering degree in 1999 with a Gold Medal and Best Thesis Award from the University College of Engineering, Osmania University, Hyderabad, with a specialization in Electronics and Communication Engineering. He obtained a direct Ph.D. degree in 2005 from the Indian Institute of Science (IISc.), Department of Electrical Communication Engineering. During April 2005--March 2006, he worked as a Technology Consultant for M/s. ESQUBE Communication Solutions Private Limited, Bangalore, and developed proprietary audio coding solutions. In April 2006, he joined the Biomedical Imaging Group, Ecole Polytechnique Federale de Lausanne, Switzerland, as postdoctoral fellow and specialized in the fields of Image Processing, Optical-Coherence Tomography, Holography, Splines, Sparse Signal Processing, and Sampling Theories. In 2009, he joined the Department of Electrical Engineering, IISc., Bangalore, where he is currently Associate Professor and directs the Spectrum Lab. He is also Associate Faculty at the Centre for Neuroscience, IISc. In 2013, he received the Prof. Priti Shankar Teaching Award from IISc. He is currently also the Vice-Chair of the IEEE Signal Processing Society Bangalore Chapter, and an Associate Editor of IEEE Signal Processing Letters and SPIE Journal of Electronic Imaging.

Satish Mulleti was born in 1982 in Kharagpur, West Bengal, India. He received a Bachelor of Engineering degree in 2005 from the Electronics and Communication Engineering Department, Jalpaiguri Government Engineering College, India. He obtained a Master of Engineering degree in Electrical Engineering in 2009, from the Department of Electrical Engineering, Indian Institute of Technology Kanpur, India. He worked as researcher in Indian Space Research Organization (ISRO), India and TCS Innovation Labs, Mumbai, India. Since August 2011, he is working in the Spectrum Lab, Department of Electrical Engineering, Indian Institute of Science, Bangalore towards his PhD. His research interest includes sampling theory, in particular, finite-rate-of-innovation signal sampling, compressive sensing, and spectral estimation.