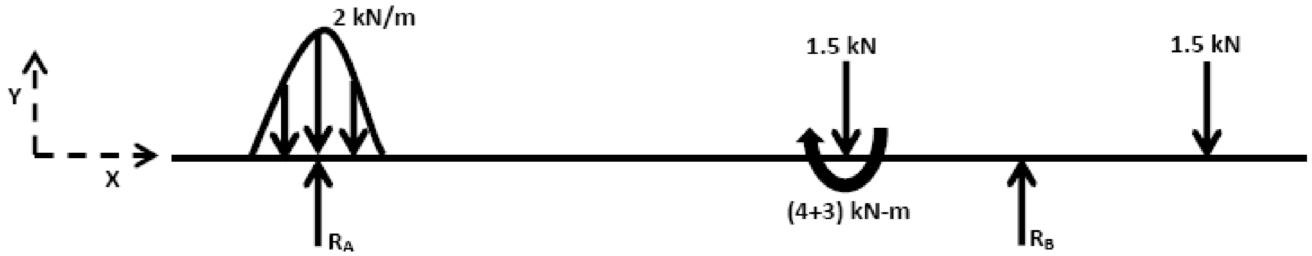
Tutorial -3

ME-101, Division I & IV (2016-2017 Semester-II)

Feb-6, 2017 **Time: 8-00 to 8-55.am**

Ans 1:

Solution: We shall start by drawing the FBD for the BEAM:



Finding Reaction Forces **R_A** and **R_B**:-

Equation for Half-Sine wave: $y = 2 * \sin\left[\frac{(x+1)\pi}{2}\right]$ ⟶ Total Load = $\int_{-1}^{1} 2 * \sin\left[\frac{(x+1)\pi}{2}\right]\, dx \;= \frac{8}{\pi}\; kN$

Taking moments about **A**,

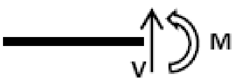
$6*R_B - 7 - 1.5*4 - 1.5*8 = 0$ ⟶ **R_B = 4.167 kN**

$R_B + R_A$ $= 3 + (8/\pi) = 5.54$ kN ⟶ **R_A = 1.38 kN**

In order to determine the SF and BM at a point, we can divide the beam into different regions:

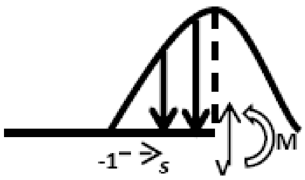
-2<x<-1 ; -1<x<0 ; 0<x<1 ; 1<x<4 ; 4<x<6 ; 6<x<8 ; 8<x<9 [Units: **m**]

- For the 1st region, i.e., **-2<x<-1** ⟶ FBD:



  Therefore, **V = 0** & **M = 0**

- For the 2nd region, i.e., **-1<x<0** ⟶ FBD:



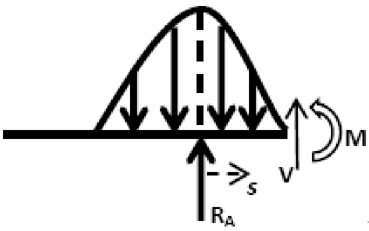
Force summation: $-\int_{-1}^{x} 2 * \sin\left[\frac{(s+1)\pi}{2}\right]\, ds + V(x) = 0$ ⟶ $V(x) = -\dfrac{4*\cos\left[\frac{(x+1)\pi}{2}\right] - 4}{\pi}$

Moment calculation about a point on the section:

$$-\int_{-1}^{x} 2 * \sin\left[\frac{(s+1)\pi}{2}\right] ds * (s-x) + M(x) = 0 \longrightarrow M(x) = \frac{8*\cos\left(\frac{\pi x}{2}\right) - 4\pi x - 4\pi}{\pi * \pi}$$

Therefore, **V = 1.27 kN & M = - 0.462 kNm**

- For the 3rd region, i.e., **0<x<1** ⟶ FBD:

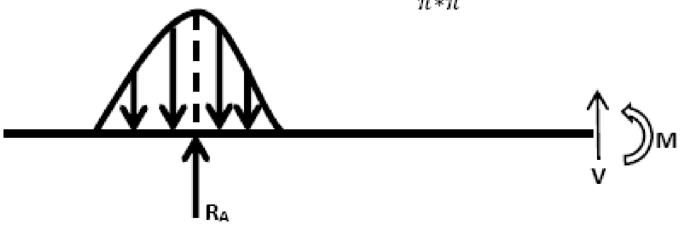


Force summation: $-\int_{-1}^{x} 2 * \sin\left[\frac{(s+1)\pi}{2}\right] ds + V(x) + R_A = 0 \longrightarrow V(x) = -\frac{4*\cos\left[\frac{(x+1)\pi}{2}\right] - 4}{\pi} - R_A$

Moment calculation about a point on the section:

$$-\int_{-1}^{x} 2 * \sin\left[\frac{(s+1)\pi}{2}\right] ds * (s-x) + M(x) - x * R_A = 0 \longrightarrow M(x) = \frac{8*\cos\left(\frac{\pi x}{2}\right) - 4\pi x - 4\pi}{\pi * \pi} + x * R_A$$

- For the 4th region, i.e., **1<x<4** ⟶ FBD:



Force summation: $-\int_{-1}^{1} 2 * \sin\left[\frac{(x+1)\pi}{2}\right] dx + V(x) + R_A = 0 \longrightarrow V(x) = 8/\pi - R_A$

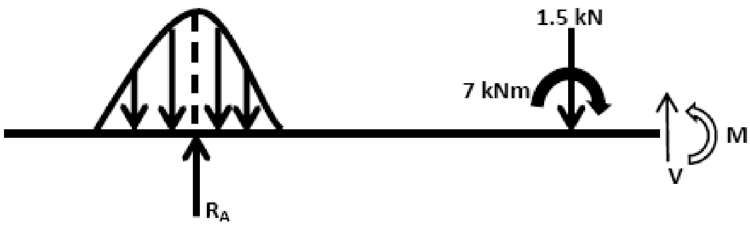
⟶ V(x) = **1.167 kN**

Moment calculation about a point on the section:

$$-\int_{-1}^{1} 2 * \sin\left[\frac{(s+1)\pi}{2}\right] ds * (s-x) + M(x) - x * R_A = 0 \longrightarrow M(x) = (8 * x)/\pi - R_A * x$$

⟶ $M(x) = -1.167 * x$ kNm

- For the 4th region, i.e., **4<x<6** ⟶ FBD:



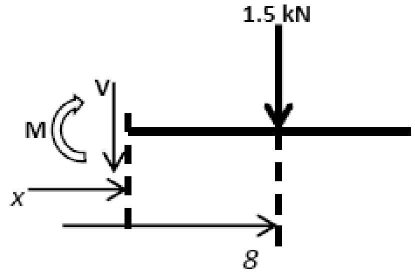
Force summation: $-\int_{-1}^{1} 2 * \sin\left[\frac{(x+1)\pi}{2}\right] dx + V(x) + R_A - 1.5 = 0 \longrightarrow V(x) = 2.667$ kN

Moment calculation about a point on the section:

$$-\int_{-1}^{1} 2 * \sin\left[\frac{(s+1)\pi}{2}\right] ds * (s-x) + M(x) - x * R_A - 7 - (4 - x)*1.5 = 0$$

⟶ M(x) = **(-2.667* x + 13) kNm**

- For the 4^{th} region, i.e., **6<x<8** ⟹ FBD: (We shall consider from the RHS, as that will make our calculations easier)

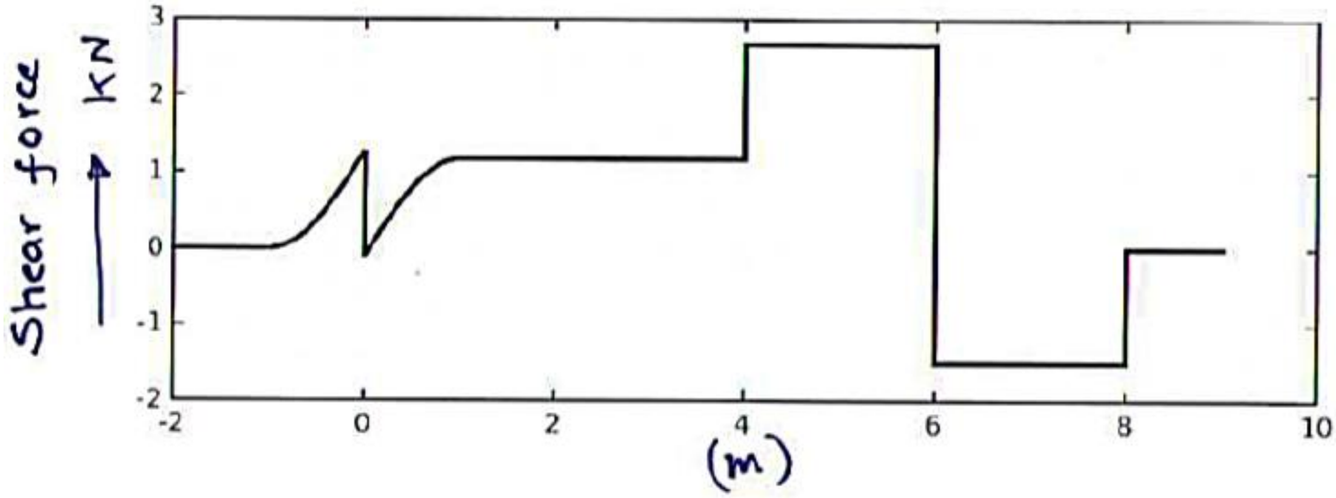


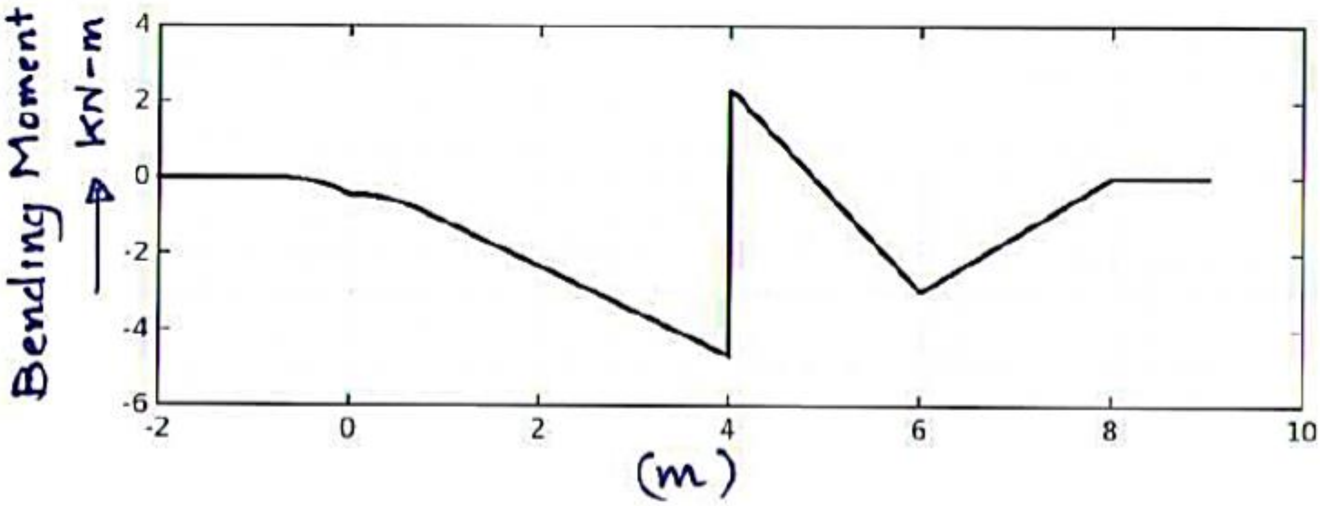
Force summation: - V – 1.5 = 0 ⟹ **V = 1.5** **kN**

Moment calculation about a point on the section:

-M(x) + (8 - x)* (-1.5) = 0 ⟹ **M(x) = - 1.5*(8 – x) kNm**

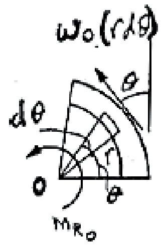
Now we can simply accumulate all the expressions and plot the SFD and BMD for the given beam.





Ans 2.

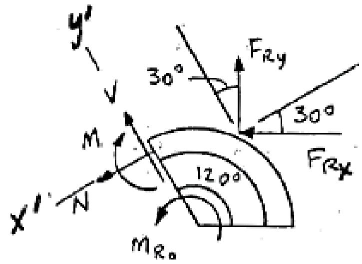
Resultants of distributed load:



$$F_{Rx} = \int_0^{\theta} w_0\,(r\,d\theta)\,\sin\theta = r\,w_0(-\cos\theta)\Big|_0^{\theta} = r\,w_0\,(1-\cos\theta)$$

$$F_{Ry} = \int_0^{\theta} w_0(r\,d\theta)\,\cos\theta = r\,w_0\,(\sin\theta)\Big|_0^{\theta} = r\,w_0\,(\sin\theta)$$

$$M_{Ro} = \int_0^{\theta} w_0\,(r\,d\theta)\,r = r^2\,w_0\theta$$



At $\theta = 120°$,

$$F_{Rx} = r\,w_0\,(1-\cos 120°) = 1.5\,r\,w_0$$

$$F_{Ry} = r\,w_0 \sin 120° = 0.86603\,r\,w_0$$

$+\swarrow\Sigma F_{x'} = 0;\qquad N + 1.5\,r\,w_0\cos 30° - 0.86603\;r\,w_0\sin 30° = 0$

$N = -0.866\,r\,w_0$ **Ans.**

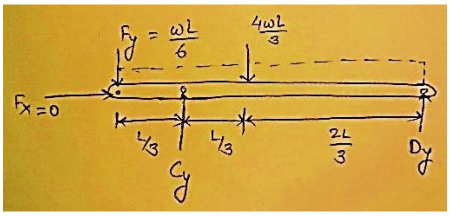
$+\nwarrow\Sigma F_{y'} = 0;\qquad V + 1.5\,r\,w_0\sin 30° + 0.86603\;r\,w_0\cos 30° = 0$

$V = -1.5\,rw_0$ **Ans.**

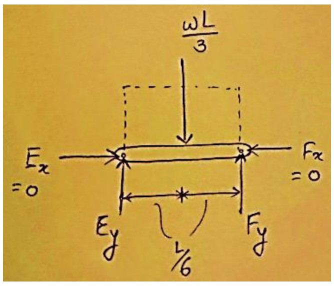
$\circlearrowleft +\Sigma M_o = 0;\qquad -M + r^2 w_0\,(\pi)\left(\dfrac{120°}{180°}\right) + (-0.866\,r\,w_0)r = 0$

$M = 1.23\,r^2 w_0$ **Ans.**

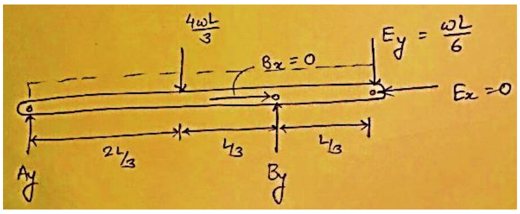
Ans 3.



FBD (a)



FBD (b)



FBD (c)

Support Reactions: From FBD (b),

$$\circlearrowleft +\sum M_E = 0; \qquad F_y\left(\frac{L}{3}\right) - \frac{wL}{3}\left(\frac{L}{6}\right) = 0 \longrightarrow F_y = \frac{wL}{6}$$

$$+\uparrow \sum F_Y = 0; \qquad E_y - \frac{wL}{6} - \frac{wL}{3} = 0 \longrightarrow E_y = \frac{wL}{6}$$

From FBD (a),

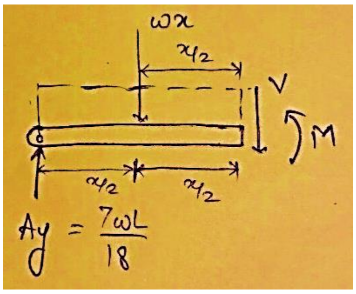
$$\circlearrowleft +\sum M_C = 0; \qquad D_y(L) - \frac{wL}{6}\left(\frac{L}{3}\right) - \frac{4wL}{3}\left(\frac{L}{3}\right) = 0 \longrightarrow D_y = \frac{7wL}{18}$$

From FBD (c),

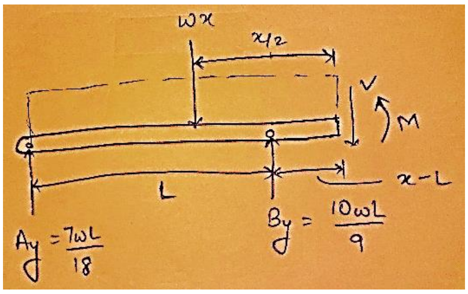
$$\circlearrowleft +\sum M_B = 0; \qquad \frac{4wL}{3}\left(\frac{L}{3}\right) - \frac{wL}{6}\left(\frac{L}{3}\right) - A_y(L) = 0 \longrightarrow A_y = \frac{7wL}{18}$$

$$+\uparrow \sum F_Y = 0; \qquad B_y + \frac{7wL}{18} - \frac{4wL}{3} - \frac{wL}{6} = 0 \longrightarrow B_y = \frac{10wL}{9}$$

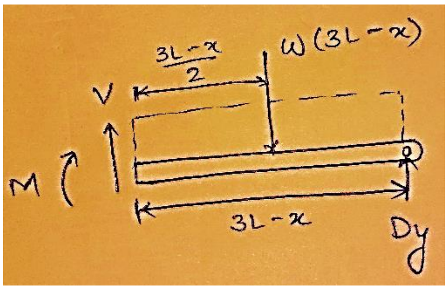
Shear and Moment Functions:



FBD (d)



FBD (e)



FBD (f)

For $0 \leq x \leq L$ [FBD (d)],

$$+\uparrow \sum F_Y = 0; \qquad \frac{7wL}{18} - wx - V = 0 \longrightarrow V = \frac{w}{18}(7L - 18x)$$ … **Ans**

$$\circlearrowleft +\sum M = 0; \qquad M - wx\left(\frac{x}{3}\right) - \frac{7wL}{18}x = 0 \longrightarrow M = \frac{w}{18}(7Lx - 9x^2)$$ … **Ans**

For $L \leq x \leq 2L$ [FBD (e)],

$+\uparrow \sum F_Y = 0;$ $\quad \frac{7wL}{18} + \frac{10wL}{9} - wx - V = 0 \Longrightarrow V = \frac{w}{2}(3L - 2x)$ … **Ans**

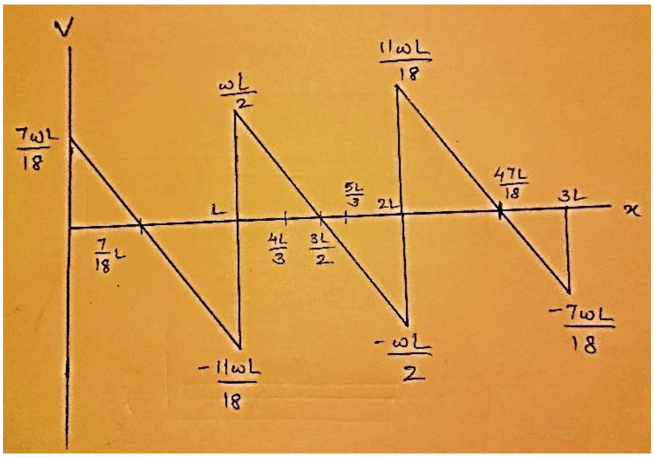
$\circlearrowleft + \sum M = 0;$ $\quad M - wx\left(\frac{x}{2}\right) - \frac{7wL}{18}x - \frac{10wL}{9}(x - L) = 0 \Longrightarrow M = \frac{w}{18}(27Lx - 20L^2 - 9x^2)$
… **Ans**

For $2L \leq x \leq 3L$ [FBD (e)],

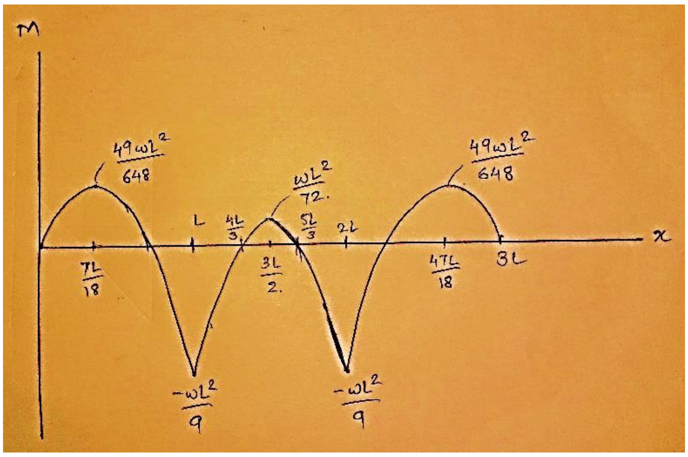
$+\uparrow \sum F_Y = 0;$ $\quad V + \frac{7wL}{18} + w(3L - x) = 0 \Longrightarrow V = \frac{w}{18}(47L - 18x)$ … **Ans**

$\circlearrowleft + \sum M = 0;$ $\quad \frac{7wL}{18}(3L - x) - w(3L - x)\left(\frac{3L - x}{2}\right) - M = 0 \Longrightarrow M = \frac{w}{18}(47Lx - 9x^2 - 60L^2)$
… **Ans**

SFD & BMD:



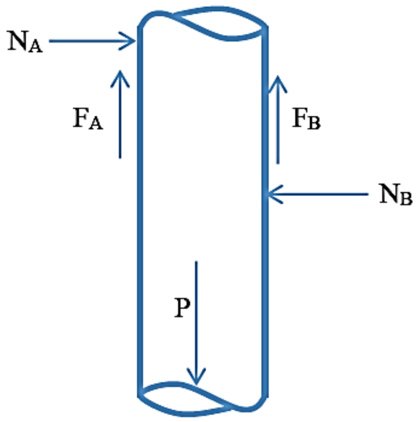
Shear Force Diagram



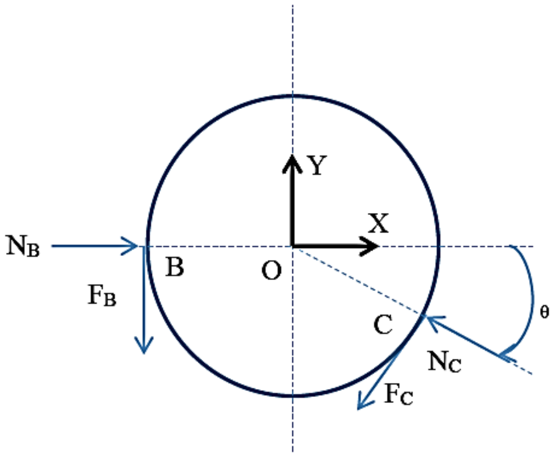
Bending Moment Diagram

Ans 4.

First consider the free body diagram,



Body.1



Body .2

We can see there are 4 unknown force components in body 2. Let us consider a situation, where the co efficient of friction is large enough to keep the road and the cylinder in equilibrium. In such a situation 3 equilibrium equations with 4 unknowns become available. In general, this leads to a situation where 3 unknowns can be written in terms of fourth unknown. This might give some information on F_B / N_B and F_C / N_C. Let us work it out.

The force equilibrium equation for body 2 is given by,

$N_B - N_C \cos\theta - F_C \sin\theta = 0$ -------- 1

$- F_B + N_C \sin\theta - F_C \cos\theta = 0$ -------- 2

Moment equilibrium equation about point O is given by,

$r F_B - r F_C = 0$ -------- 3

[Line of action of normal forces passes through O. hence they do not contribute in the equation. 'r' is the radius of the cylinder.]

From equation 3 we get

$F_B = F_C$ ------- 3a

Replacing F_C by F_B in equation 1 and 2, we get,

$- N_C \cos\theta - F_B \sin\theta = -N_B$ ------- 4

$+N_C \sin\theta - F_B - F_B \cos\theta = 0$ ------- 5

‘4* Sin θ + 5* Cos θ’ yields,

$-F_B \text{Sin}^2 \theta - F_B \text{Cos}^2 \theta - F_B \text{Cos } \theta = -N_B \text{Sin } \theta$

$F_B (1+\text{Cos } \theta) = N_B \text{Sin } \theta$

$$\frac{F_B}{N_B} = \frac{Sin\theta}{1+Cos\theta} \qquad \text{------- 6}$$

We will later show that for the rod to be in equilibrium for $P>0$, N_B is greater than zero.

‘-4* (1+Cos θ) + 5* Sin θ’ yields,

$N_C (\text{Cos } \theta + \text{Cos}^2 \theta) + F_B \text{Sin } \theta (1+\text{Cos } \theta) + N_C \text{Sin}^2 \theta - F_B \text{Sin } \theta (1+\text{Cos } \theta) = N_B (1+\text{Cos } \theta)$

$N_C (1+\text{Cos } \theta) = N_B (1+\text{Cos } \theta)$

Assuming $0 \le \theta < 180$,

$N_C = N_B$ ------- 7

From equation 3a, 6, and 7 we get,

$$\frac{F_B}{N_B} = \frac{F_C}{N_C} = \frac{Sin\theta}{1+Cos\theta} \qquad \text{------- 8}$$

Let us now move on to the free body diagram of rod (Body-1)

Since the body is in static equilibrium, we can impose the force equilibrium equation:

$N_A - N_B = 0$ ------- 9

That is, $N_A = N_B$

$P = F_A + F_B$ ------ 10

If P>0, either F_A or F_B should be more than zero. F_B>0 is possible only if N_B >0, and F_A >0 only if N_A>0, that is N_B >0 (from eq.9). Hence, if P>0 at equilibrium, N_B>0.

Applying 8 in 10, we get,

$$P = \frac{Sin\theta}{1+Cos\theta} N_B + F_A \qquad \text{-------} \ 11$$

If μ_A, μ_B, μ_C, are coefficient of static friction, at A,B and C. Then by law of dry friction,

$$\frac{F_B}{N_B} \le \mu_B \,, \qquad \frac{F_C}{N_C} \le \mu_C \,, \ \& \qquad \frac{F_A}{N_A} \le \mu_A$$

So the minimum values of If μ_A, μ_B, and μ_C, that does not violate the equilibrium equations, in particular, eq.8 and eq.11 are

$$\mu_B, = \mu_C = \frac{Sin\theta}{1+Cos\theta}$$

$$\mu_A = 0$$

Note:

Moment equilibrium equations for the rod give a relation for the offset between the line of action of F_A & F_B. We are not interested in it.