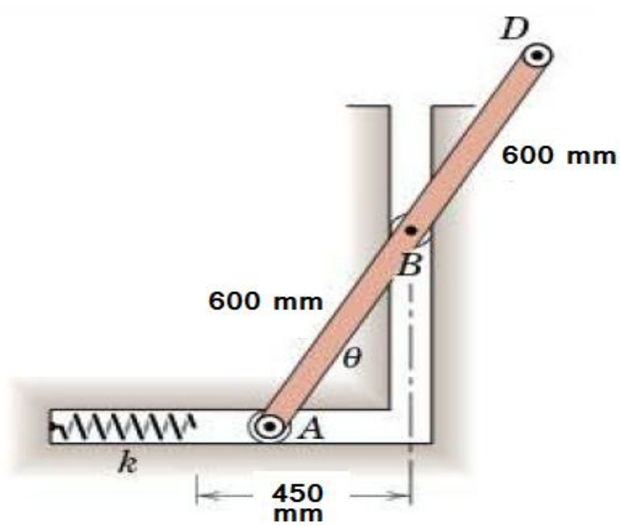
Indian Institute of Technology Guwahati

ME 101: Engineering Mechanics (2016-2017, Sem II)

Tutorial 10 (17.04.2017) (Div 1 & 4)

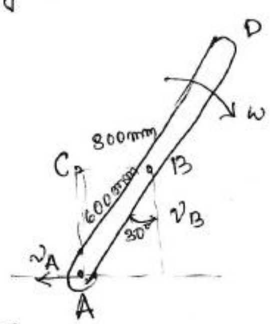
Time: 8:00 AM – 8:55 AM **Full Marks**: 40

Ques.1 – The 1200-mm slender bar has a mass of 20 kg with mass center at *B* and is released from rest in the position for which θ is essentially zero. Point *B* is confined to move in the smooth vertical guide, while end *A* move in the smooth horizontal guide and compressed the spring as the bar falls. Determine *(a)* the angular velocity of bar as the position $\theta = 30^o$ is passed and *(b)* the velocity with which *B* strikes the horizontal surfaces if the stiffness of the spring is 5 kN/m.



<u>Solution</u>: With the friction and mass of the small rollers at A and B neglected, the system may be treated as being conservative.

<u>Part(a)</u> For the first interval of motion from $\theta = 0$ (state 1) to $\theta = 30^\circ$ (state 2), the spring is not engaged, so that there is no V_e term in the energy equation. If we adopt the alternative of treating the work of the weight in the V_g term, then there are no other forces which do work, and $U'_{1-2} = 0$.



Since we have a constrained plane motion, there is a kinematic relation between the velocity v_B of the centre of mass and the angular velocity ω of the bar. This relation is easily obtained by using the instantaneous centre C of zero velocity and noting that $v_B = \overline{CB}\omega$. Thus the kinetic energy of the bar in the 30° position becomes

$$T = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2}20(0.300\omega)^2 + \frac{1}{2}\left(\frac{1}{12}20(1.2)^2\right)\omega^2 = 2.10\,\omega^2$$

With a datum established at the initial position of the mass centre B, our initial and final gravitational potential energies are

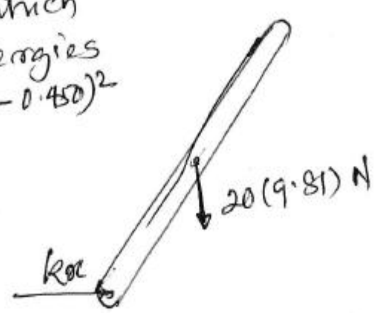
$$V_1 = 0, \quad V_2 = 20(9.81)(0.600\cos 30^\circ - 0.600) = -15.77\ \text{J}$$

We now substitute into the energy equation and obtain

$$T_1 + V_1 + U'_{1-2} = T_2 + V_2 \Rightarrow 0 + 0 + 0 = 2.10\,\omega^2 - 15.77$$

$$\Rightarrow \boxed{\omega = 2.74\ \text{rad/s.}} \quad \text{— Ans}$$

Part (b) We define state 3 as that for which $\theta = 90^\circ$. The initial and final spring potential energies are $V_e = \frac{1}{2}kx^2$, $V_1 = 0$, $V_3 = \frac{1}{2}(5000)(0.600 - 0.450)^2 = 56.3\ \text{J}$



(Alternative Active-force Diagram)

In the final horizontal position, point A has no velocity, so that the bar is, in effect, rotating about A. Hence, it's final kinetic energy is

$$T = \frac{1}{2}I_A\omega^2 \Rightarrow T_3 = \frac{1}{2}\left(\frac{1}{3}20(1.2)^2\right)\left(\frac{v_B}{0.6}\right)^2 = 13.33\,v_B^2$$

The final gravitational potential energy is

$V_g = Wh \Rightarrow V_3 = 20(9.81)(-0.6) = -117.2\ J$

Substituting into the energy equation gives

$T_1 + V_1 + U'_{1-3} = T_3 + V_3$

$\Rightarrow 0 + 0 + 0 = 13.33\, v_B^2 + 56.3 - 117.2$

$\Rightarrow \boxed{v_B = 2.15\ m/s.}$ —— Ans

Alternatively, if the bar alone constitutes the system, the active-force diagram shows the weight, which does positive work, and the spring force kx, which does negetive work. We would then write

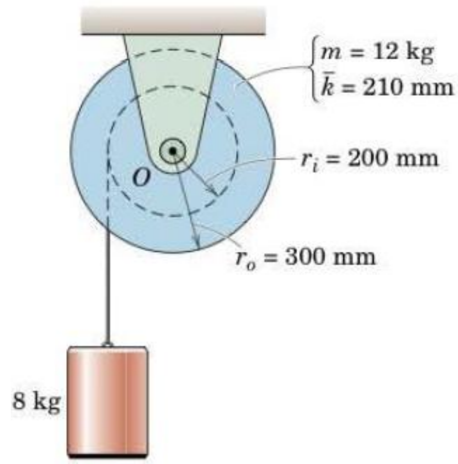
$T_1 + U_{1-3} = T_3$

$\Rightarrow 117.2 - 56.3 = 13.33 v_B^2$

$\Rightarrow v_B = 2.1374\ m/s.$

which is identical with the previous result.

Ques.2 – The velocity of the 8-kg cylinder is 0.3 m/s at a certain instant. What is its speed *v* after dropping as additional 1.5 m? The mass of the grooved drum is 12 kg, its centroidal radius of the gyration is $\bar{k}$ =210 mm, and the radius of its groove is r_i = 200 mm. The frictional moment at *O* is a constant 3 N-m.



Sol.2

$$T_1 + U_{1-2} = T_2$$

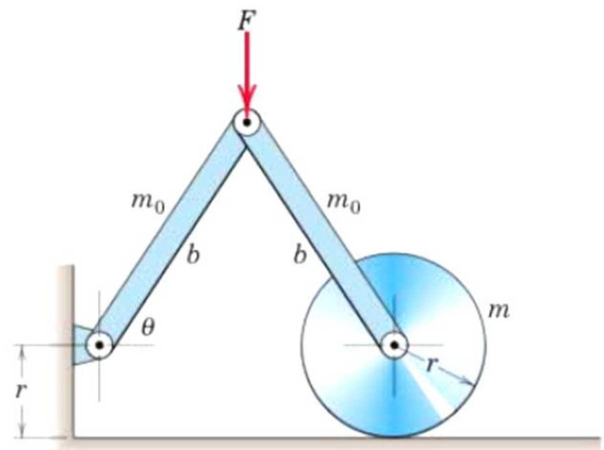
$$T_1 = \frac{1}{2} 8(0.3)^2 + \frac{1}{2} 12(0.210)^2 \left(\frac{0.3}{0.2}\right)^2 = 0.955 \text{ J}$$

$$U_{1-2} = 8(9.81)(1.5) - 3\left(\frac{1.5}{0.2}\right) = 95.2 \text{ J}$$

$$T_2 = \frac{1}{2} 8 v^2 + \frac{1}{2} 12(0.210)^2 \left(\frac{v}{0.2}\right)^2 = 10.62 v^2$$

So $0.955 + 95.2 = 10.62 v^2$, $\underline{v = 3.01 \text{ m/s}}$

Ques.3- A constant force F is applied in the vertical direction to the symmetrical linkage starting from the rest position shown. Determine the angular velocity ω which the links acquire as they reach the position $\theta = 0$. Each link has a mass m_0. The wheel is a solid circular disk of mass m and rolls on the horizontal surface without slipping



Sol. 3 Note: the wheel has no motion in initial or final positions so $\Delta T_{wheel} = 0$

$U' = \Delta V_g + \Delta T$; $\quad U' = Fb\sin\theta$

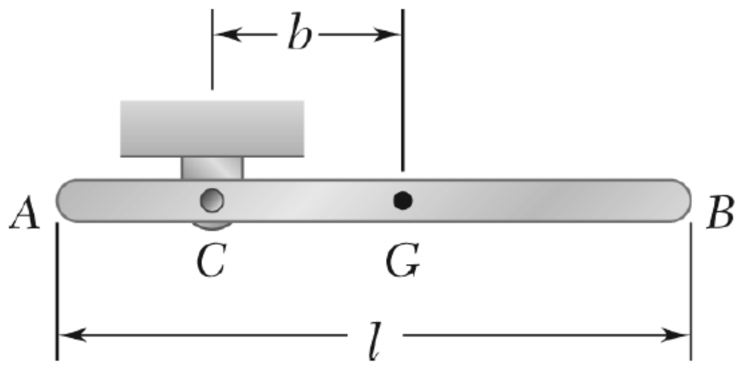
$$\Delta V_g = -2m_0 g \frac{b}{2}\sin\theta$$

$$\Delta T = 2\left(\frac{1}{2} I_C \omega^2\right) = \frac{1}{3} m_0 b^2 \omega^2$$

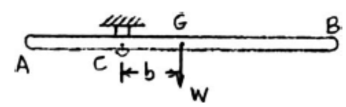
Thus $\quad Fb\sin\theta = -m_0 g b \sin\theta + \frac{1}{3} m_0 b^2 \omega^2$

$$\omega = \sqrt{\frac{3(F + m_0 g)\sin\theta}{m_0 b}}$$

Ques.4– A slender rod of length l is pivoted about a Point C located at a distance b from its center G. It is released from rest in a horizontal position and swings freely. Determine (a) the distance b for which the angular velocity of the rod as it passes through a vertical position is maximum, (b) the corresponding values of its angular velocity and of the reaction at C.



SOLUTION 4



Position 1. $\bar{v} = 0, \quad \omega = 0 \quad T_1 = 0$

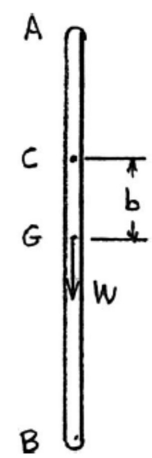
Elevation: $h = 0 \quad V_1 = mgh = 0$

Position 2. $\bar{v}_2 = b\omega_2$

$$I = \frac{1}{12}ml^2$$

$$T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2$$

$$= \frac{1}{2}m\left(b^2 + \frac{1}{12}l^2\right)\omega_2^2$$



Elevation: $h = -b \quad V_2 = -mgb$

Principle of conservation of energy.

$$T_1 + V_1 = T_2 + V_2: \quad 0 + 0 = \frac{1}{2}m\left(b^2 + \frac{1}{12}l^2\right)\omega_2^2 - mgb$$

$$\omega_2^2 = \frac{2gb}{b^2 + \frac{1}{12}l^2}$$

(a) Value of b for maximum ω_2.

$$\frac{d}{db}\left(\frac{b}{b^2+\frac{1}{12}l^2}\right)=\frac{\left(b^2+\frac{1}{12}l^2\right)-b(2b)}{\left(b^2+\frac{1}{12}l^2\right)^2}=0 \qquad b^2=\frac{1}{12}l^2 \qquad b=\frac{l}{\sqrt{12}} \blacktriangleleft$$

(b) Angular velocity.

$$\omega_2^2=\frac{2g\frac{l}{\sqrt{12}}}{\frac{l^2}{12}+\frac{l^2}{12}}$$

$$=\sqrt{12}\frac{g}{l}$$

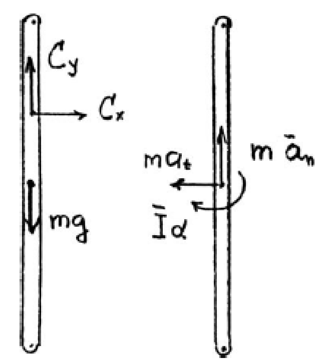
$$\omega_2=12^{1/4}\sqrt{\frac{g}{l}} \qquad \omega_2=1.861\sqrt{\frac{g}{l}} \blacktriangleleft$$

Reaction at C.

$$a_n=b\omega_2^2$$

$$=\frac{l}{\sqrt{12}}\sqrt{12}\frac{g}{l}$$

$$=g$$



$$+\uparrow \Sigma F_y=ma_n: \quad C_y-mg=mg$$

$$C_y=2mg$$

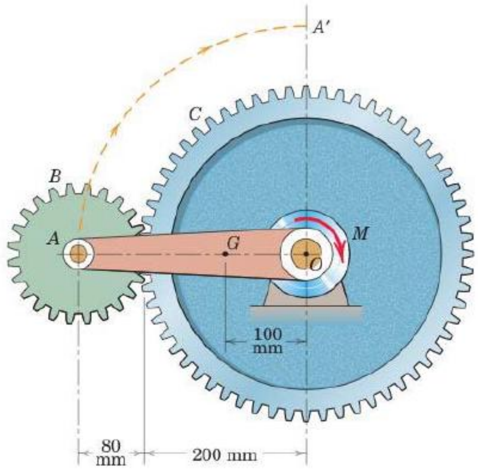
$$+\circlearrowright \Sigma M_C=mba_t+\bar{I}\alpha: \quad 0=(mb^2+\bar{I})\alpha$$

$$\alpha=0, \quad a_t=0$$

$$\xrightarrow{+} \Sigma F_x=ma_t: \qquad C_x=-ma_t=0$$

$$\mathbf{C}=2mg\uparrow \blacktriangleleft$$

Ques.5 – For the assembly shown, arm *OA* has a mass of 0.8 kg and a radius of gyration about *O* of 140 mm. Gear *B* has a mass of 0.9 kg and may be treated as a solid circular disk. Gear *C* is fixed in the vertical plane and cannot rotate. If a constant moment $M = 4$ N. m is applied to arm *OA*, initially at rest in the horizontal position shown, calculate the velocity *v* of point *A* as it reaches the top A'.



Sol. 5

For the top position $\omega_B = \dfrac{v}{0.080}$, $\omega_{OA} = \dfrac{v}{0.280}$

For entire system $U'_{1\text{-}2} = \Delta T + \Delta V_g$

$$U'_{1\text{-}2} = M\theta = 4(\pi/2) = 6.28 \text{ J}$$

$$\Delta T_{OA} = \tfrac{1}{2} I_O \omega_{OA}^2 = \tfrac{1}{2} 0.8(0.140^2)(v/0.280)^2 = 0.1\, v^2 \text{ J}$$

$$\Delta T_B = \tfrac{1}{2} m v^2 + \tfrac{1}{2}\bar{I}\omega^2 = \tfrac{1}{2} 0.9 v^2 + \tfrac{1}{2}\left[\tfrac{1}{2} 0.9 \times 0.080^2\right]\left(\frac{v}{0.080}\right)^2$$
$$= 0.675\, v^2 \text{ J}$$

$$(\Delta V_g)_{OA} = mgh = 0.8(9.81)(0.100) = 0.785 \text{ J}$$

$$(\Delta V_g)_B = mgh = 0.9(9.81)(0.280) = 2.47 \text{ J}$$

Thus $6.28 = 0.1 v^2 + 0.675 v^2 + 0.785 + 2.47$,

$$v^2 = 3.90 \ (\text{m/s})^2$$

$$\underline{v = 1.976 \text{ m/s}}$$