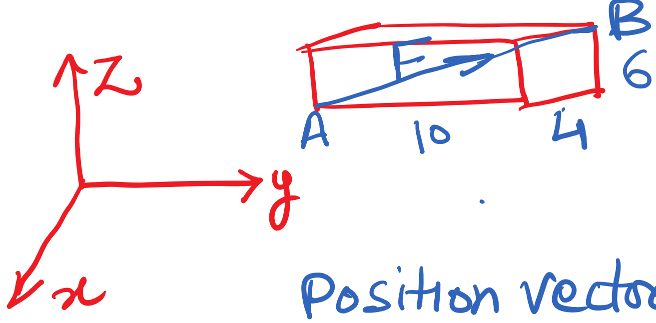
# Force: Representing in 3D



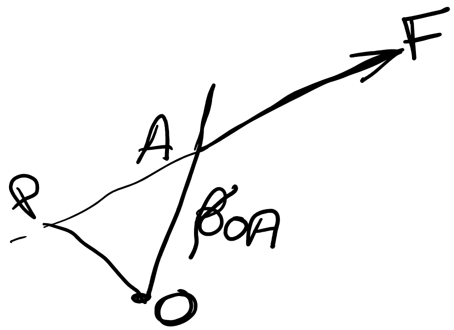

Position vector $P_{AB} = -4\hat{i} + 10\hat{j} + 6\hat{k}$

$$|P_{AB}| = \sqrt{4^2 + 10^2 + 6^2}$$

$$\hat{P}_{AB} = \frac{P_{AB}}{|P_{AB}|} = \frac{-4\hat{i} + 10\hat{j} + 6\hat{k}}{12.32}$$

$$= .324\hat{i} + .811\hat{j} + .4867\hat{k}$$

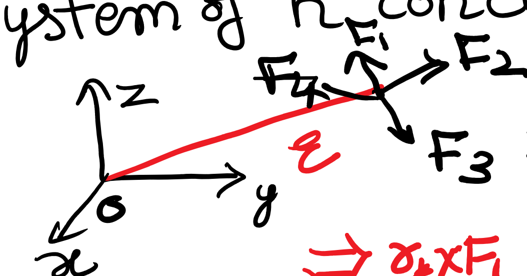
# Moment of a force about a Point




$= \quad M_O = r \times F$

$= r_{OA} \times F$

# System of n concurrent force




$\Rightarrow M = M_1 + M_2 + M_3 + M_4$

$\Rightarrow r_1 \times F_1 + r \times F_2 + r \times F_3 + r \times F_4$

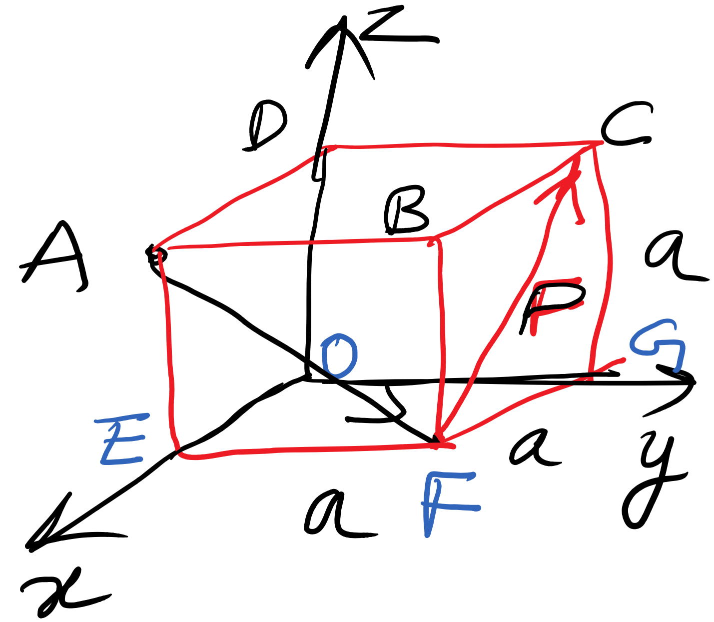
## Problem I:

a cube of side a

(i) Moment of P about A

(ii) About the Edge AB

(iii) About the diagonal AG




## Solution:

$$r_{F/A} = (a-a)\hat{i} + (a-0)\hat{j} + (0-a)\hat{k}$$

$$= a\hat{j} - a\hat{k}$$

$$P_{F/C} = (0-a)\hat{i} + (a-a)\hat{j} + (a-0)\hat{k} = -a\hat{i} + a\hat{k}$$

$$\hat{p}_{F/C} = \frac{p_{F/C}}{|p_{F/C}|} \Rightarrow \frac{-a\hat{i} + a\hat{k}}{a\sqrt{2}} \Rightarrow -\frac{\hat{i}}{\sqrt{2}} + \frac{\hat{k}}{\sqrt{2}}$$

Now

$$P = P\,\hat{p}_{F/C} = -\frac{P\hat{i}}{\sqrt{2}} + \frac{P\hat{k}}{\sqrt{2}}$$

Moment about A

$$M_A = r_{F/A} \times$$

$$= a(j - \hat{k}) \times \left(\frac{P}{\sqrt{2}}(-\hat{i} + \hat{k})\right)$$

$$= a\hat{k} + a\hat{j} + a\hat{i})\frac{P}{\sqrt{2}}$$

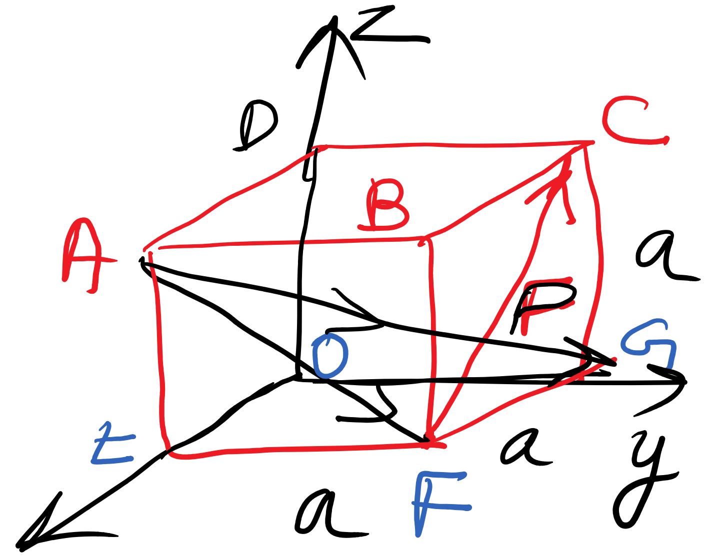
# Moment about a line

$$m_L = M \cdot \lambda$$

# ~~Moment about~~ AB

$$M_{AB} = i \cdot M_A =$$

Moment about diagonal




$$\hat{P}_{AG} = \frac{P_{AG}}{|P_{AG}|} = \frac{(0-a)\hat{i} + (a-0)\hat{j} + (0-a)\hat{k}}{\sqrt{a^2+a^2+a^2}}$$

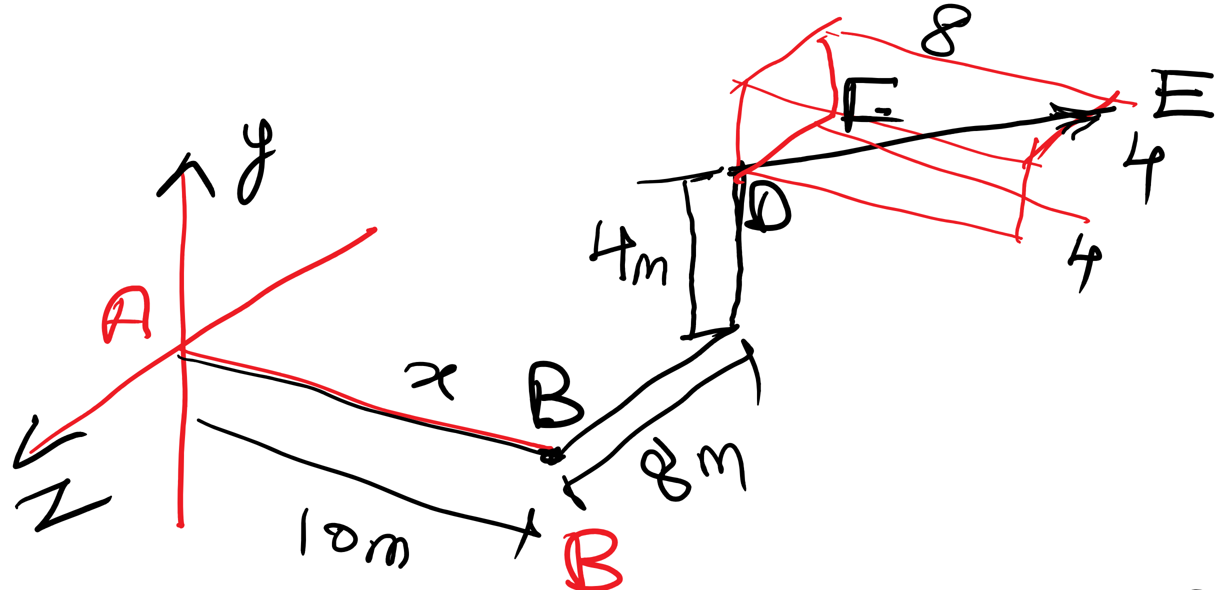
$$= \frac{-a\hat{i} + a\hat{j} - a\hat{k}}{a\sqrt{3}} = \frac{1}{\sqrt{3}}(-\hat{i} + \hat{j} - \hat{k})$$

$$M_{AG} = \hat{\rho}_{AG} \cdot M_A$$

$$= \frac{1}{\sqrt{3}}(-\hat{i} + \hat{j} - \hat{k}) \cdot \frac{aP}{\sqrt{2}}(\hat{i} + \hat{j} + \hat{k})$$

$$\frac{aP}{\sqrt{6}}(-1 + 1 - 1) = -\frac{aP}{\sqrt{6}}$$

# Class Problem




(i) Determine the moment of the 100 N F as shown in Fig.

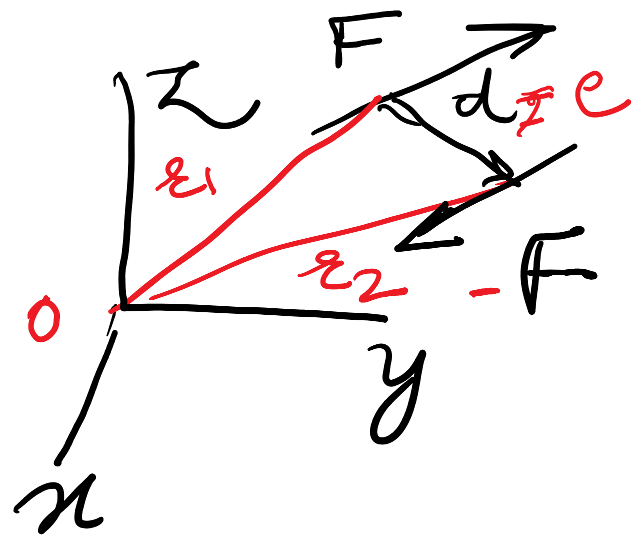
(a) about point A

(b) About point B

# Couple & couple moment

The couple is formed by any two equal parallel forces that have opposite senses

$$e = \vec{r}_1 - \vec{r}_2$$



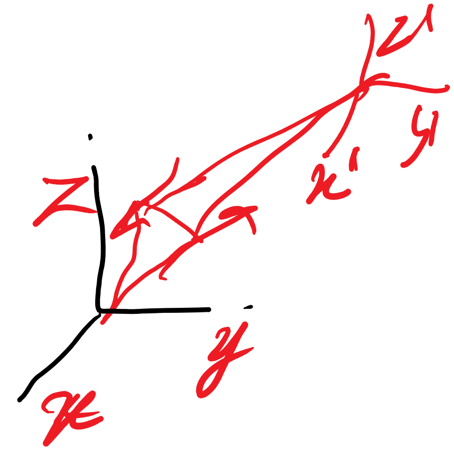

$$M = r_1 \times F_1 + r_2 \times (-F)$$

$$= (r_1 - r_2) \times F$$

$$M = e \times F$$

* Couple moment as a free vector

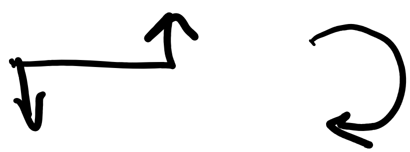
\# Couple has the same moment about every point in space

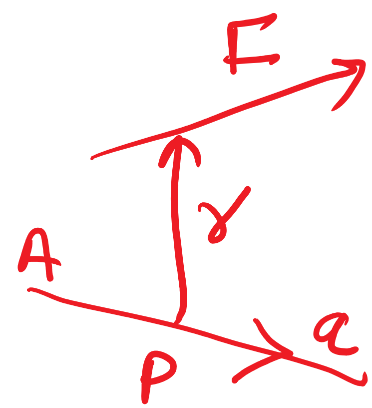



$$M = r \times F$$

Its direction which is the direction of moment. $|C| = |F| d$

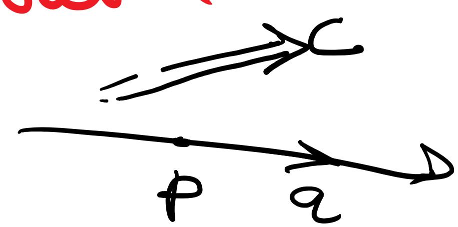
A common notation for couple in a plane






* Moment of a couple about a line

$$M_{AA} = (r \times F) \cdot a$$




$$M_{AA} = C \cdot a$$

# Equivalent force System

1. Equality & Equivalance of Vectors

* Two vectors are equal if they have the same dimensions, magnitude and direction.

# Two vectors are equivalent in a certain capacity if each produces the very same effect in this capacity

# Equal vectors need not always be equivalent, it dependends entirely on the situation at hand.

* The equality of two vectors is determined by the vectors themselves, and the equivalence between two vectors is determined by the ~~task~~ knowing the vectors.

Three classes concerning equivalance of vectors.

1. Free Vectors:

2. Transmissible Vectors:

3. Bound Vectors-:

**Free vectors:** — Vectors may be positioned any where in space without loss or change

**Transmissible Vectors (Sliding Vectors):** Vector may be moved along their line of action without change of meaning.

**Bound Vectors:-** Situations in which the vectors must be applied at definite points (Fixed vectors)

# Equivalent Force Systems

Two force systems are equivalent if they are capable of inducting the same motion of the rigid body

Two capacity of the force system

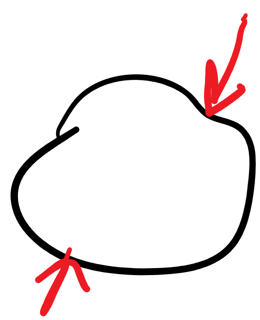
(a) Push or pull on the body in any direction

(B) Turning action about any point in space

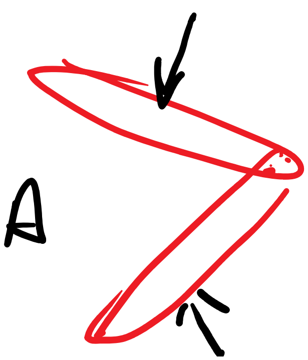
Restsictions

single body Multibody Deformable body

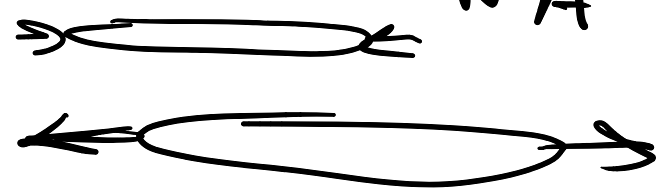
A.



NA

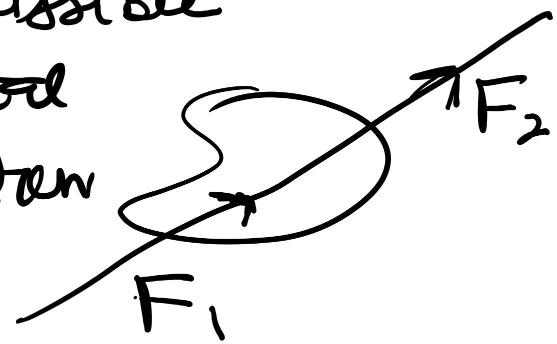


NA

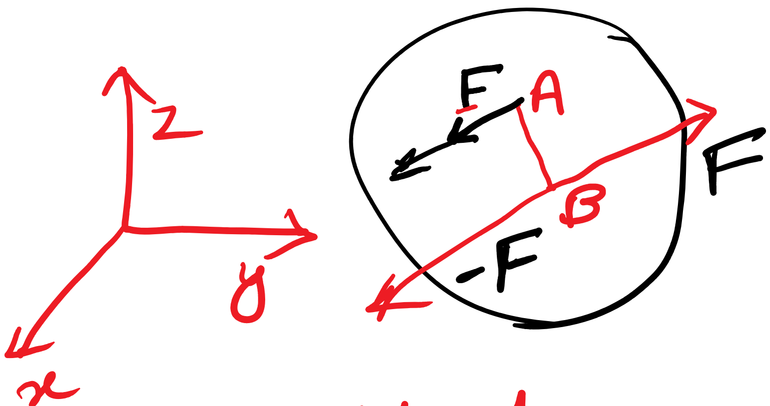


A single force on a single rigid body

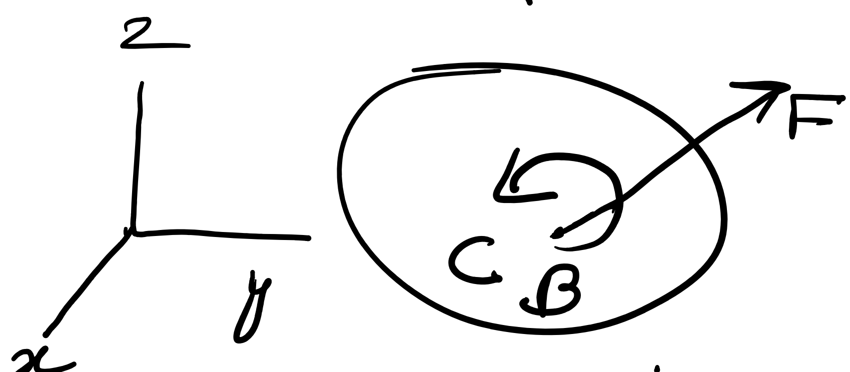
force on a rigid body is transmissible vector. Line of action plays a vital role not the point of application




$|F_1| = |F_2|$




$\underline{F}$ applied at a point




$\underline{M} = \underline{C} = \underline{r}_{BA} \times \underline{F}$

i) $\underline{F}$ applied at a point B

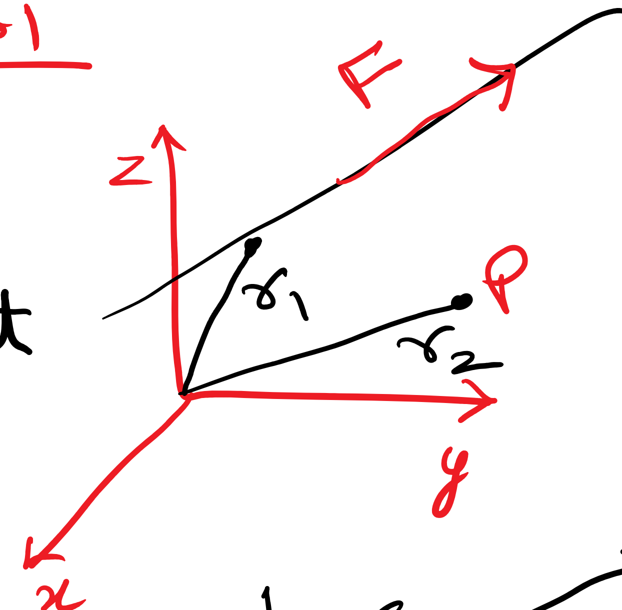
ii) $\underline{M} = \underline{r}_{BA} \times \underline{F}$

## Mathematical Definition

Two force systems are said to be equivalent if they have same total force sum $\underline{F}$ and same total moment sum $\underline{M}_A$ about one point A.

Example 4.1

$$\underline{F} = 6\hat{i} + 3\hat{j} + 6\hat{k}$$
$$r_1 = 2\hat{i} + \hat{j} + 10\hat{k}$$




Replace this force by an equivalent force system, going through P.

$$r_2 = 6\hat{i} + 10\hat{j} + 12\hat{k}$$

$$\underline{C} = (r_2 - r_1) \times F,$$

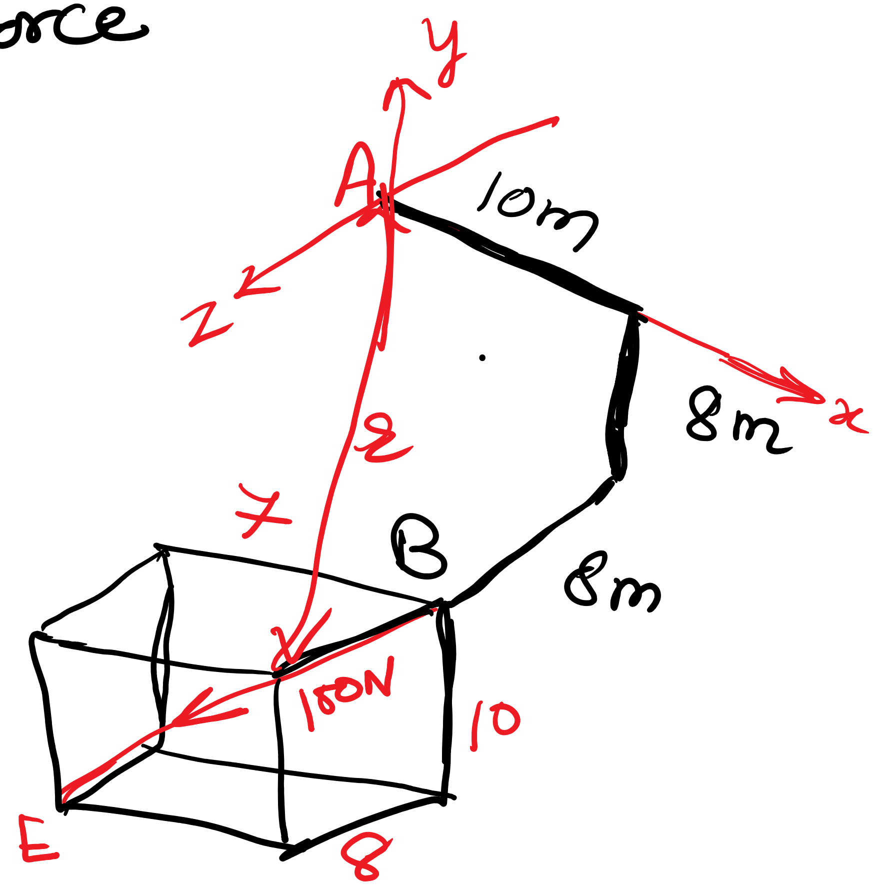
2nd Problem: What is the equivalent force system at position A for the 100 N force

Steps:

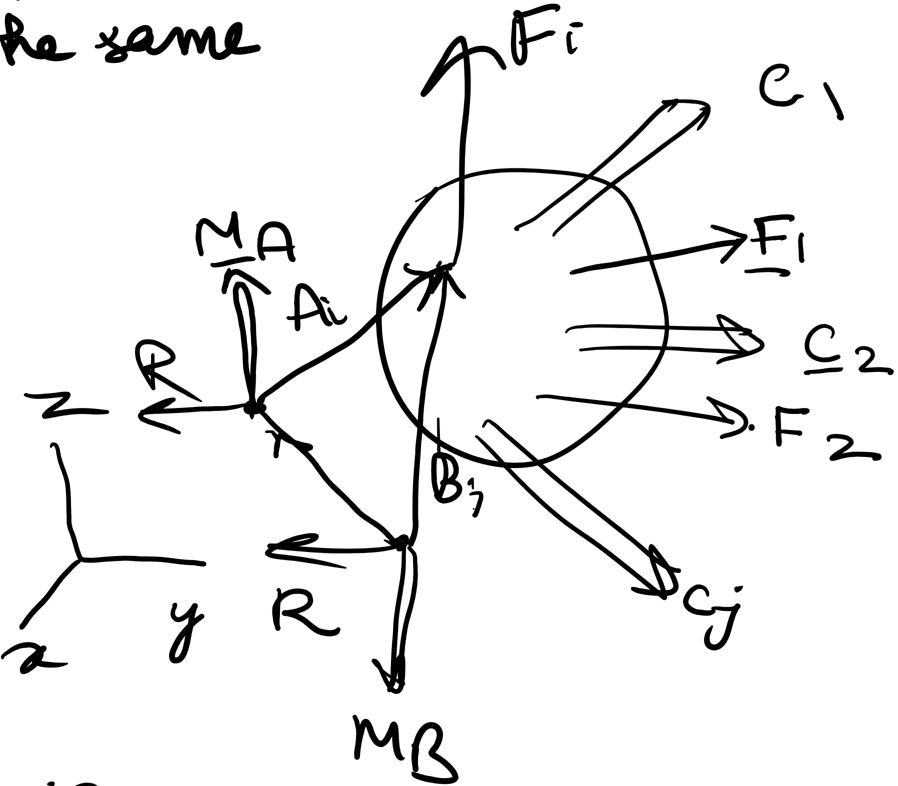
$$\underline{F} = F \frac{\overline{BE}}{|BE|}$$

$$C = \underline{r} \times F$$

$$= r_{BA} \times F$$



Moment sum $\underline{M}_B$ of two equivalent force system about any point B in the same

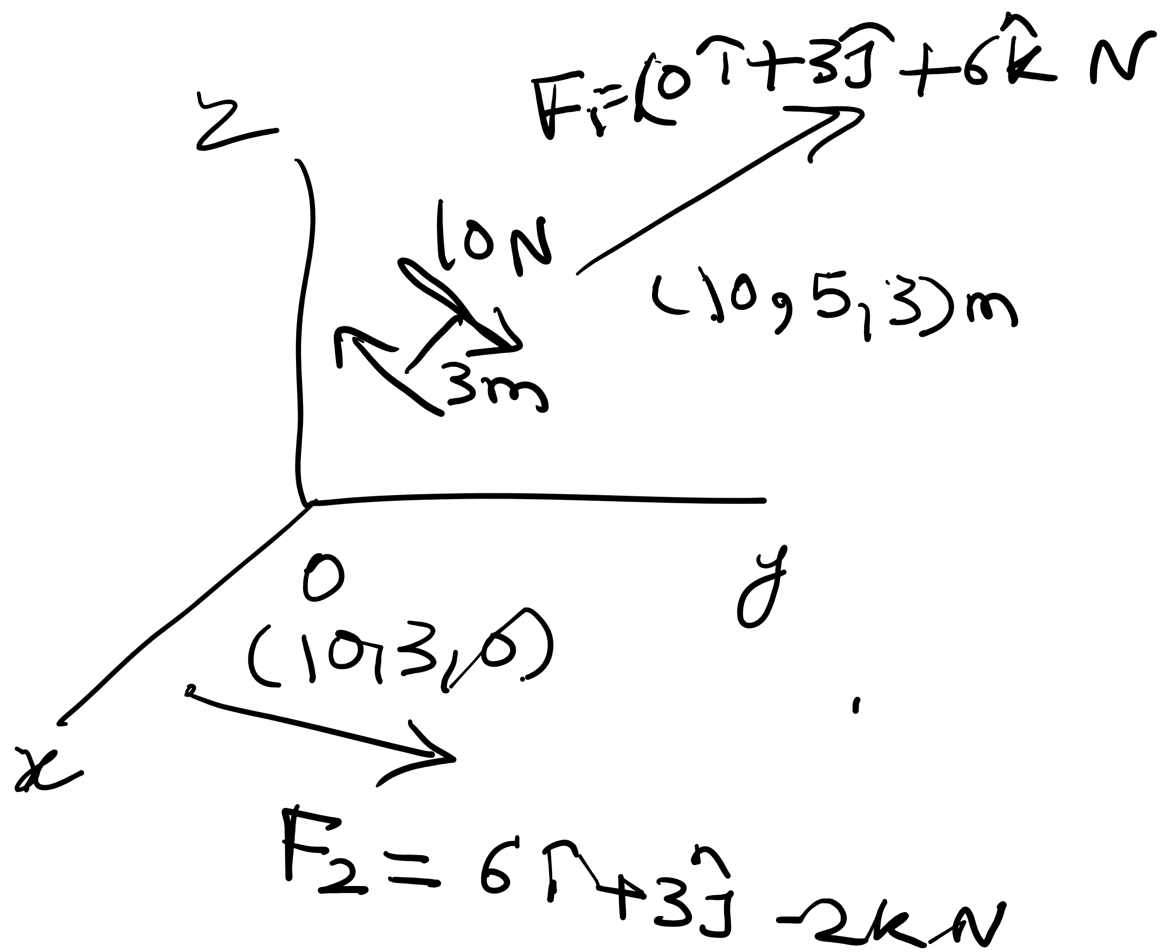



$$\underline{M}_B = \sum_{i=1}^{n} \underline{B}_i \times \underline{F}_i + \sum_{j=1}^{n} \underline{C}_j$$

$$= \sum_{i=1}^{n} (A_i + BA) \times F_i + \sum C_j$$

$$= BA \times \sum F_i + \sum \underline{\underline{A_i \times F_i}} + \sum C_j$$

$$= BA \times R + \underline{M}A$$




Find the resultant of the system at the origin O.

The Simplest resultant of a given force system is a wrench

Proof: Let the resultant at A be $\underline{R}$, $\underline{M}A$. The resultant at point B is $\underline{R}$, $\underline{M}_B$ with

$$\underline{M}_B = \underline{M}A + \underline{BA} \times \underline{R}$$

$$= (\underline{M}A)_{\parallel} + \underline{M}A_{\perp} + \underline{BA} \times \underline{R}$$

$MA_{\parallel}$ = component of parallel to R

$MA_{\perp}$ = component $\perp$ to R

It is possible to choose an approach point B such that

$$M_B = (MA)_{\parallel}$$

if $(M_A)_{\parallel} = 0$ i.e if $\underline{M}_A \cdot R = 0$ then

The simplest resultant is a single force / single couple / null system

(a) A single force if $R \neq 0$

(b) A single couple $M_A = C$ if $R = 0$

(c) A null system if $R = 0$, $M_A = 0$

## Simplest Resultant of special force system

(a) Concurrent force system

the simplest resultant of a concurrent force system is a single force through the point of concurrence

(b) Coplanar force system.

— force in one plane

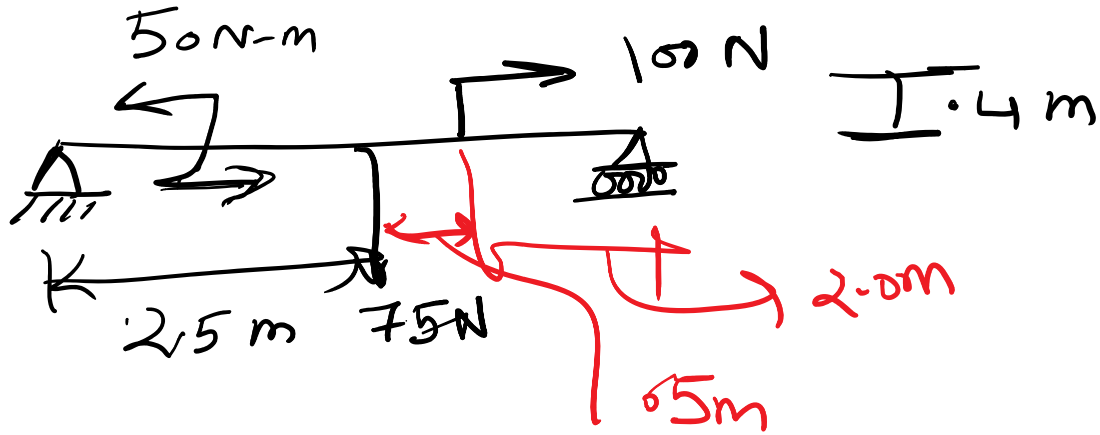
→ couples with moment along normal to the plane

$R \cdot M_A = 0$ , $R \perp M_A$

(c) Parallel force system

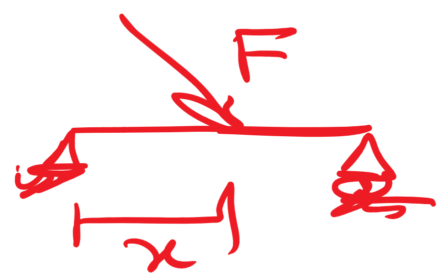
forces parallel to a line

couples with moment normal to this plane.




Compute the simplest resultant for the loads shown

$$F_R = 100i - 75j \ N$$




Let $x$ be the intercept.

$$-75\bar{x} = 50 - 2.5 \times 75 - .4 \backslash 1$$

## Equation of Equilibrium

Necessary Conditions for a rigid body

$$F_R = 0, \quad C_R = 0$$

This condition is sufficient to maintain an initially stationary body in a state of equilibrium.

Further

$$\sum_{i=1} F_i = 0$$

$$\sum \rho_i \times F_i + \sum_i C_i = 0$$

$$\underline{F} = 0$$

$$\underline{M}_A = 0 \Rightarrow \underline{M}_A = \underline{M}_O + AO \times \underline{F}$$

$$\sum_i (F_x)_i = 0; \quad \sum_i (F_y)_i = 0; \quad \sum_i (F_z)_i = 0$$

$$\sum (M_x)_i = 0; \quad \sum_i (M_y)_i = 0; \quad \sum_i (M_z)_i = 0$$

# Free Body Diagram

For the purpose of proper application of Euler's axioms to a system consisting of a set of n bodies or particle, a body, a finite or an infinitesimal part of body. It is necessary that

1. The system should be well-identified and its sketch be drawn in isolation from its surroundings, and
2. The external forces exerted by the surroundings on the system should be drawn on it.

.5 Such a diagram is called a free body diagram (FBD)

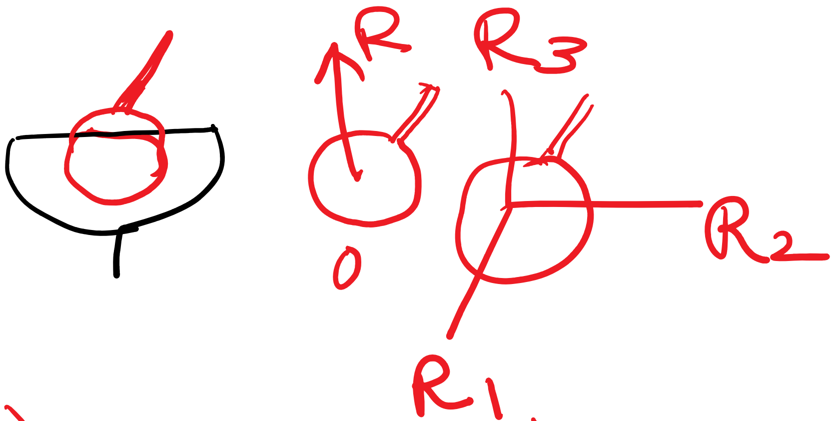
The force exerted by one part of the system on the another part of the system are called internal forces and these should not be shown in the FBD.

Common Support and Reactions

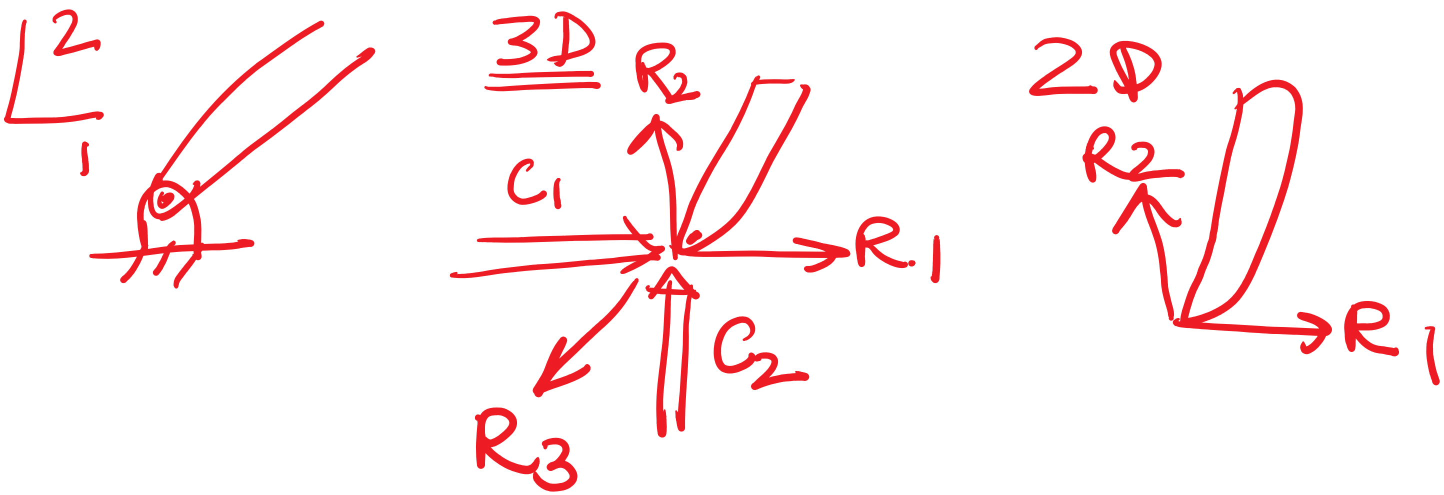
F

The type of load exerted by a support on a body depends on the type of constraint provided by it to the body. It excerts a force component corresponding to each of the displacement components constrained by it, and a moment component corresponding to each of the rotation component constrainted by it.

(a) Smooth ball and socket joint

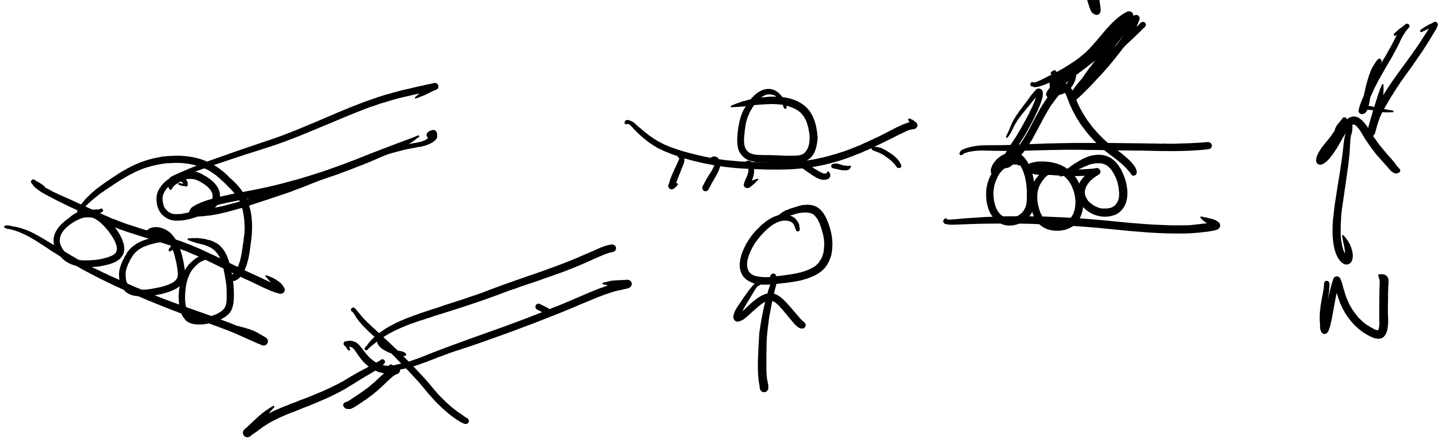



(b) Smooth hinge joint with axis $e_3$

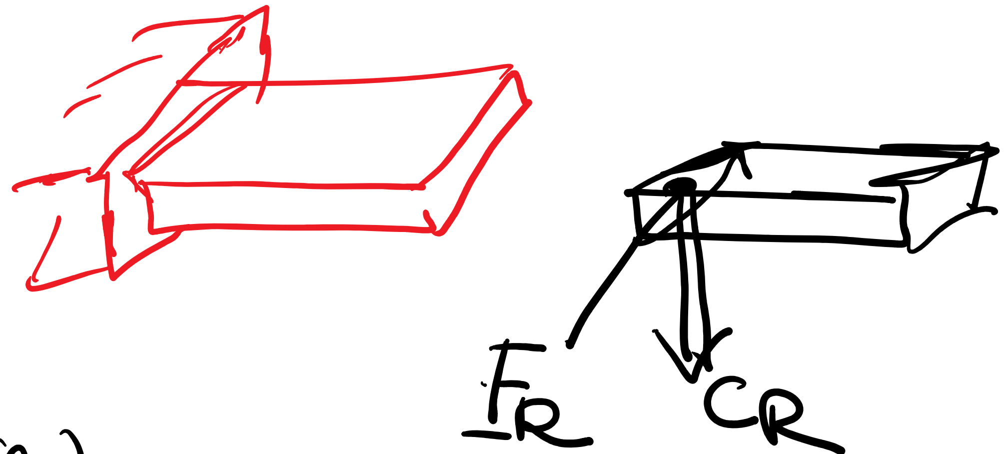


(c) Pin in Smooth slot

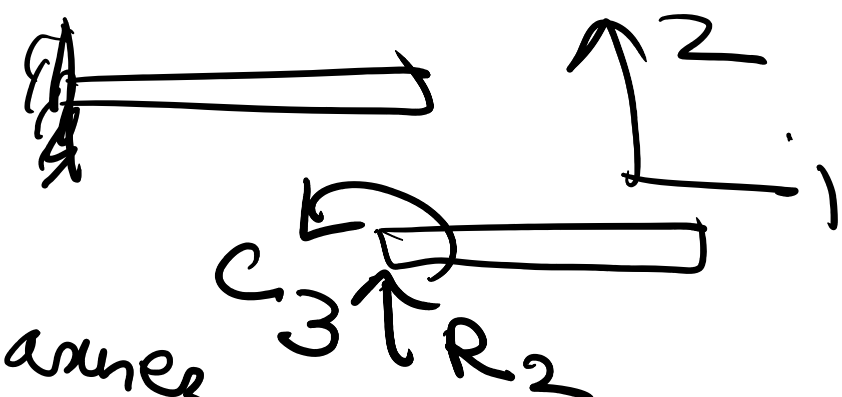

(d) Smooth Roller Support



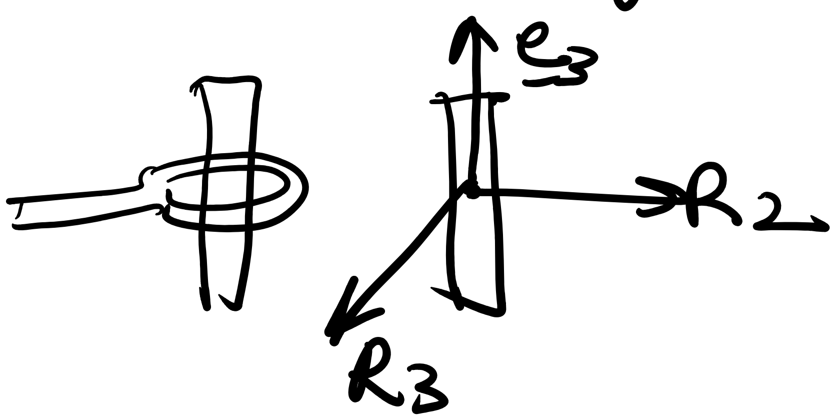
(e) Fixed built-in end (3D load)



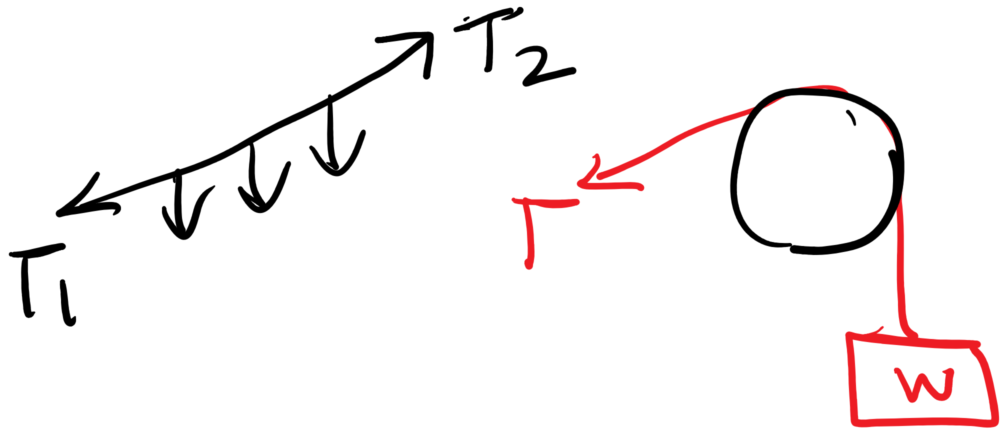

2D load



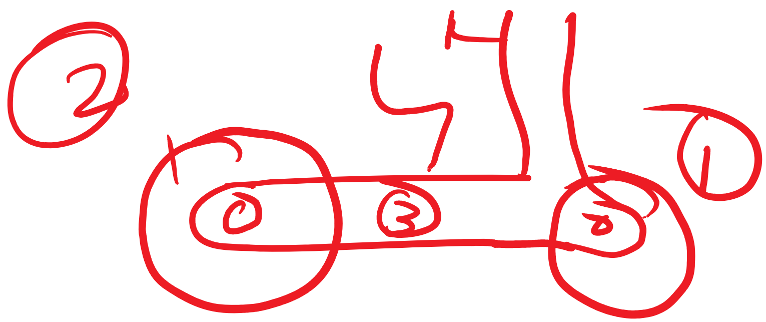

(f) Smooth eye bolt with axes

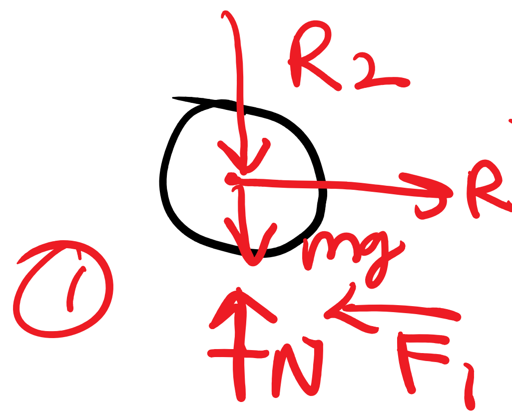


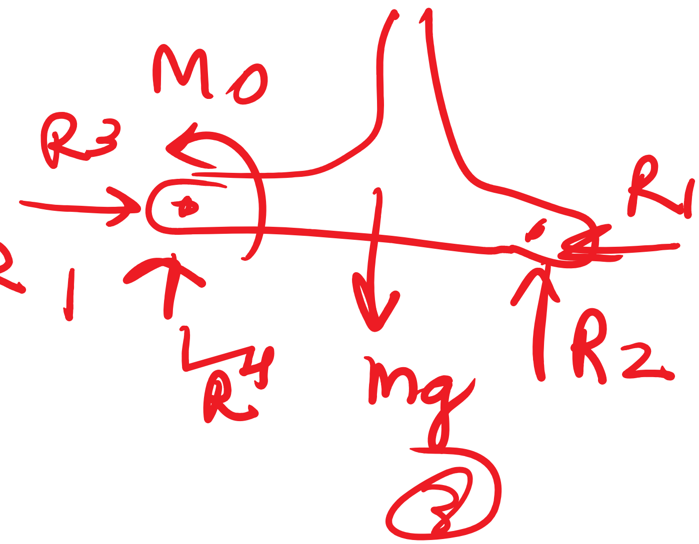
# h. Flexible Cable







For an internal section of body, The distributed force across the section can be replaced by its resultant at the centroid $C^*$ of the cross section, consisting of a resultant force $F_R$ and a resultant couple with moment $C_R$ at $C^*$. For the case of the bar, can be resolved in terms of right handed unit triad.

$$F_R = F_t e_t + F_n e_n + F_b e_b, \quad C_R = C_t e_t + C_n e_n + C_b e_b$$

Axial component of $F_R$ = Normal force N

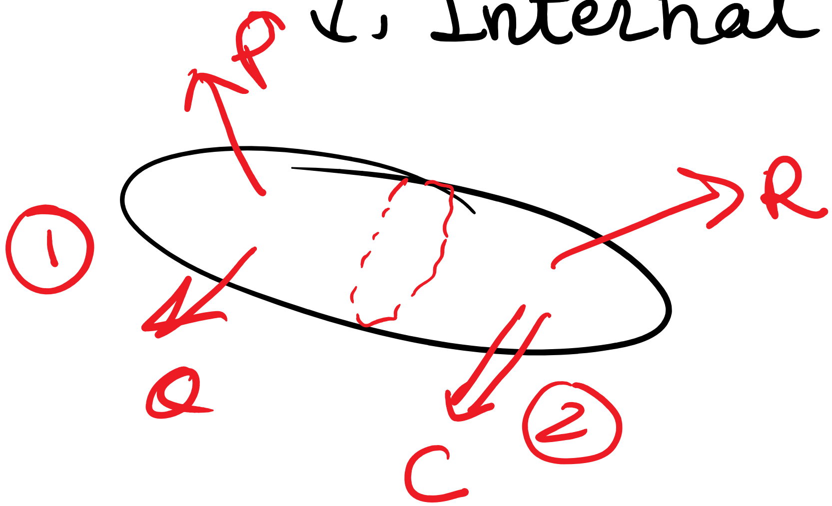
Transverse component = Shear force S.

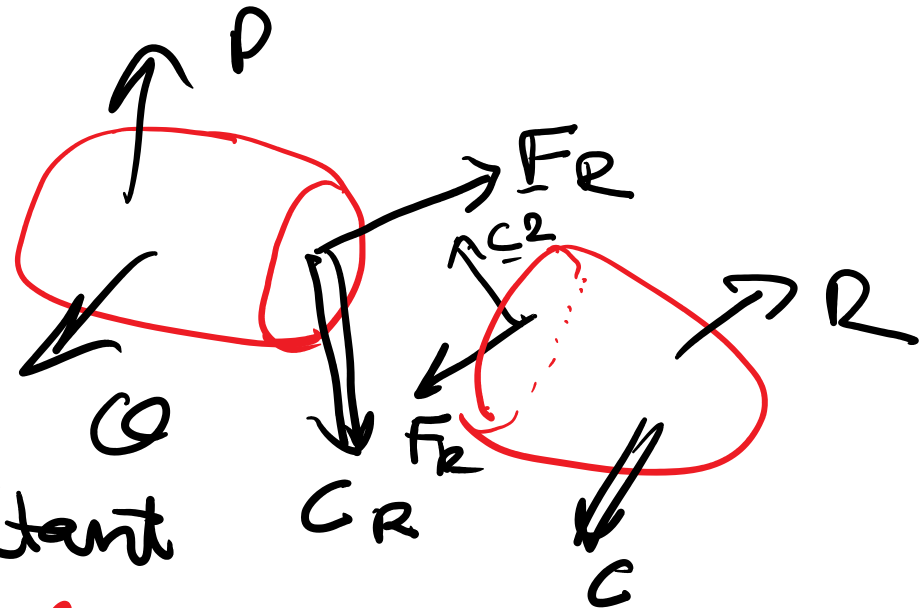
axial component of $C_R$ = Twisting Moment $M_t$

Transverse component of $C_R = M_b$

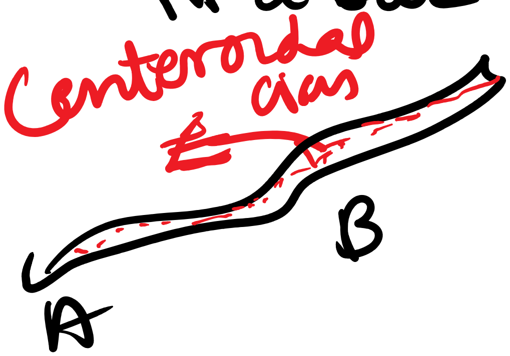
$$\left.\begin{aligned} N &= F_R \cdot e_t = F_t \\ S &= F_n e_n + F_b e_b \end{aligned}\right\} \begin{aligned} M_t &= C_R \cdot e_t = C_t \\ M &= C_n e_n + C_b e_b \end{aligned}$$

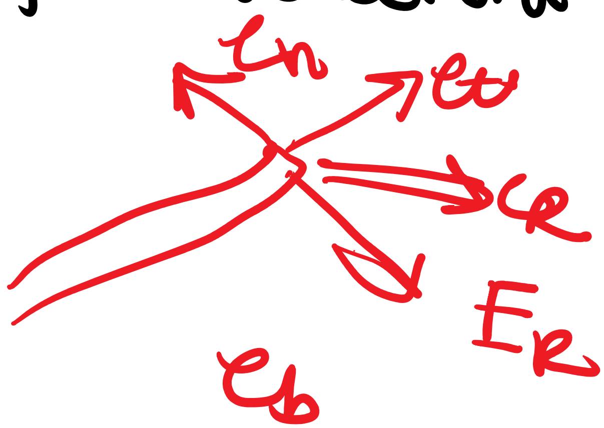
(i) Internal Section of body



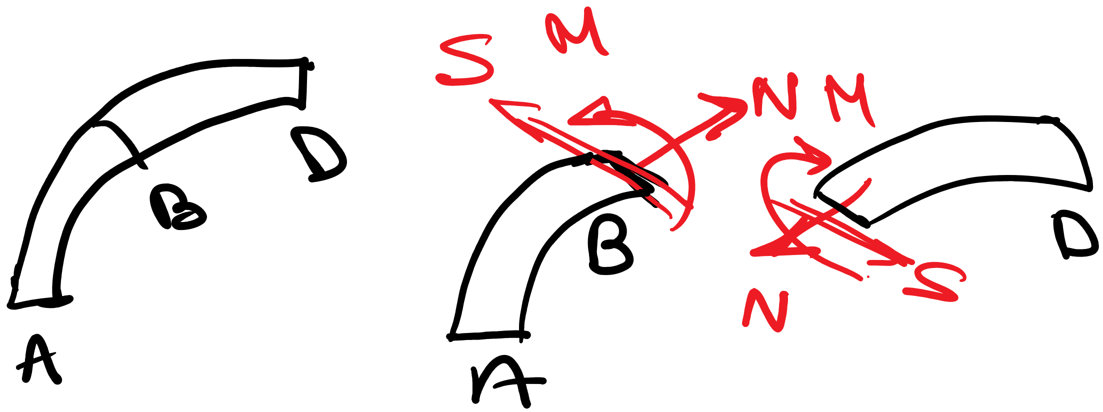




(j) Internal force resultant in a bar

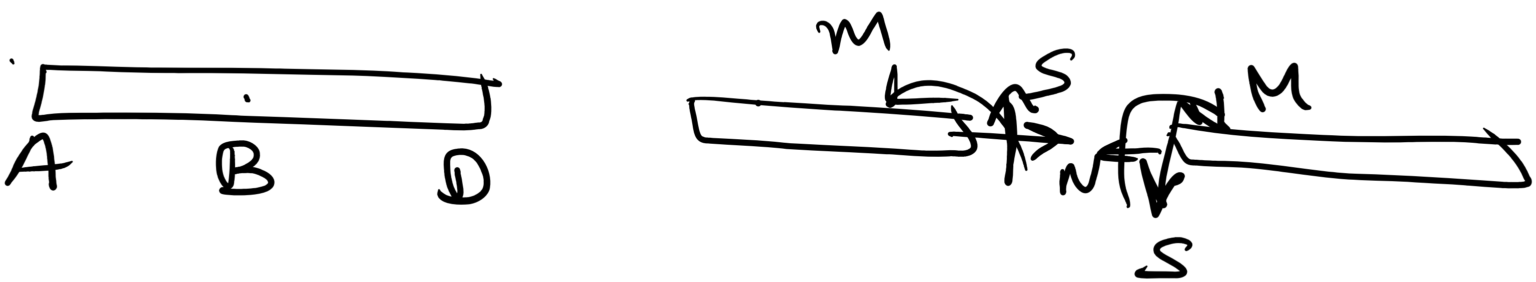


(k) Coplanar load on coplanar beam



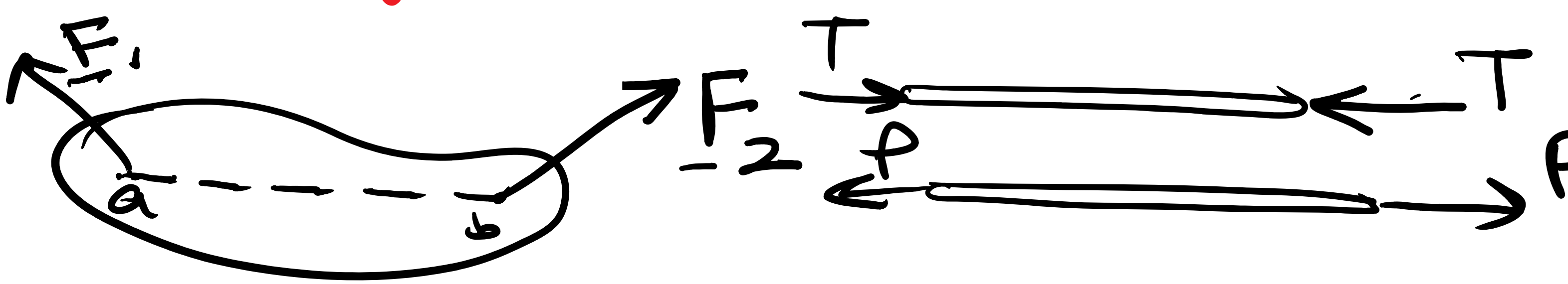

(i) Coplanar load on straight beam.



# Two Force Member

## Two point Equivalent loadings

A two force member is a member subjected to only two forces. If a two force member is in equilibrium or a two force inertialess member in in motion then two forces have equal magnitude opposite direction and act along the line joining their points of application.



Proof

$$\underline{M}_A = \underline{AB} \times F_2 = 0$$


$F_2$ acts through A along AB.

Similarly $F_1$ acts along AB

$$\underline{F} = F_1 + F_2 = 0$$

$$F_1 = -F_2$$