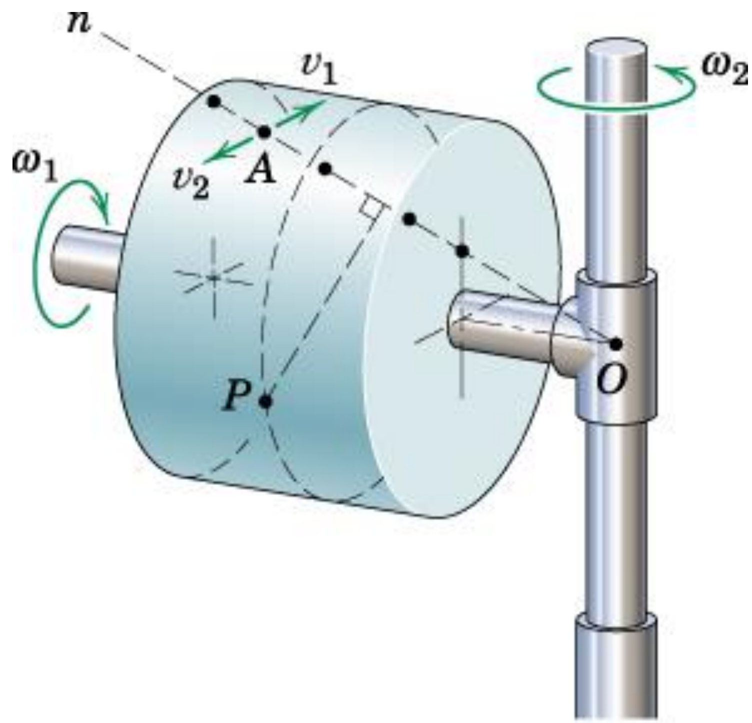
3-D Kinematics of Rigid Bodies

Rotation about a Fixed Point



Instantaneous Axis of Rotation

:: Rotation of solid rotor with black particles embedded on its surface

:: ω_1 and ω_2 are steady angular velocities

:: Along line *O-n*, velocity of particles will be momentarily zero (the line will be sharply defined)

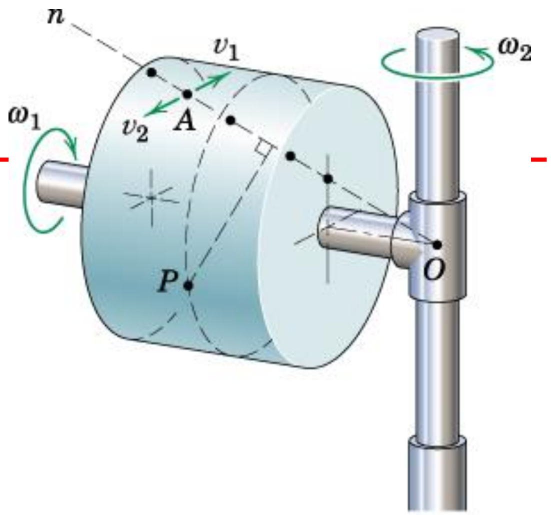
:: The line of points with no velocity
→ Instantaneous position of the Axis of Rotation

:: Any point on this line, e.g., *A*, would have equal and opposite velocity components v_1 and v_2 due to ω_1 and ω_2, respectively

All particles of the body, except those on line *O-n*, rotate momentarily in circular arcs @ the instantaneous axis of rotation

3-D Kinematics

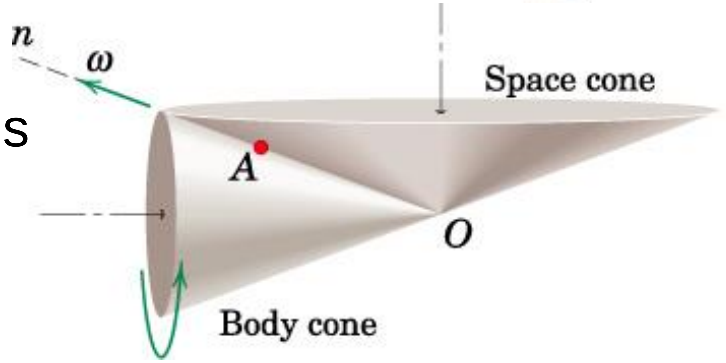
Rotation about a Fixed Point



Instantaneous Axis of Rotation

:: Change of position of rotation axis both in space and relative to the body

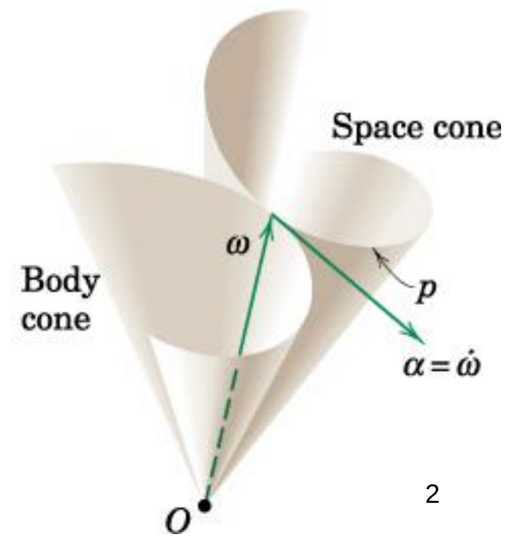
Body and Space Cones



:: Relative to the cylinder, the instantaneous axis of rotation *O*-*A*-*n* generates a right circular cone @ the cylinder axis *O* → **Body Cone**

:: Instantaneous axis of rotn also generates a right circular cone @ the vertical axis → **Space Cone**

:: Rolling of body cone on space cone



:: Angular vel ω of the body located along the common element of the two cones

3-D Kinematics

Rotation about a Fixed Point

Angular Acceleration

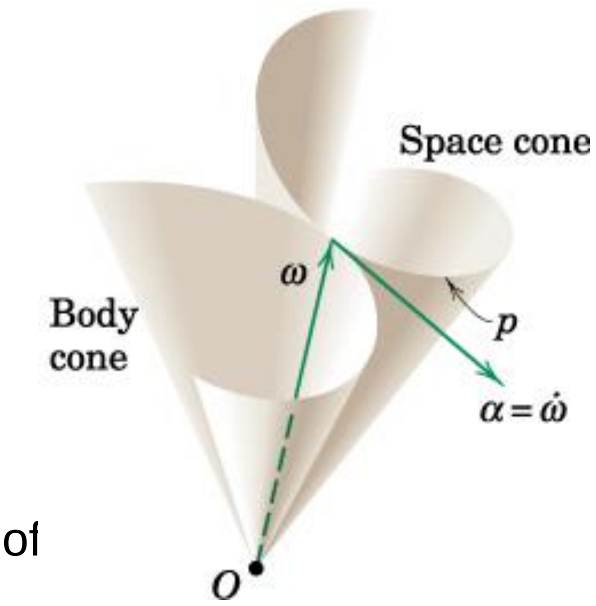
:: Planar motion

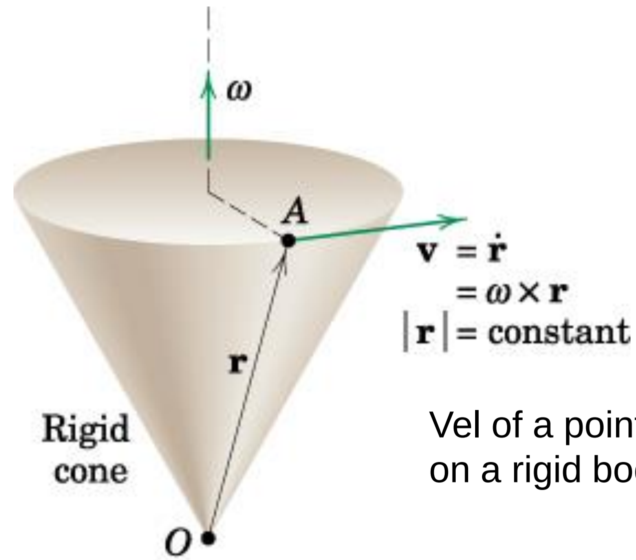
- scalar α reflects the change in magnitude of $\boldsymbol{\omega}$

:: 3-D motion

- vector $\boldsymbol{\alpha}$ reflects the change in the direction and magnitude of $\boldsymbol{\omega}$

:: $\boldsymbol{\alpha}$ - tangent vector to the space curve *p* in the direction of the change in $\boldsymbol{\omega}$

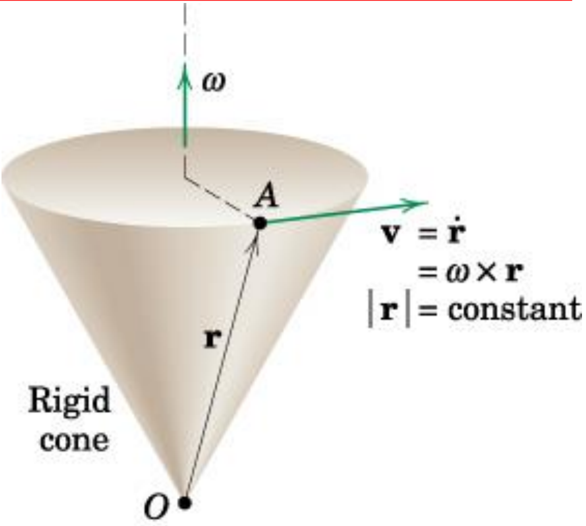




Vel of a point
on a rigid body

3-D Kinematics

Rotation about a Fixed Point

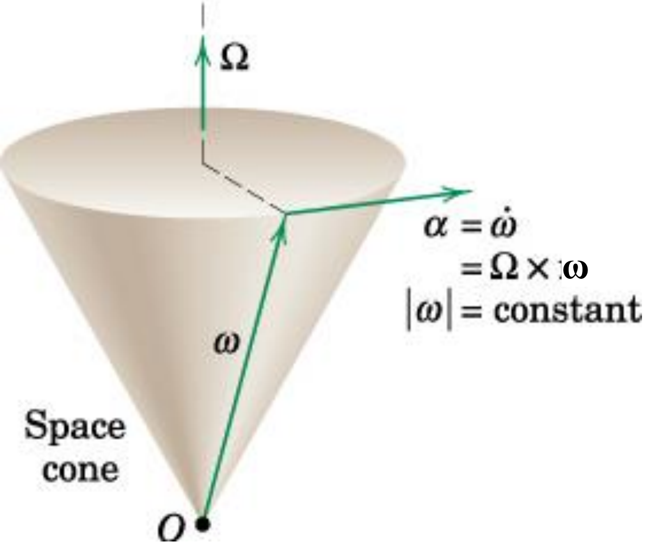


If the magnitude of **ω** remains constant, **α** is normal to **ω**

Let **Ω** be the angular velocity with which the vector **ω** itself rotates (precesses) as it forms the space cone

→ $$\boldsymbol{\alpha} = \boldsymbol{\Omega} \times \boldsymbol{\omega}$$

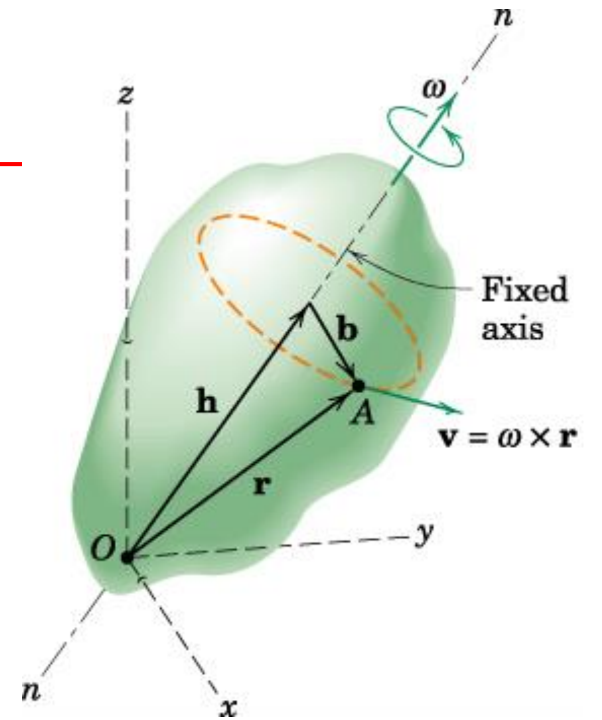
:: **α**, **Ω**, and **ω** have the same relationship as that of **v**, **ω**, and **r** defining the vel of a point *A* in a rigid body



3-D Kinematics

Rotation about a Fixed Point

When a rigid body rotates @ a fixed point *O* with the instantaneous axis of rotation *n*-*n*, vel **v** and accln **a** = $\dot{\mathbf{v}}$ of any point *A* in the body are given by the same expressions derived when the axis was fixed:



$$\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$$

$$\mathbf{a} = \dot{\boldsymbol{\omega}} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$$

Rotation about a Fixed Axis vs Rotation about a Fixed Point

For rotation @ a fixed point:

- angular accln will have a comp normal to $\boldsymbol{\omega}$ due to change in dirn of $\boldsymbol{\omega}$, as well as a comp in the dirn of $\boldsymbol{\omega}$ to reflect any change in the magnitude of $\boldsymbol{\omega}$
- Although any point on the rotation axis *n*-*n* momentarily will have zero velocity, it will not have zero accln as long as $\boldsymbol{\omega}$ is changing its direction

For rotation @ a fixed axis:

- angular accln will have only one component along the fixed axis to reflect any change in the magnitude of $\boldsymbol{\omega}$.
- Points which lie on the fixed axis of rotation will have no velocity or accln

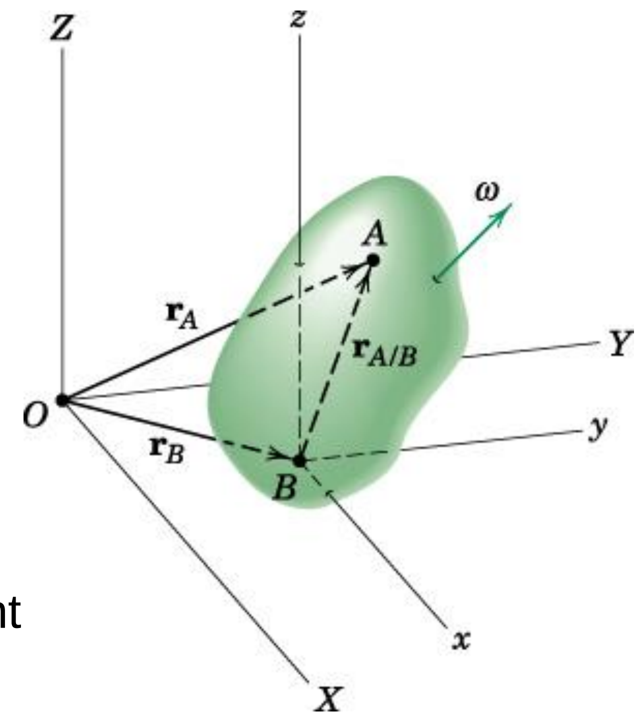
3-D Kinematics

General Motion in Three Dimension

:: Principles of relative motion (both translating & rotating reference axes)

Translating Reference Axes

- *X-Y-Z* :: fixed reference frame
- *x-y-z* :: translating reference frame with any point *B* as the origin
- ω :: angular velocity of the rigid body



Linear velocity and accln of any point *A* in the body

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$$

-3 coplanar vectors in each eqn

- Distance $\overline{AB}$ should remain constant

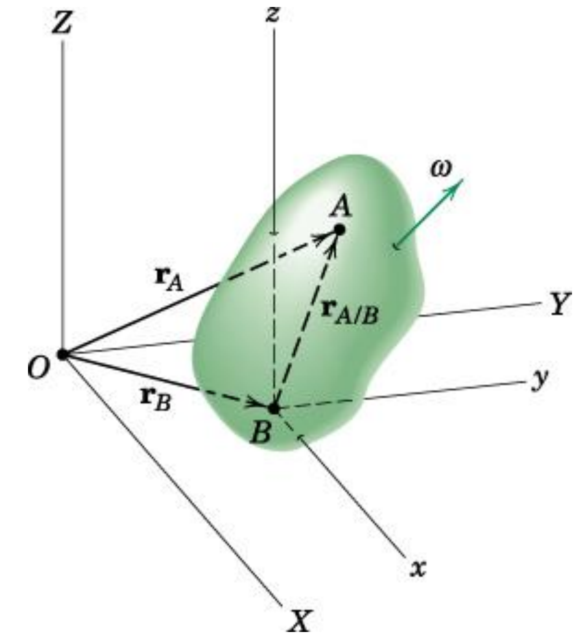
- To an observer on *x-y-z*, the body will appear to rotate @ *B*
- Point *A* will appear to lie on a spherical surface with *B* as the center

→ General Motion ≡ translation of body with motion of B + rotation of body @ B

3-D Kinematics :: General motion

Translating Reference Axes

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$
$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$$



Relative motion terms represent effect of rotation about point *B*

→ These are identical to the vel and accln relations obtained for a rigid body rotating @ a fixed point:

$$\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$$
$$\mathbf{a} = \dot{\boldsymbol{\omega}} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$$

Relative terms

→
$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{A/B}$$
$$\mathbf{a}_A = \mathbf{a}_B + \dot{\boldsymbol{\omega}} \times \mathbf{r}_{A/B} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{A/B})$$

$\boldsymbol{\omega}$ and $\dot{\boldsymbol{\omega}}$ :: instantaneous angular vel and angular accln of the body

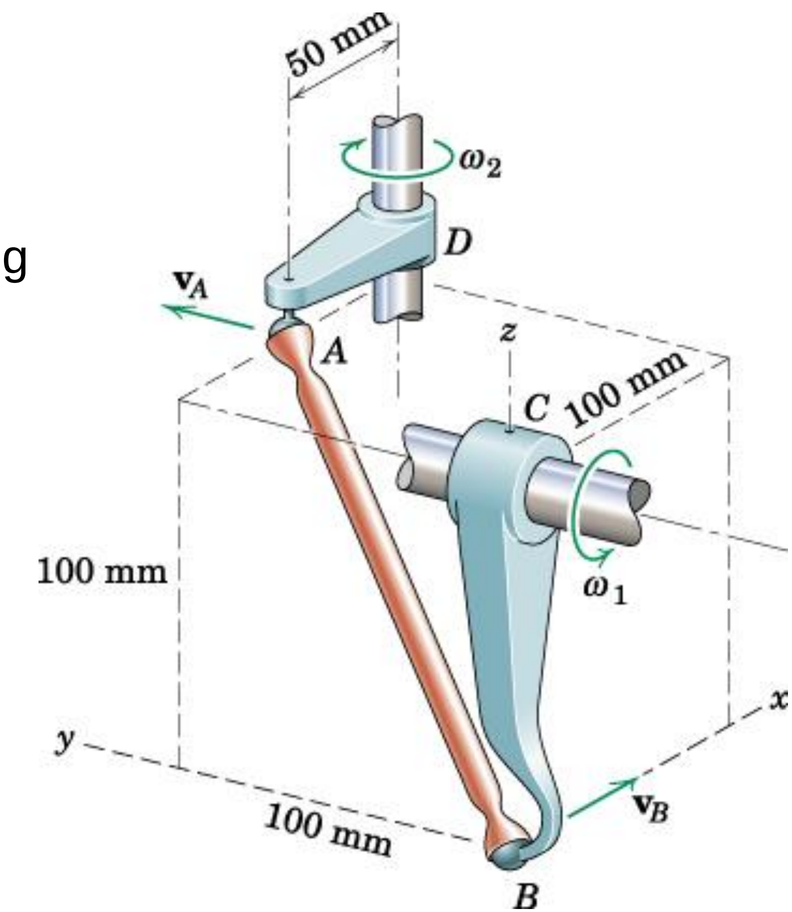
If point *A* is chosen as a reference point then vel and accln relations for point *B*:

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A}$$
$$\mathbf{a}_B = \mathbf{a}_A + \dot{\boldsymbol{\omega}} \times \mathbf{r}_{B/A} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{B/A})$$

$\mathbf{r}_{B/A} = -\mathbf{r}_{A/B}$. $\boldsymbol{\omega}$ & $\dot{\boldsymbol{\omega}}$ will be same for both cases

Example (1) on general motion

Example: Crank *CB* rotates @ the horz axis with an ang vel ω_1 = 6 rad/s, which is constant for a short interval of motion including the position shown. Link *AB* has a ball-and-socket fitting on each end and connects crank *DA* with *CB*. For the instant shown, det the ang vel ω_2 of crank *DA* and ang vel $\boldsymbol{\omega}_n$ of link *AB*. Further determine, the ang accln $\dot{\omega}_2$ of crank *DA* and ang accln $\dot{\boldsymbol{\omega}}_n$ of link *AB.*



Solution:

Choose point *B* as the origin of translating reference axes.

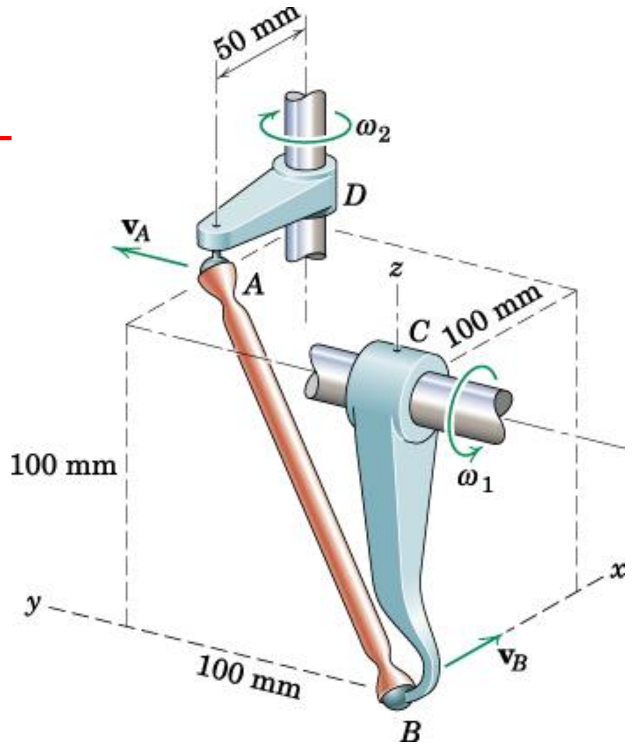
Relative velocity relation:

$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega}_n \times \mathbf{r}_{A/B}$$

Example (1) on general motion

Relative velocity relation:

$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega}_n \times \mathbf{r}_{A/B}$$



$\boldsymbol{\omega}_n$ is the angular vel of link *AB* taken normal to *AB*

:: The angular vel of *AB* is taken as normal to *AB* since any rotation of the link @ its own axis *AB* has no influence on the behaviour of the link.

→ $\boldsymbol{\omega}_n$ and $\mathbf{r}_{A/B}$ must be at right angles

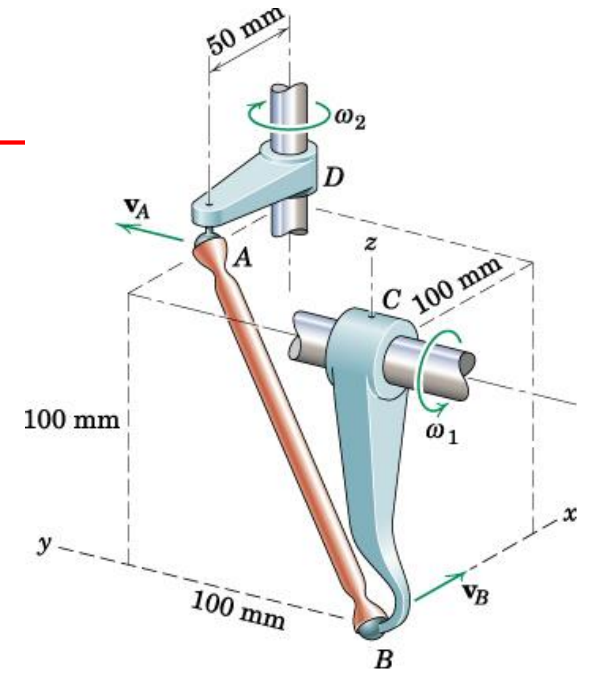
→ $\boldsymbol{\omega}_n \cdot \mathbf{r}_{A/B} = 0$

→ Additional eqn generated from kinematic requirement

$\mathbf{v}_{A/B}$ is also perpendicular to both $\boldsymbol{\omega}_n$ and $\mathbf{r}_{A/B}$ (since $\mathbf{v}_A - \mathbf{v}_B = \mathbf{v}_{A/B} = \boldsymbol{\omega}_n \times \mathbf{r}_{A/B}$)

Example (1) on general motion

$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega}_n \times \mathbf{r}_{A/B}$$



Velocities of *A* and *B*: $v = r\,\omega$

$\mathbf{v}_A = 50\omega_2\mathbf{j} \qquad \mathbf{v}_B = 100(6)\mathbf{i} = 600\mathbf{i}$ mm/s

$\mathbf{r}_{A/B} = 50\mathbf{i} + 100\mathbf{j} + 100\mathbf{k}$ mm (from *B* to *A*)

$$\rightarrow \; 50\omega_2\mathbf{j} = 600\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_{n_x} & \omega_{n_y} & \omega_{n_z} \\ 50 & 100 & 100 \end{vmatrix}$$

Expanding and equating the coefficients:

$$-6 = \quad + \omega_{n_y} - \omega_{n_z}$$
$$\omega_2 = -2\omega_{n_x} \quad + \omega_{n_z}$$
$$0 = 2\omega_{n_x} - \omega_{n_y}$$

Solving: $\omega_2 = 6$ rad/s

$\boldsymbol{\omega}_n \cdot \mathbf{r}_{A/B} = 0$

$50\omega_{nx} + 100\omega_{ny} + 100\omega_{nz} = 0$

Substituting in previous two eqns:

$\omega_{nx} = -4/3$ rad/s

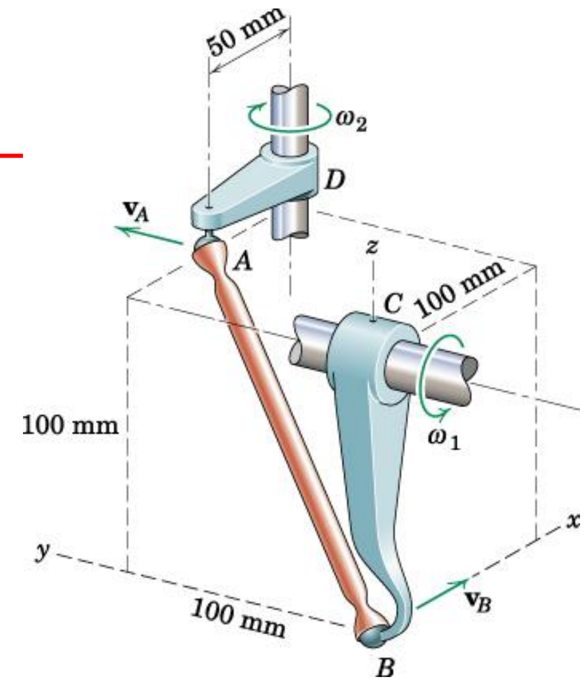
$\omega_{ny} = -8/3$ rad/s

$\omega_{nz} = 10/3$ rad/s

$$\boldsymbol{\omega}_n = \frac{2}{3}(-2\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}) \text{ rad/s}$$

$$\omega_n = \frac{2}{3}\sqrt{2^2 + 4^2 + 5^2} = 2\sqrt{5} \text{ rad/s}$$

Example (1) on general motion



Angular Acceleration

Linear accln of the links can be found from:

$$\mathbf{a}_A = \mathbf{a}_B + \dot{\boldsymbol{\omega}}_n \times \mathbf{r}_{A/B} + \boldsymbol{\omega}_n \times (\boldsymbol{\omega}_n \times \mathbf{r}_{A/B})$$

$\boldsymbol{\omega}_n$ :: angular velocity of *AB* taken normal to *AB*

$\dot{\boldsymbol{\omega}}_n$ :: the angular accln of link *AB*

: Comp of ang vel along *AB* has no influence on the motion of the cranks *C* and *D*

∴ Contribution of change in magnitude and dirn of this comp of ang vel on actual angular accln of link will not be considered here

→ Only the effect of $\dot{\boldsymbol{\omega}}_n$

Accln of *A* and *B* in terms of normal & tangential comp: $a_n = |\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})| = b\omega^2$

$$\mathbf{a}_A = 50\omega_2{}^2\mathbf{i} + 50\dot{\omega}_2\mathbf{j} = 1800\mathbf{i} + 50\dot{\omega}_2\mathbf{j}\ \text{mm/s}^2$$

$a_t = |\dot{\boldsymbol{\omega}} \times \mathbf{r}| = b\alpha$

$$\mathbf{a}_B = 100\omega_1{}^2\mathbf{k} + (0)\mathbf{i} = 3600\mathbf{k}\ \text{mm/s}^2$$

$\dot{\boldsymbol{\omega}}_1 = 0$, since $\boldsymbol{\omega}_1$ is constant

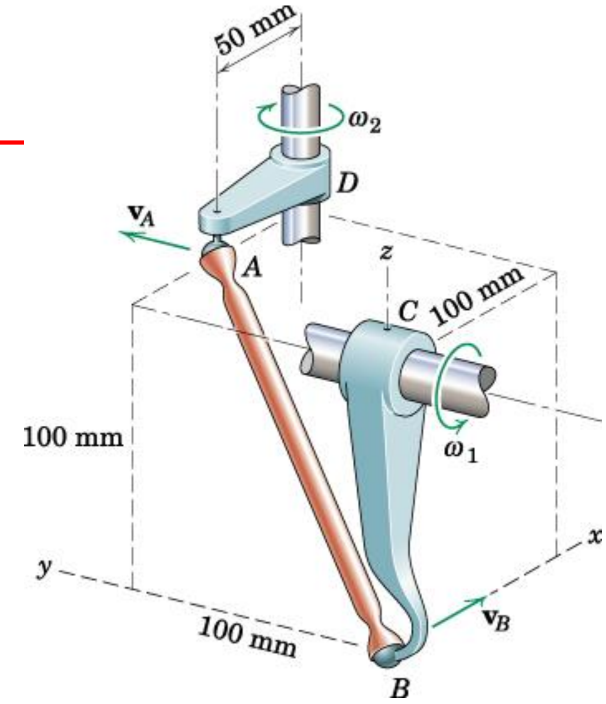
Further: $\boldsymbol{\omega}_n \times (\boldsymbol{\omega}_n \times \mathbf{r}_{A/B}) = -\omega_n{}^2\mathbf{r}_{A/B} = -20(50\mathbf{i} + 100\mathbf{j} + 100\mathbf{k})\ \text{mm/s}^2$

P x (**Q** x **R**) = (**P·R)Q** – (**P·Q)R** (first term will be zero because $\boldsymbol{\omega}_n$ is normal to $\mathbf{r}_{A/B}$)

Example (1) on general motion

$$\mathbf{a}_A = \mathbf{a}_B + \dot{\boldsymbol{\omega}}_n \times \mathbf{r}_{A/B} + \boldsymbol{\omega}_n \times (\boldsymbol{\omega}_n \times \mathbf{r}_{A/B})$$

$$\dot{\boldsymbol{\omega}}_n \times \mathbf{r}_{A/B} = (100\dot{\omega}_{n_y} - 100\dot{\omega}_{n_z})\mathbf{i} + (50\dot{\omega}_{n_z} - 100\dot{\omega}_{n_x})\mathbf{j} + (100\dot{\omega}_{n_x} - 50\dot{\omega}_{n_y})\mathbf{k}$$



Substituting into rel. accln. eqn and equating respective coefficients:

$$\begin{aligned} 28 &= \dot{\omega}_{n_y} - \dot{\omega}_{n_z} \\ \dot{\omega}_2 + 40 &= -2\dot{\omega}_{n_x} + \dot{\omega}_{n_z} \\ -32 &= 2\dot{\omega}_{n_x} - \dot{\omega}_{n_y} \end{aligned}$$

→ $\dot{\omega}_2 = -36 \text{ rad/s}^2$

The vector $\dot{\boldsymbol{\omega}}_n$ is normal to $\mathbf{r}_{A/B}$ (but is not normal to $\mathbf{v}_{A/B}$). The component of $\dot{\boldsymbol{\omega}}_n$ which is not normal to $\mathbf{v}_{A/B}$ gives rise to the change in direction of $\mathbf{v}_{A/B}$.

$[\dot{\boldsymbol{\omega}}_n \cdot \mathbf{r}_{A/B} = 0]$ $\quad 2\dot{\omega}_{n_x} + 4\dot{\omega}_{n_y} + 4\dot{\omega}_{n_z} = 0$

Substituting in previous two eqns: $\dot{\omega}_{n_x} = -8 \text{ rad/s}^2$ $\quad \dot{\omega}_{n_y} = 16 \text{ rad/s}^2$ $\quad \dot{\omega}_{n_z} = -12 \text{ rad/s}^2$

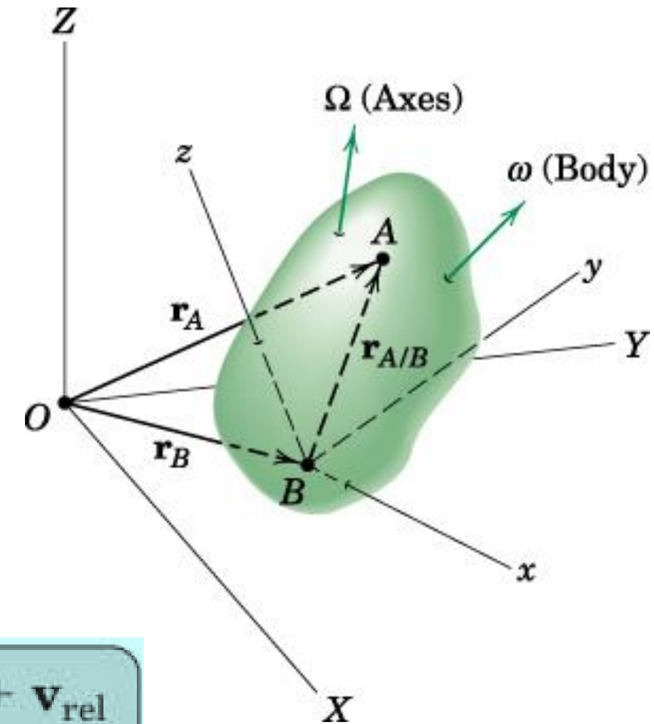
$\dot{\boldsymbol{\omega}}_n = 4(-2\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}) \text{ rad/s}^2$ $\qquad |\dot{\boldsymbol{\omega}}_n| = 4\sqrt{2^2 + 4^2 + 3^2} = 4\sqrt{29} \text{ rad/s}^2$

3-D Kinematics

Rotating Reference Axes

: Reference axes translate as well as rotate

: Origin of the reference axes attached to *B*



The reference axes x-y-z rotate with an absolute angular velocity $\boldsymbol{\Omega}$, which may be different from absolute angular velocity $\boldsymbol{\omega}$ of the body

For plane motion:

$$\dot{\mathbf{i}} = \boldsymbol{\omega} \times \mathbf{i} \quad \text{and} \quad \dot{\mathbf{j}} = \boldsymbol{\omega} \times \mathbf{j}$$

$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r} + \mathbf{v}_{\text{rel}}$$

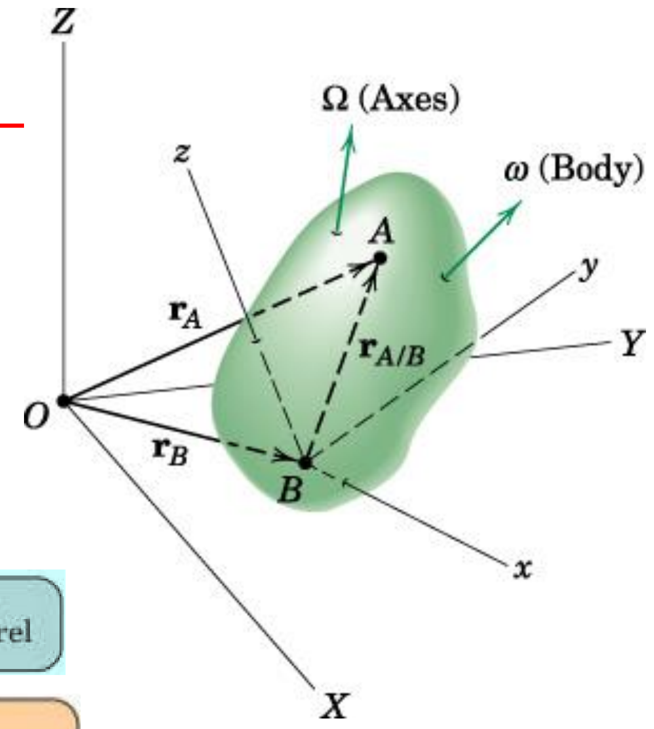
$$\left(\frac{d\mathbf{V}}{dt}\right)_{XY} = \left(\frac{d\mathbf{V}}{dt}\right)_{xy} + \boldsymbol{\omega} \times \mathbf{V}$$

$$\mathbf{a}_A = \mathbf{a}_B + \dot{\boldsymbol{\omega}} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + 2\boldsymbol{\omega} \times \mathbf{v}_{\text{rel}} + \mathbf{a}_{\text{rel}}$$

For 3-D motion:

- include the *z*-component of the vectors

3-D Kinematics



Rotating Reference Axes

$$\dot{\mathbf{i}} = \boldsymbol{\omega} \times \mathbf{i} \quad \text{and} \quad \dot{\mathbf{j}} = \boldsymbol{\omega} \times \mathbf{j}$$

$$\dot{\mathbf{i}} = \boldsymbol{\Omega} \times \mathbf{i} \qquad \dot{\mathbf{j}} = \boldsymbol{\Omega} \times \mathbf{j} \qquad \dot{\mathbf{k}} = \boldsymbol{\Omega} \times \mathbf{k}$$

→ Time derivatives of the unit vectors attached to *x-y-z*

Vel and accln of point *A*:

$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r} + \mathbf{v}_{\text{rel}}$$

$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\Omega} \times \mathbf{r}_{A/B} + \mathbf{v}_{\text{rel}}$$

$$\mathbf{a}_A = \mathbf{a}_B + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{A/B} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{A/B}) + 2\boldsymbol{\Omega} \times \mathbf{v}_{\text{rel}} + \mathbf{a}_{\text{rel}}$$

Vel and accln of point A measured rel. to *x-y-z* by an observer attached to *x-y-z*

$$\mathbf{v}_{\text{rel}} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j} + \dot{z}\mathbf{k} \text{ and } \mathbf{a}_{\text{rel}} = \ddot{x}\mathbf{i} + \ddot{y}\mathbf{j} + \ddot{z}\mathbf{k}$$

$$\mathbf{a}_A = \mathbf{a}_B + \dot{\boldsymbol{\omega}} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + 2\boldsymbol{\omega} \times \mathbf{v}_{\text{rel}} + \mathbf{a}_{\text{rel}}$$

For different **Ω** and **ω**, the term $r_{A/B}$ remains **constant in magnitude** but will **change its direction wrt *x-y-z***

: If *x-y-z* are rigidly attached to the body,

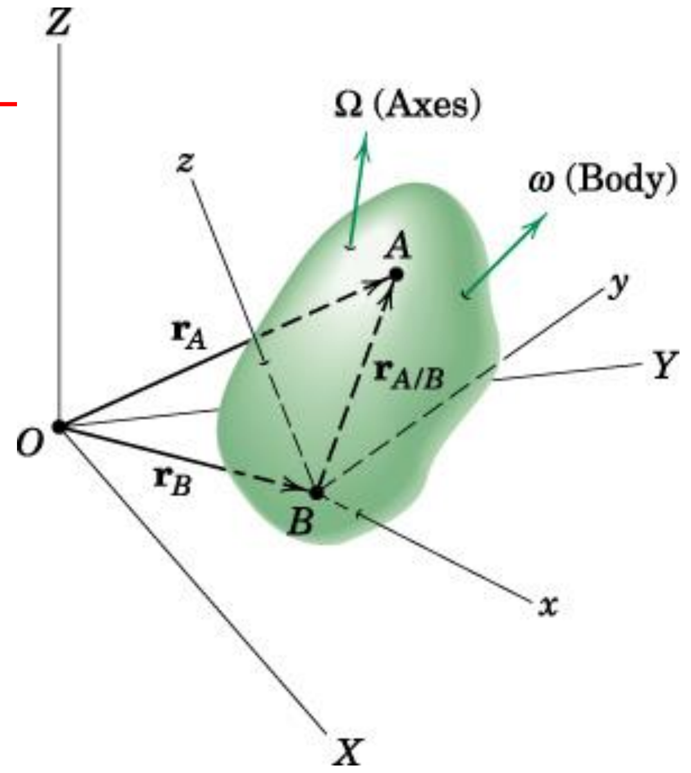
→ $\boldsymbol{\Omega} = \boldsymbol{\omega}$, and $\mathbf{v}_{\text{rel}} = \mathbf{a}_{\text{rel}} = 0$ → motion will be the same as for translating reference axes

3-D Kinematics

Rotating Reference Axes

$$\left(\frac{d\mathbf{V}}{dt}\right)_{XY} = \left(\frac{d\mathbf{V}}{dt}\right)_{xy} + \boldsymbol{\omega} \times \mathbf{V}$$

for any vector



Eqn relating time derivative of a vector **V** measured in fixed *X-Y-Z* system and the time derivative of **V** measured relative to rotating *x-y-z* system:

$$\left(\frac{d\mathbf{V}}{dt}\right)_{XYZ} = \left(\frac{d\mathbf{V}}{dt}\right)_{xyz} + \boldsymbol{\Omega} \times \mathbf{V}$$

Applying this transformation to the relative posn vector:

$\mathbf{r}_{A/B} = \mathbf{r}_A - \mathbf{r}_B$

$$\rightarrow \left(\frac{d\mathbf{r}_A}{dt}\right)_{XYZ} = \left(\frac{d\mathbf{r}_B}{dt}\right)_{XYZ} + \left(\frac{d\mathbf{r}_{A/B}}{dt}\right)_{xyz} + \boldsymbol{\Omega} \times \mathbf{r}_{A/B}$$

→ The same eqn is obtained

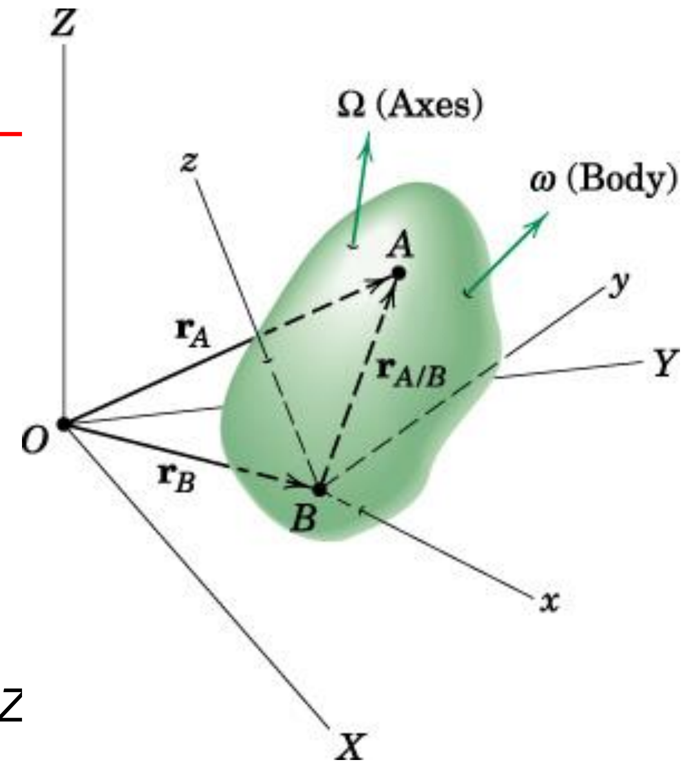
$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{rel} + \boldsymbol{\Omega} \times \mathbf{r}_{A/B}$$

$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\Omega} \times \mathbf{r}_{A/B} + \mathbf{v}_{rel}$$

$$\mathbf{a}_A = \mathbf{a}_B + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{A/B} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{A/B}) + 2\boldsymbol{\Omega} \times \mathbf{v}_{rel} + \mathbf{a}_{rel}$$

3-D Kinematics

Rotating Reference Axes



$$\left(\frac{d\mathbf{V}}{dt}\right)_{XYZ} = \left(\frac{d\mathbf{V}}{dt}\right)_{xyz} + \boldsymbol{\Omega} \times \mathbf{V}$$

This eqn may be recast as the vector operator for general use:

$$\left(\frac{d[\quad]}{dt}\right)_{XYZ} = \left(\frac{d[\quad]}{dt}\right)_{xyz} + \boldsymbol{\Omega} \times [\quad]$$

[ ] stands for any vector **V** expressible both in *X-Y-Z*

Applying the operator to itself → second time derivative.

$$\left(\frac{d^2[\quad]}{dt^2}\right)_{XYZ} = \left(\frac{d^2[\quad]}{dt^2}\right)_{xyz} + \dot{\boldsymbol{\Omega}} \times [\quad] + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times [\quad]) + 2\boldsymbol{\Omega} \times \left(\frac{d[\quad]}{dt}\right)_{xyz}$$

→ The same eqn is for accln

$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\Omega} \times \mathbf{r}_{A/B} + \mathbf{v}_{\text{rel}}$$

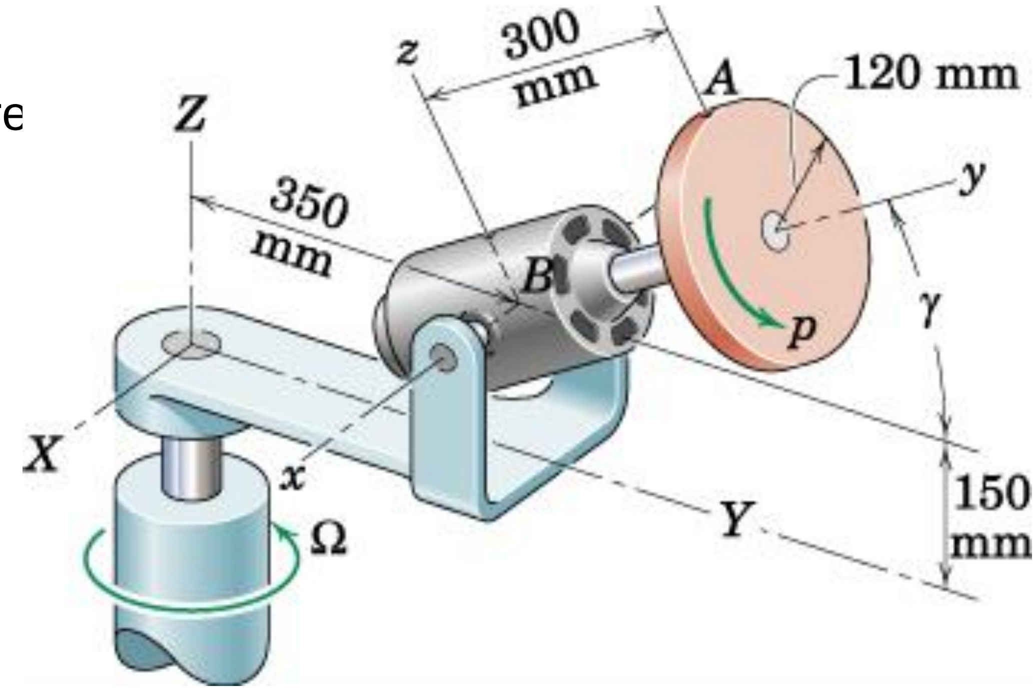
$$\mathbf{a}_A = \mathbf{a}_B + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{A/B} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{A/B}) + 2\boldsymbol{\Omega} \times \mathbf{v}_{\text{rel}} + \mathbf{a}_{\text{rel}}$$

Example on Rotating Reference Axes

The motor housing and its bracket rotate about the Z-axis at the constant rate $\Omega = 3$ rad/s. The motor shaft and disk have a constant angular velocity of spin $p = 8$ rad/s with respect to the motor housing in the direction shown. If γ is constant at 30°, determine the velocity and acceleration of point A at the top of the disk and the angular acceleration $\boldsymbol{\alpha}$ of the disk.

Solution

The rotating reference axes *x*-*y*-*z* are attached to the motor housing.



Unit vectors for *X*-*Y*-*Z*: **I**, **J**, **K**

Unit vectors for *x*-*y*-*z*: **i**, **j**, **k**

Angular vel of *x*-*y*-*z*:

$$\mathbf{\Omega} = \Omega\mathbf{K} = 3\mathbf{K} \text{ rad/s}$$

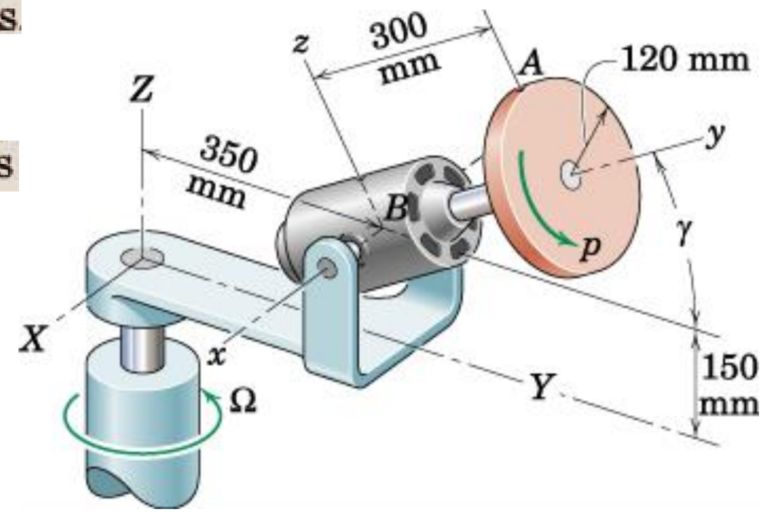
$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{\Omega} \times \mathbf{r}_{A/B} + \mathbf{v}_{rel}$$

$$\mathbf{a}_A = \mathbf{a}_B + \dot{\mathbf{\Omega}} \times \mathbf{r}_{A/B} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}_{A/B}) + 2\mathbf{\Omega} \times \mathbf{v}_{rel} + \mathbf{a}_{rel}$$

Example on Rotating Reference Axes

Velocity of *A*:

$$\boldsymbol{\Omega} = \Omega \mathbf{K} = 3\mathbf{K} \text{ rad/s}$$



$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\Omega} \times \mathbf{r}_{A/B} + \mathbf{v}_{rel}$$

$$\mathbf{v}_B = \boldsymbol{\Omega} \times \mathbf{r}_B = 3\mathbf{K} \times 0.350\mathbf{J} = -1.05\mathbf{I} = -1.05\mathbf{i} \text{ m/s}$$

In $\mathbf{r}_B$, **K** component is not considered because **K** x **K** = 0

$$\boldsymbol{\Omega} \times \mathbf{r}_{A/B} = 3\mathbf{K} \times (0.300\mathbf{j} + 0.120\mathbf{k})$$

K x **i** = **J** = **j** cosγ – **k** sinγ
K x **j** = - **i** cosγ
K x **k** = **i** sinγ

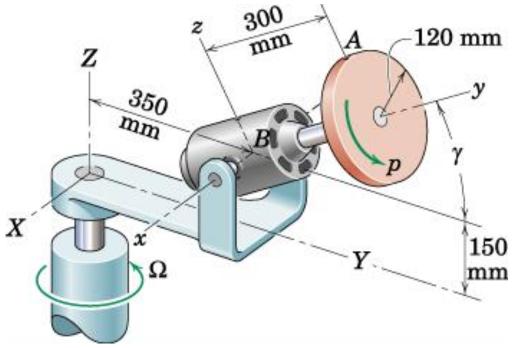
$$\boldsymbol{\Omega} \times \mathbf{r}_{A/B} = (-0.9 \cos 30°)\mathbf{i} + (0.36 \sin 30°)\mathbf{i} = -0.599\mathbf{i} \text{ m/s}$$

$$\mathbf{v}_{rel} = \mathbf{p} \times \mathbf{r}_{A/B} = 8\mathbf{j} \times (0.300\mathbf{j} + 0.120\mathbf{k}) = 0.960\mathbf{i} \text{ m/s}$$

→ $\mathbf{v}_A = -1.05\mathbf{i} - 0.599\mathbf{i} + 0.960\mathbf{i} = -0.689\mathbf{i} \text{ m/s}$

Example on Rotating Reference Axes

Acceleration of A



$$\mathbf{a}_A = \mathbf{a}_B + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{A/B} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{A/B}) + 2\boldsymbol{\Omega} \times \mathbf{v}_{rel} + \mathbf{a}_{rel}$$

Here:

$$\mathbf{a}_B = \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_B) = 3\mathbf{K} \times (3\mathbf{K} \times 0.350\mathbf{J}) = -3.15\mathbf{J}$$

$$= 3.15(-\mathbf{j}\cos 30° + \mathbf{k}\sin 30°) = -2.73\mathbf{j} + 1.575\mathbf{k} \text{ m/s}^2$$

$$\dot{\boldsymbol{\Omega}} = \mathbf{0}$$

$$\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{A/B}) = 3\mathbf{K} \times [3\mathbf{K} \times (0.300\mathbf{j} + 0.120\mathbf{k})]$$

$$= 3\mathbf{K} \times (-0.599\mathbf{i}) = -1.557\mathbf{j} + 0.899\mathbf{k} \text{ m/s}^2$$

$$2\boldsymbol{\Omega} \times \mathbf{v}_{rel} = 2(3\mathbf{K}) \times 0.960\mathbf{i} = 5.76\mathbf{J}$$

$$= 5.76(\mathbf{j}\cos 30° - \mathbf{k}\sin 30°) = 4.99\mathbf{j} - 2.88\mathbf{k} \text{ m/s}^2$$

$$\mathbf{a}_{rel} = \mathbf{p} \times (\mathbf{p} \times \mathbf{r}_{A/B}) = 8\mathbf{j} \times [8\mathbf{j} \times (0.300\mathbf{j} + 0.120\mathbf{k})] = -7.68\mathbf{k} \text{ m/s}^2$$

→ $\mathbf{a}_A = 0.703\mathbf{j} - 8.09\mathbf{k} \text{ m/s}^2$ $\quad$ $a_A = \sqrt{(0.703)^2 + (8.09)^2} = 8.12 \text{ m/s}^2$

Angular Accln:

$$\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}} = \boldsymbol{\Omega} \times \boldsymbol{\omega} = 3\mathbf{K} \times (3\mathbf{K} + 8\mathbf{j}) \quad = \mathbf{0} + (-24\cos 30°)\mathbf{i} = -20.8\mathbf{i} \text{ rad/s}^2$$