Engineering Mechanics: Dynamics

- Dynamics
  - Branch of mechanics that deals with the motion of bodies under the action of forces ***(Accelerated Motion)***
- Two distinct parts:
  - Kinematics
    - study of motion without reference to the forces that cause motion or are generated as a result of motion
  - Kinetics
    - relates the action of forces on bodies to their resulting motions

Engineering Mechanics: Dynamics

- Basis of rigid body dynamics
  - Newton's 2nd law of motion
    - A **particle** of **mass "m" acted upon by an unbalanced force "*F*"** experiences an **acceleration "*a*"** that has the **same direction as the force** and a **magnitude** that is **directly proportional to the force**
    - **a** is the resulting **acceleration** measured in a **non-accelerating frame of reference**

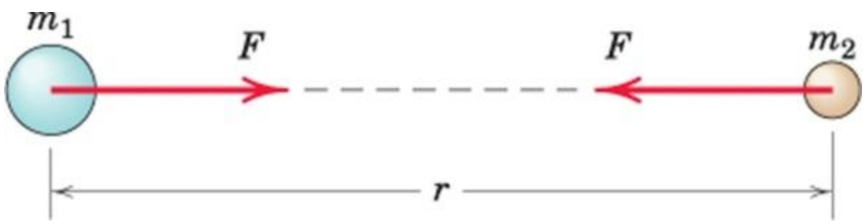
Engineering Mechanics: Dynamics

- Space
  - Geometric region occupied by bodies
    - *Reference system*
      - Linear or angular measurements
    - **Primary reference system** or **astronomical frame of reference**
      - Imaginary set of rectangular axes fixed in space
      - Validity for measurements for velocity < speed of light
      - Absolute measurements
    - **Reference frame attached to the earth??**
- Time
- Mass

Engineering Mechanics: Dynamics

- **Newton's law of gravitation**

$$F = G\frac{m_1 m_2}{r^2}$$



– ***F*** :: mutual force of attraction between two particles

– ***G*** :: universal gravitational constant

  - 6.673x10-11 $m^3/(kg.s^2)$

Engineering Mechanics: Dynamics

- **Weight**
  - Only significant gravitational force between the earth and a particle located near the surface

$$W = G\frac{mM_e}{r^2} \qquad W = mg$$

- $g = GM_e/r^2$ :: acceleration due to gravity (9.81m/s^2)
- Variation of g with altitude

$$g = g_0 \frac{R^2}{(R+h)^2}$$

g is the absolute acceleration due to gravity at altitude h

g_0 is the absolute acceleration due to gravity at sea level

R is the radius of the earth

Effect of Altitude on Gravitation

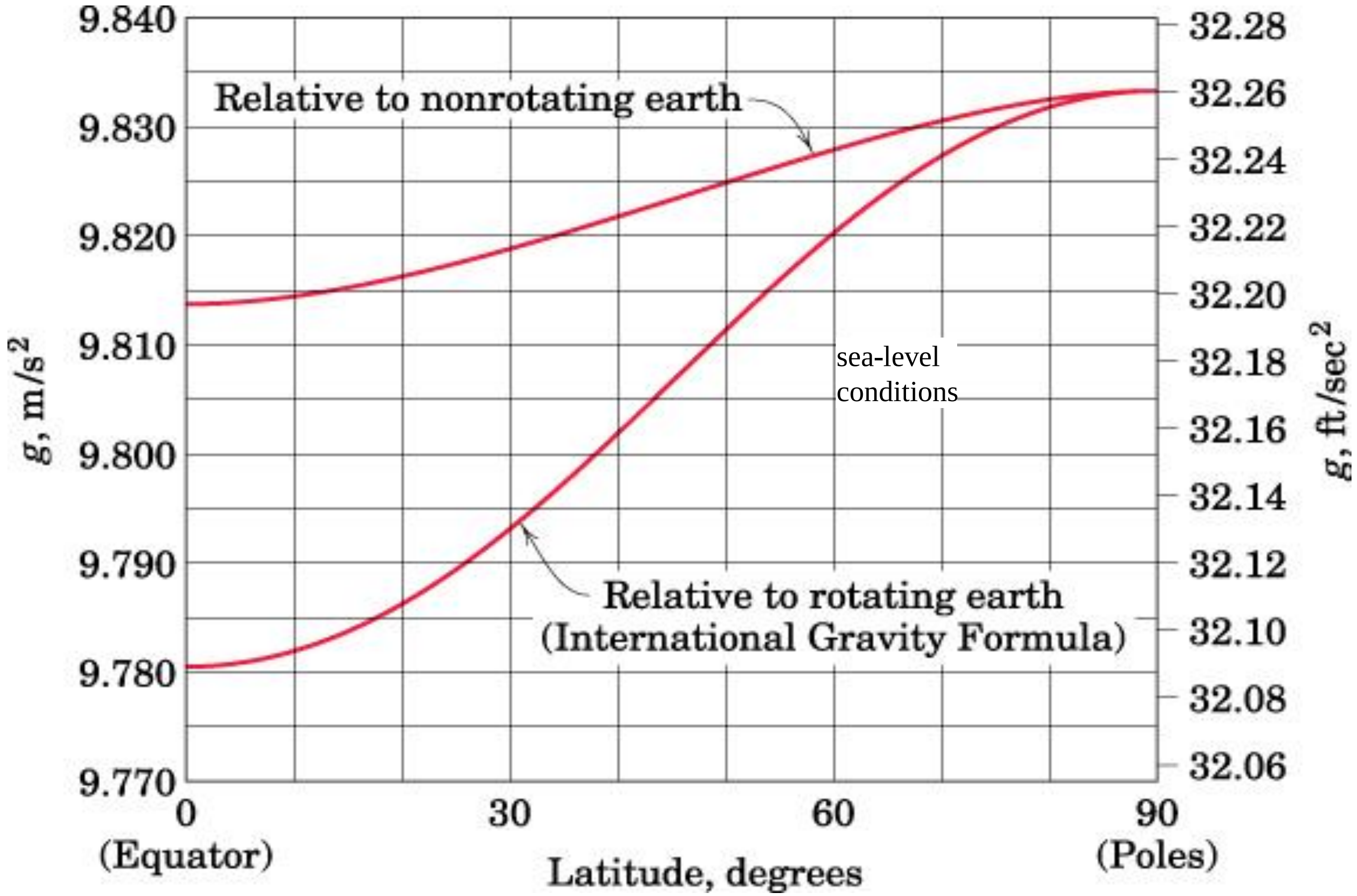
- **Force** of **gravitational attraction of the earth on a body** depends on the **position of the body relative to the earth**
- **Assuming** the **earth** to be a **perfect homogeneous sphere**, a **mass** of **1 kg** would be attracted to the earth by a force of:
  - **9.825 N** if the mass is on the surface of the earth
  - **9.822 N** if the mass is at an altitude of 1 km
  - **9.523 N** if the mass is at an altitude of 100 km
  - **7.340 N** if the mass is at an altitude of 1000 km
  - **2.456 N** if the mass is at an altitude of equal to the mean radius of the earth, 6371 km

Engineering Mechanics: Dynamics

- Effect of earth's rotation
  - $\boldsymbol{g}$ from law of gravitation
    - Fixed set of axes at the centre of the earth
      - Absolute value of $\boldsymbol{g}$
    - Earth's rotation
      - **Actual acceleration** of a **freely falling body** is **less than absolute** $\boldsymbol{g}$
      - **Measured** from a **position attached to the surface of the earth**

Engineering Mechanics: Dynamics

- Effect of earth's rotation



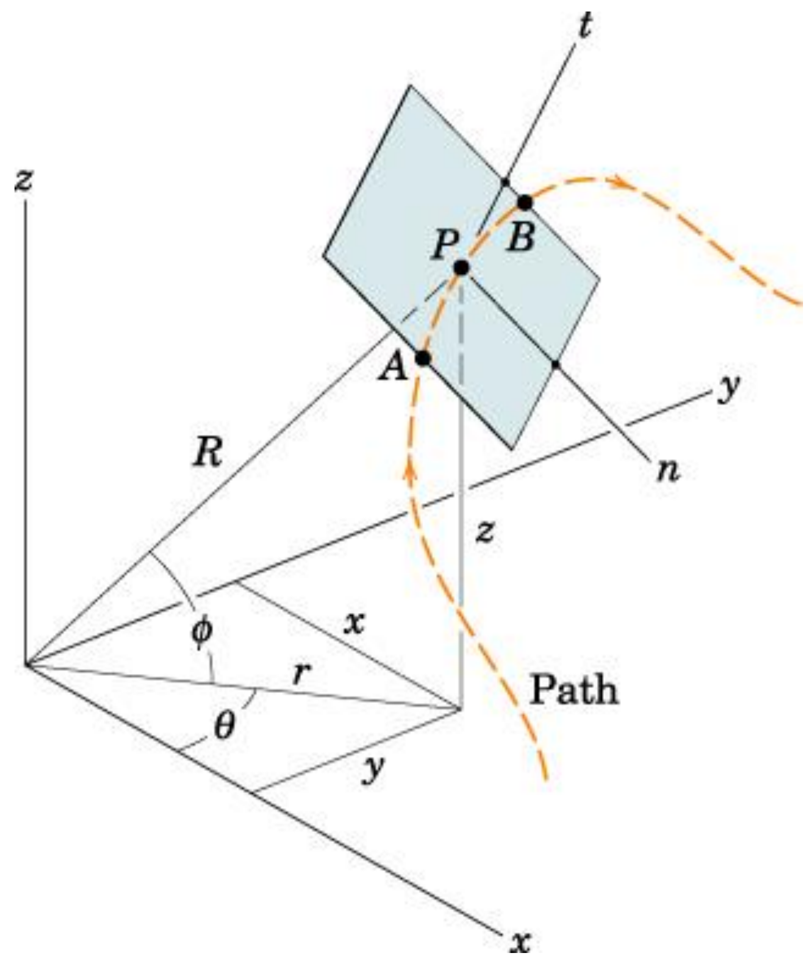
  - **Engineering applications** :: **variation of g is ignored**

Kinematics of Particles

- **Motion**
  - Constrained :: confined to a specific path
  - Unconstrained :: not confined to a specific path
- **Choice of coordinates**
  - Position of P at any time t
    - rectangular (i.e., Cartesian) coordinates x, y, z
    - cylindrical coordinates r, θ, z
    - spherical coordinates R, θ, Φ
  - Path variables
    - Measurements along the tangent t and normal n to the curve

Kinematics of Particles

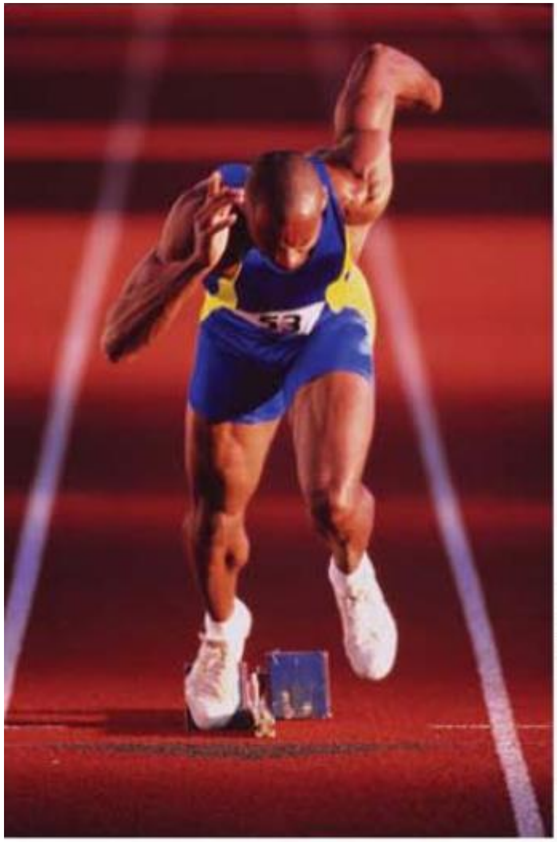
- Choice of coordinates



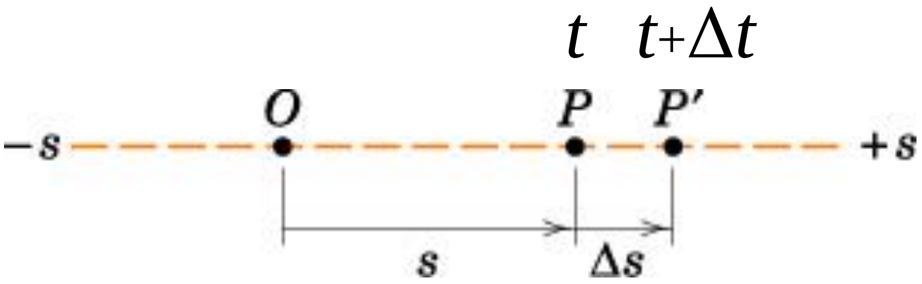
Kinematics of Particles

Rectilinear Motion

- Motion along a straight line

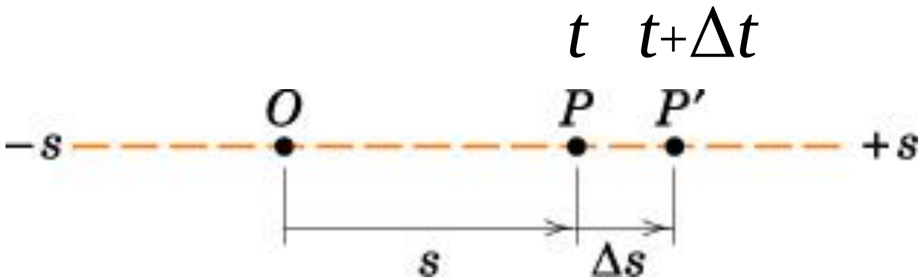


This sprinter will undergo rectilinear acceleration until he reaches his terminal speed.



Kinematics of Particles :: Rectilinear Motion

•Motion along a straight line



Position at any instance of time *t*

:: specified by its distance *s* measured from some convenient reference point *O* fixed on the line

:: (disp. is negative if the particle moves in the negative *s*-direction).

Velocity of the particle:

$$v = \frac{ds}{dt} = \dot{s}$$

Both are vector quantities

+ve or –ve depending on +ve or –ve displacement

Acceleration of the particle:

$$a = \frac{dv}{dt} = \dot{v} \quad \text{or} \quad a = \frac{d^2 s}{dt^2} = \ddot{s}$$

+ve or –ve depending on whether velocity is increasing or decreasing

$$v\,dv = a\,ds \qquad or \qquad \dot{s}\,d\dot{s} = \ddot{s}\,ds$$

Kinematics of Particles

Rectilinear Motion:

Graphical Interpretations

Using ***s-t*** curve, ***v-t*** & ***a-t*** curves can be plotted.

Area under *v-t* curve during time *dt* = *vdt* == *ds*

- Net disp from t_1 to t_2 = corresponding area under *v-t* curve →

$$\int_{s_1}^{s_2} ds = \int_{t_1}^{t_2} v dt$$

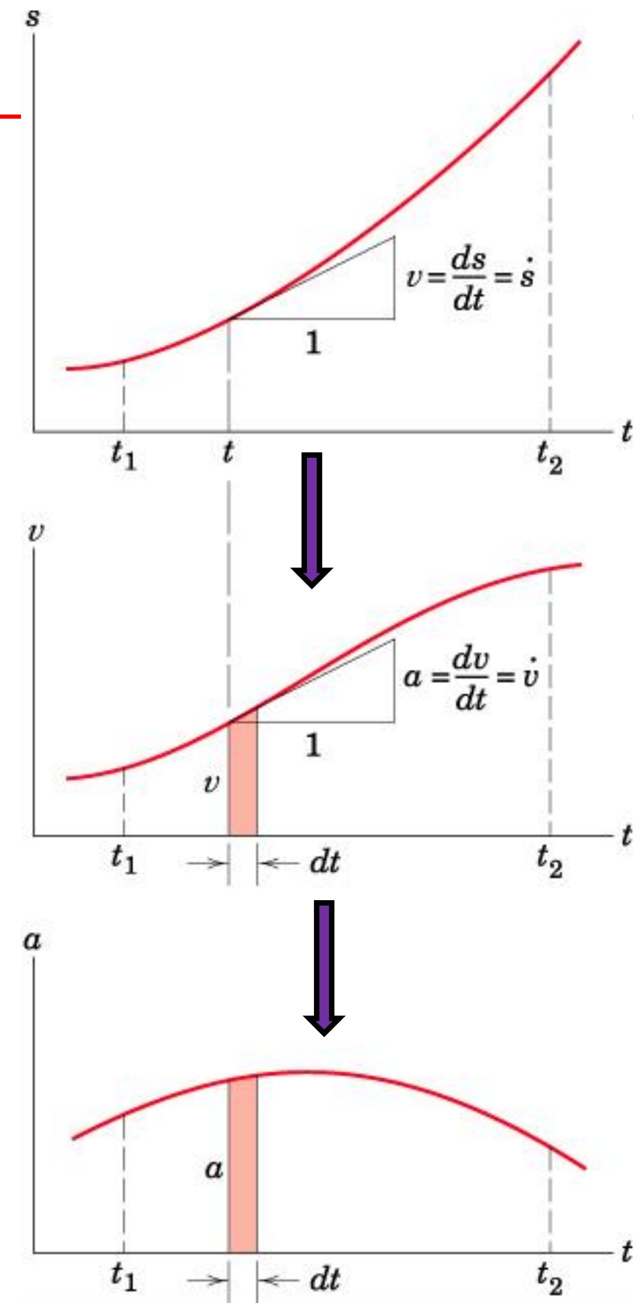
or **$s_2 - s_1$ = (area under *v-t* curve)**

Area under *a-t* curve during time *dt* = *adt* == *dv*

- Net change in vel from t_1 to t_2 = corresponding area under *a-t* curve →

$$\int_{v_1}^{v_2} dv = \int_{t_1}^{t_2} a dt$$

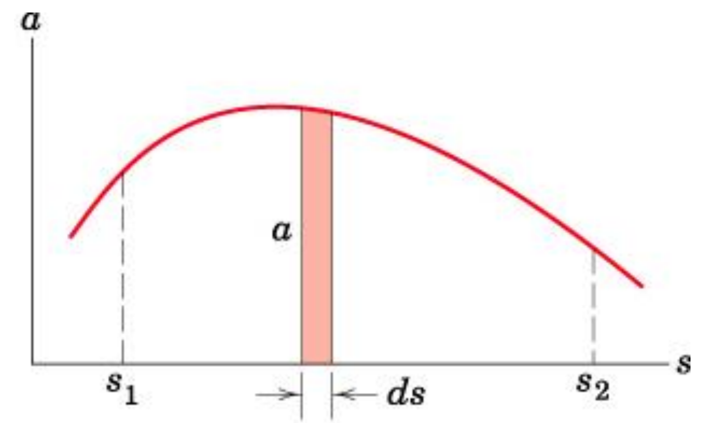
or **$v_2 - v_1$ = (area under *a-t* curve)**



Kinematics of Particles

Rectilinear Motion:

Graphical Interpretations



Two additional graphical relations:

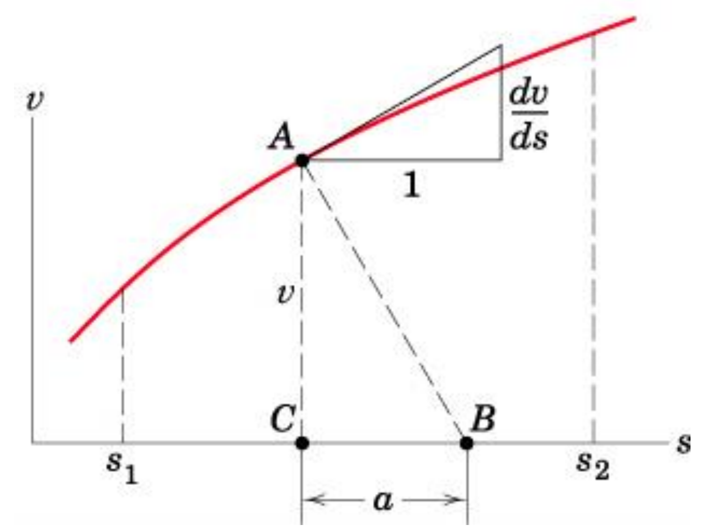
Area under *a*-*s* curve during disp *ds*= *ads* == *vdv*

- Net area under *a*-*s* curve betn position coordinates s_1 and s_2 →

$$\int_{v_1}^{v_2} v dv = \int_{s_1}^{s_2} a ds$$

or ½ **$(v_2^2 - v_1^2)$ = (area under *a*-*s* curve)**

Slope of *v*-*s* curve at any point *A* = *dv*/*ds*



- Construct a normal *AB* to the curve at *A*. From similar triangles:

$$\frac{\overline{CB}}{v} = \frac{dv}{ds} \quad \rightarrow \quad \overline{CB} = v\frac{dv}{ds} = a \;\; (acceleration)$$

- Vel and posn coordinate axes should have the same numerical scales so that the accln read on the x-axis in meters will represent the actual accln in m/s[2]

Kinematics of Particles :: Rectilinear Motion

Analytical Integration to find the position coordinate

Acceleration may be specified as a function of time, velocity, or position coordinate, or as a combined function of these.

(a) Constant Acceleration

At the beginning of the interval → $t = 0$, $s = s_0$, $v = v_0$

For a time interval t: integrating the following two equations

$$a = \frac{dv}{dt} \qquad \int_{v_0}^{v} dv = a\int_0^t dt \quad \text{or} \quad v = v_0 + at$$

$$v\,dv = a\,ds \qquad \int_{v_0}^{v} v\,dv = a\int_{s_0}^{s} ds \quad \text{or} \quad v^2 = {v_0}^2 + 2a(s - s_0)$$

Substituting in the following equation and integrating will give the position coordinate:

$$v = \frac{ds}{dt} \qquad \int_{s_0}^{s} ds = \int_0^t (v_0 + at)\,dt \quad \text{or} \quad s = s_0 + v_0 t + \frac{1}{2}at^2$$

Equations applicable for Constant Acceleration and for time interval 0 to t

Kinematics of Particles :: Rectilinear Motion

Analytical Integration to find the position coordinate

(b) Acceleration given as a function of time, $a = f(t)$

At the beginning of the interval → $t = 0$, $s = s_0$, $v = v_0$
For a time interval t: integrating the following equation

$$a = \frac{dv}{dt} \rightarrow f(t) = \frac{dv}{dt} \qquad \int_{v_0}^{v} dv = \int_0^t f(t)\,dt \quad \text{or} \quad v = v_0 + \int_0^t f(t)\,dt$$

Substituting in the following equation and integrating will give the position coordinate:

$$v = \frac{ds}{dt} \qquad \int_{s_0}^{s} ds = \int_0^t v\,dt \quad \text{or} \quad s = s_0 + \int_0^t v\,dt$$

Alternatively, following second order differential equation may be solved to get the position coordinate:

$$a = \frac{d^2 s}{dt^2} = \ddot{s} \rightarrow \ddot{s} = f(t)$$

Kinematics of Particles :: Rectilinear Motion

Analytical Integration to find the position coordinate

(c) Acceleration given as a function of velocity, $a = f(v)$

At the beginning of the interval → $t = 0$, $s = s_0$, $v = v_0$

For a time interval t: Substituting a and integrating the following equation

$$a = \frac{dv}{dt} \rightarrow f(v) = \frac{dv}{dt} \qquad t = \int_0^t dt = \int_{v_0}^{v} \frac{dv}{f(v)}$$

Solve for v as a function of t and integrate the following equation to get the position coordinate:

$$v = \frac{ds}{dt}$$

Alternatively, substitute $a = f(v)$ in the following equation and integrate to get the position coordinate :

$$v\,dv = a\,ds \qquad \int_{v_0}^{v} \frac{v\,dv}{f(v)} = \int_{s_0}^{s} ds \quad \text{or} \quad s = s_0 + \int_{v_0}^{v} \frac{v\,dv}{f(v)}$$

Kinematics of Particles: Rectilinear Motion

Analytical Integration to find the position coordinate

(d) Acceleration given as a function of displacement, $a = f(s)$

At the beginning of the interval → $t = 0$, $s = s_0$, $v = v_0$

For a time interval t: substituting a and integrating the following equation

$$v\,dv = a\,ds \qquad \int_{v_0}^{v} v\,dv = \int_{s_0}^{s} f(s)\,ds \quad \text{or} \quad v^2 = v_0^2 + 2\int_{s_0}^{s} f(s)\,ds$$

Solve for v as a function of s : $v = g(s)$, substitute in the following equation and integrate to get the position coordinate:

$$v = \frac{ds}{dt} \qquad \int_{s_0}^{s} \frac{ds}{g(s)} = \int_{0}^{t} dt \quad \text{or} \quad t = \int_{s_0}^{s} \frac{ds}{g(s)}$$

It gives t as a function of s. Rearrange to obtain s as a function of t to get the position coordinate.

In all these cases, if integration is difficult, graphical, analytical, or computer methods can be utilized.

Kinematics of Particles: Rectilinear Motion

Example

Position coordinate of a particle confined to move along a straight line is given by $s = 2t^3 - 24t + 6$, where s is measured in meters from a convenient origin and t is in seconds. Determine: (a) time reqd for the particle to reach a velocity of 72 m/s from its initial condition at $t = 0$, (b) acceleration of the particle when $v = 30$ m/s, and (c) net disp of the particle during the interval from $t = 1$ s to $t = 4$ s.

Solution

Differentiating $s = 2t^3 - 24t + 6$ → $v = 6t^2 - 24$ m/s
→ $a = 12t$ m/s^2

(a) $v = 72$ m/s → $t = \pm 4$ s
(- 4 s happened before initiation of motion → no physical interest.)
→ $t = 4$ s

(b) $v = 30$ m/s → $t = 3$ sec → $a = 36$ m/s^2

(c) t = 1 s to 4 s. Using $s = 2t^3 - 24t + 6$
$\Delta s = s_4 - s_1 = [2(4^3) - 24(4) + 6] - [2(1^3) - 24(1) + 6]$
$\Delta s = 54$ m

Kinematics of Particles

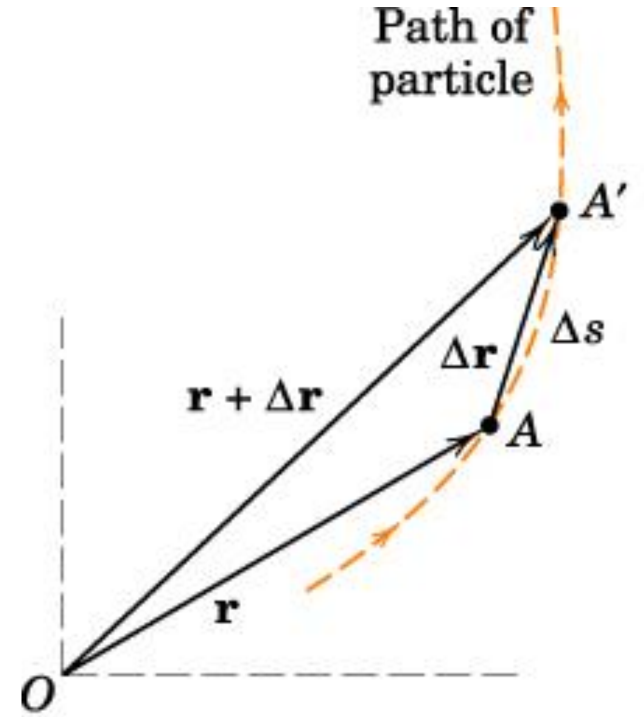
Plane Curvilinear Motion

Motion of a particle along a curved path which lies in a single plane.

For a short time during take-off and landing, planes generally follow plane curvilinear motion

Kinematics of Particles

Plane Curvilinear Motion:



Between *A* and *A'*:
Average velocity of the particle : $\mathbf{v}_{av} = \Delta\mathbf{r}/\Delta t$
→ A vector whose direction is that of $\Delta\mathbf{r}$ and whose magnitude is magnitude of $\Delta\mathbf{r}/\Delta t$
Average speed of the particle = $\Delta s/\Delta t$

Instantaneous velocity of the particle is defined as the limiting value of the average velocity as the time interval approaches zero →

$$\mathbf{v} = \lim_{\Delta t \to 0} \frac{\Delta\mathbf{r}}{\Delta t}$$

→ **v** is always a vector tangent to the path

Extending the definition of derivative of a scalar to include vector quantity:

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \dot{\mathbf{r}}$$

Derivative of a vector is a vector having a magnitude and a direction.

Magnitude of v is equal to speed (scalar)

$$v = |\mathbf{v}| = \frac{ds}{dt} = \dot{s}$$

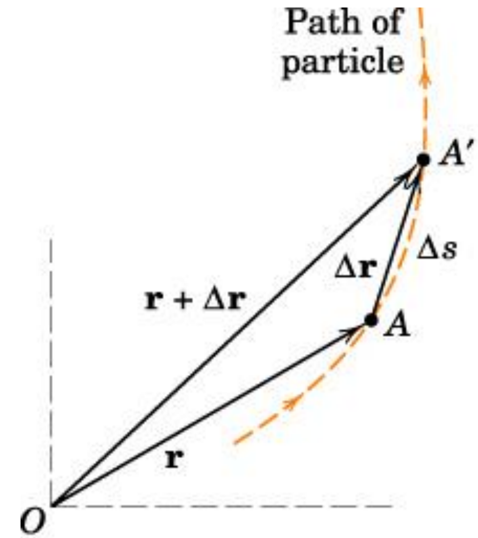
Kinematics of Particles

Plane Curvilinear Motion

Magnitude of the derivative:

$$|d\mathbf{r}/dt| = |\dot{\mathbf{r}}| = \dot{s} = |\mathbf{v}| = v$$

→ Magnitude of the velocity or the speed

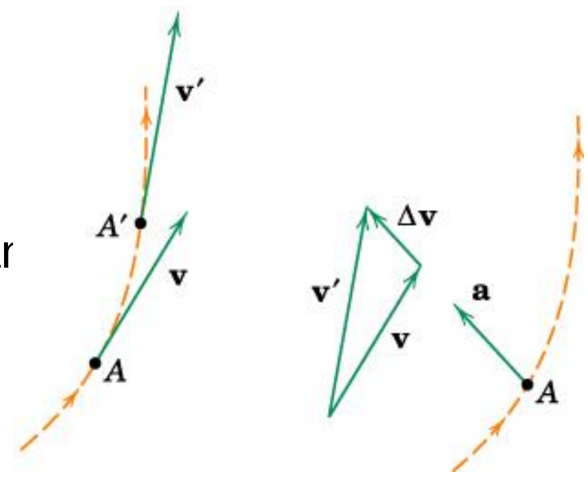


Derivative of the magnitude:

$$d|\mathbf{r}|/dt = dr/dt = \dot{r}$$

→ Rate at which the length of the position vector is changing

Velocity of the particle at *A* → tangent vector **v**
Velocity of the particle at *A'* → tangent vector **v'**
→ **v'** – **v** = Δ**v**
→ Δ**v** Depends on both the change in magnitude of **v** ar
on the change in direction of **v**.



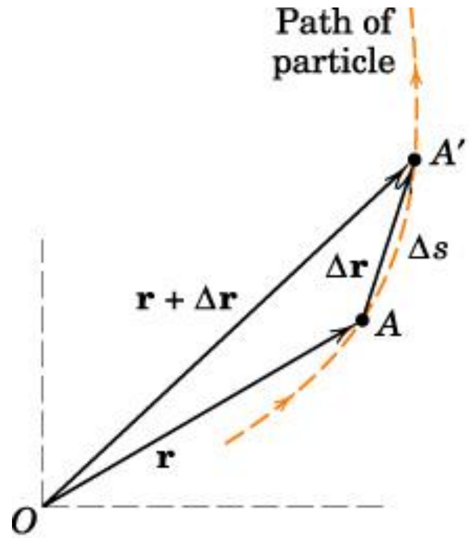
Kinematics of Particles

Plane Curvilinear Motion

Between *A* and *A'*:

Average acceleration of the particle : $\mathbf{a}_{av} = \Delta\mathbf{v}/\Delta t$

→ A vector whose direction is that of $\Delta\mathbf{v}$ and whose magnitude is the magnitude of $\Delta\mathbf{v}/\Delta t$

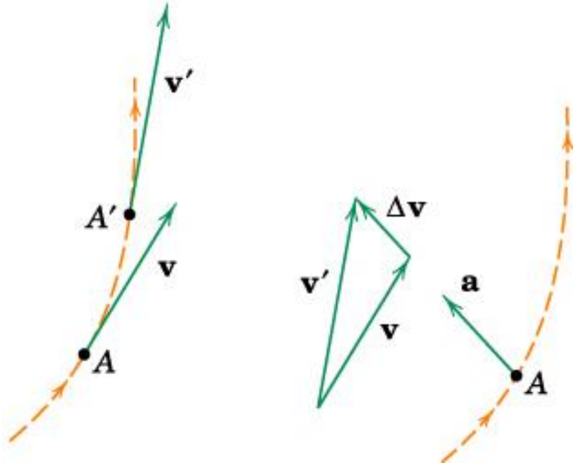


Instantaneous accln of the particle is defined as the limiting value of the average accln as the time interval approaches zero →

$$\mathbf{a} = \lim_{\Delta t \to 0} \frac{\Delta\mathbf{v}}{\Delta t}$$

By definition of the derivative:

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \dot{\mathbf{v}}$$



→ In general, direction of the acceleration of a particle in curvilinear motion neither tangent to the path nor normal to the path.

→ Acceleration component normal to the path points toward the center of curvature of the path.