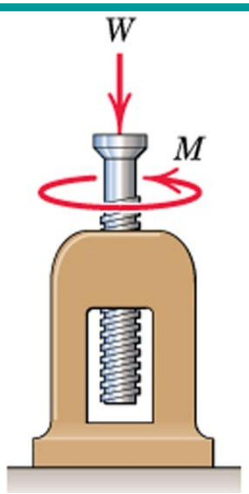
Applications of Friction in Machines

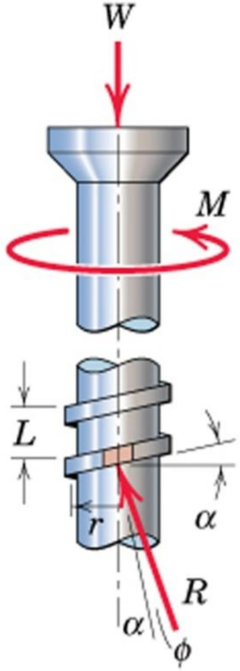
Square Threaded Screws

- Used for fastening and for transmitting power or motion
- Square threads are more efficient
- Friction developed in the threads largely determines the action of the screw



FBD of the Screw: R exerted by the thread of the jack frame on a small portion of the screw thread is shown

Lead = L = advancement per revolution
L = Pitch – for single threaded screw
L = 2xPitch – for double threaded screw (twice advancement per revolution)
Pitch = axial distance between adjacent threads on a helix or screw
Mean Radius = r ; α = Helix Angle



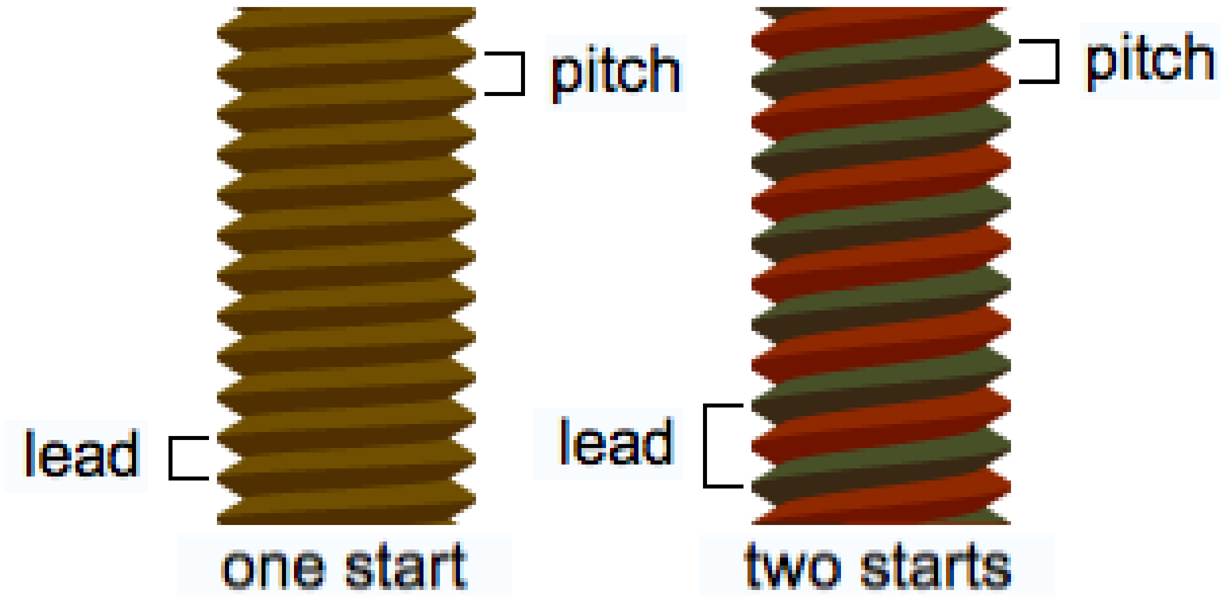
Similar reactions exist on all segments of the screw threads

Analysis similar to block on inclined plane since friction force does not depend on area of contact.

- Thread of base can be "unwrapped" and shown as straight line. Slope is $2\pi r$ horizontally and lead L vertically.

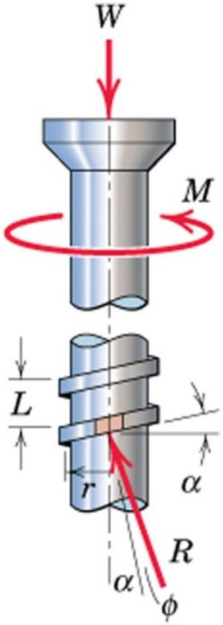
Applications of Friction in Machines

- Thread of a screw



Single thread **Double thread**

Applications of Friction in Machines: Screws



If M is just sufficient to turn the screw → Motion Impending
Angle of friction = ϕ (made by R with the axis normal to the thread)
→ $\tan\phi = \mu$

Moment of R @ vertical axis of screw = $R\sin(\alpha+\phi)r$
→ Total moment due to all reactions on the thread = $\sum R\sin(\alpha+\phi)r$
→ Moment Equilibrium Equation for the screw:

$$\rightarrow M = [r\sin(\alpha+\phi)] \sum R$$

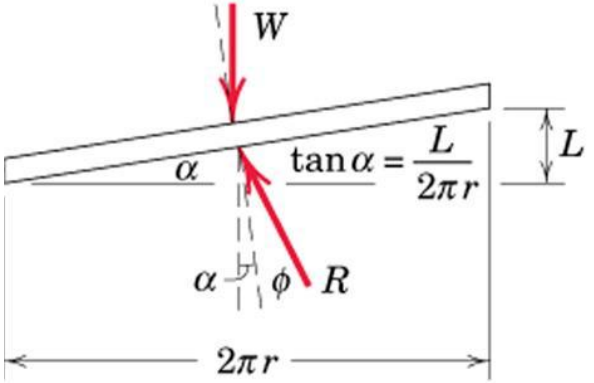
Equilibrium of forces in the axial direction: $W = \sum R\cos(\alpha+\phi)$

$$\rightarrow W = [\cos(\alpha+\phi)] \sum R$$

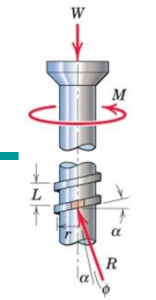
Finally → $\mathbf{M = W\, r \tan(\alpha+\phi)}$

Helix angle α can be determined by unwrapping the thread of the screw for one complete turn

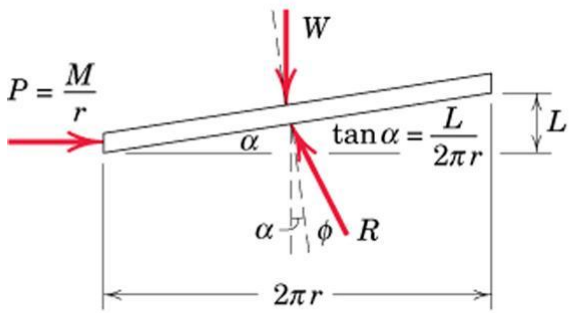
$$\alpha = \tan^{-1}(L/2\pi r)$$



Applications of Friction in Machines: Screws



Alternatively, action of the entire screw can be simulated using unwrapped thread of the screw



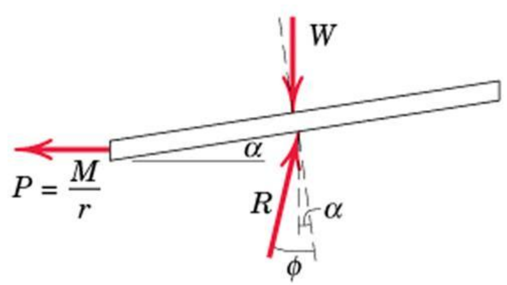
To Raise Load

Equivalent force required to push the movable thread up the fixed incline is:
$P = M/r$

From Equilibrium:

$$M = W\, r \tan(\alpha + \phi)$$

If M is removed: the screw will remain in place and be self-locking provided $\alpha<\phi$ and will be on the verge of unwinding if $\alpha=\phi$



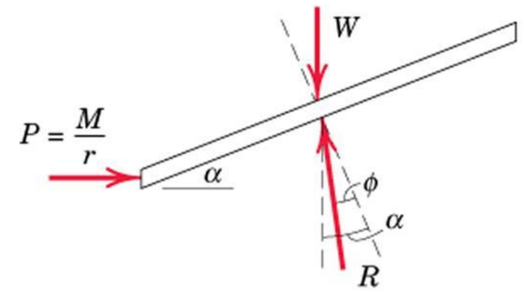
To Lower Load ($\alpha<\phi$)

To lower the load by unwinding the screw, We must reverse the direction of M as long as $\alpha<\phi$

From Equilibrium:

$$M = W\, r \tan(\phi - \alpha)$$

→ This is the moment required to unwind the screw



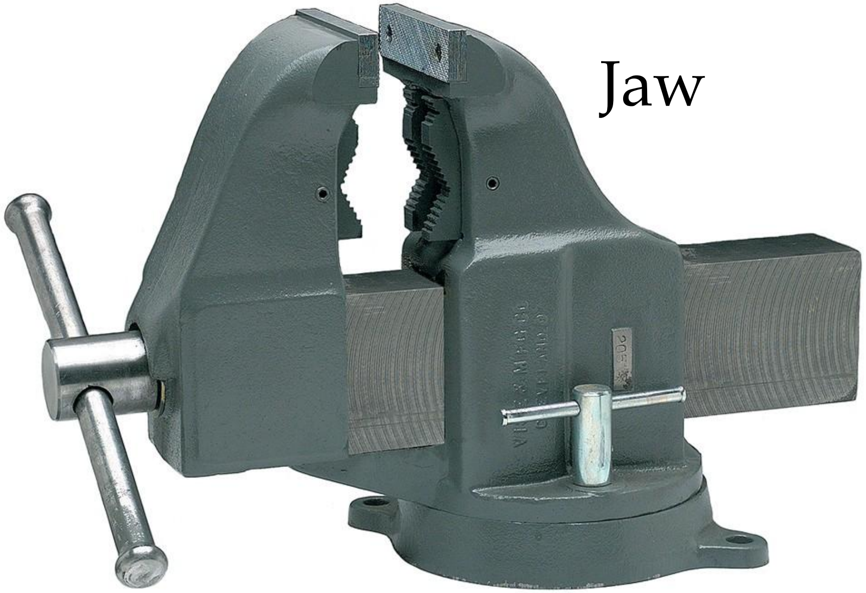
To Lower Load ($\alpha>\phi$)

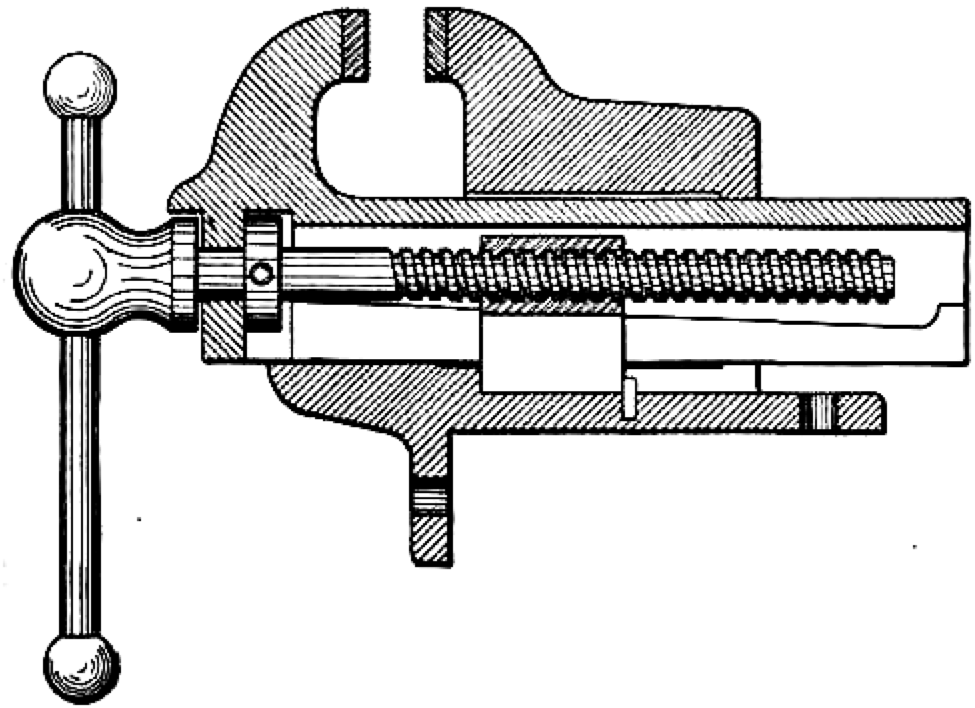
If $\alpha > \phi$, the screw will unwind by itself. Moment required to prevent unwinding:

From Equilibrium:

$$M = W\, r \tan(\alpha - \phi)$$

Example of Screw Action: Vise





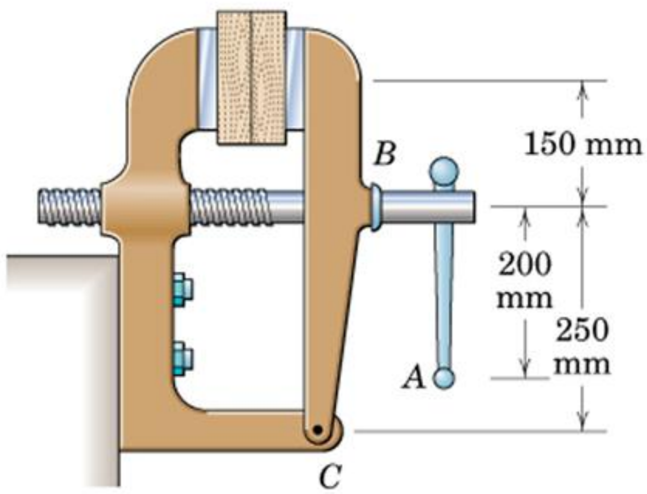
Example of Screw Action: Vise

Single threaded screw of the vise has a mean diameter of 25 mm and a lead of 5 mm. A 300 N pull applied normal to the handle at *A* produces a clamping force of 5 kN between the jaws of the vise. Determine:

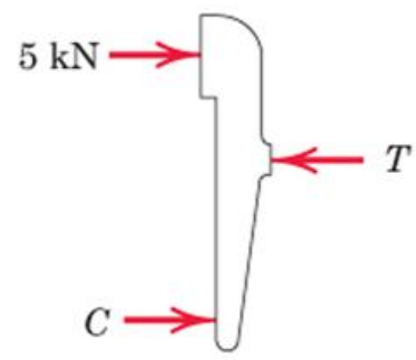
(a)Frictional moment M_B developed at *B* due to thrust of the screw against body of the jaw

(b)Force *Q* applied normal to the handle at *A* required to loosen the vise

μ_s in the threads = 0.20



Solution: Draw FBD of the jaw to find tension in the screw



$\sum M_C = 0$

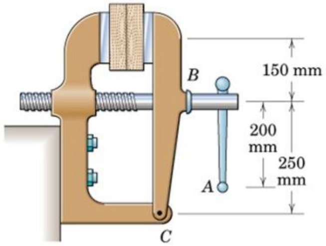
$\rightarrow$ T = 8 kN

Find the helix angle *α* and the friction angle *ϕ*

$\alpha = tan^{-1} (L/2\pi r) = 3.64^o$

$\tan \phi = \mu \rightarrow \phi = 11.31^o$

Example of Screw Action: Vise

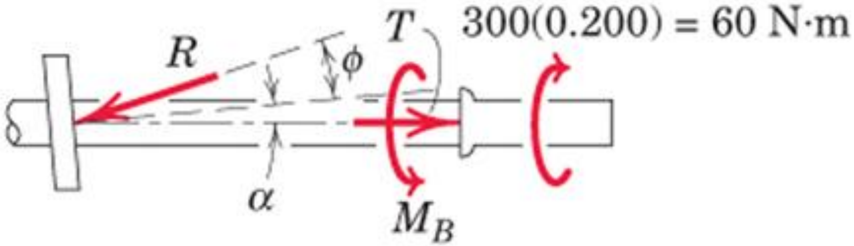


Example: Screw

Solution:

(a) To tighten the vise

Draw FBD of the screw



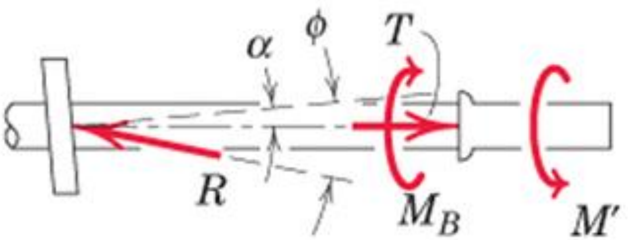
$M = T\, r \tan(\alpha + \phi)$

$60 - M_B = 8000(0.0125)\tan(3.64° + 11.31°)$

$\boldsymbol{M_B = 33.3}$ **Nm**

(a) To loosen the vise (on the verge of being loosened)

Draw FBD of the screw: Net moment = (Applied moment) - M_B



$M = T\, r \tan(\phi - \alpha)$

$M' - 33.3 = 8000(0.0125)\tan(11.31° - 3.64°)$

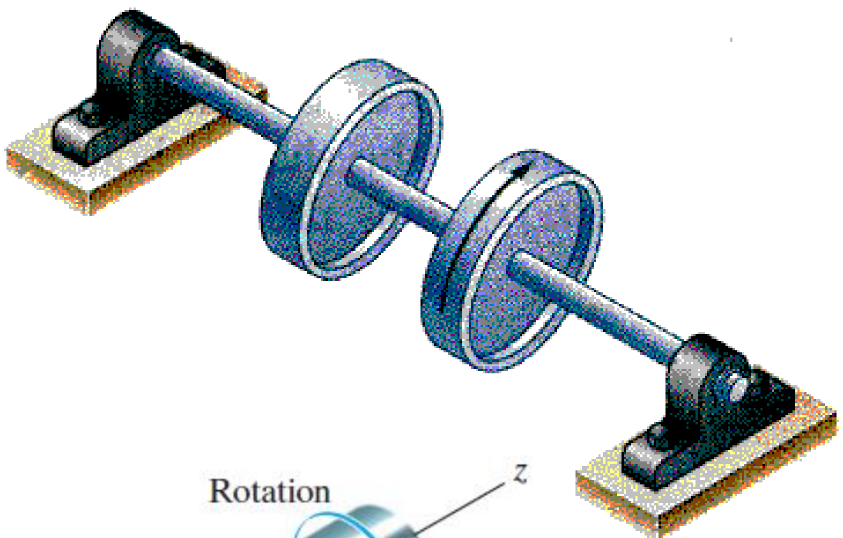
$M' = 46.8$ Nm

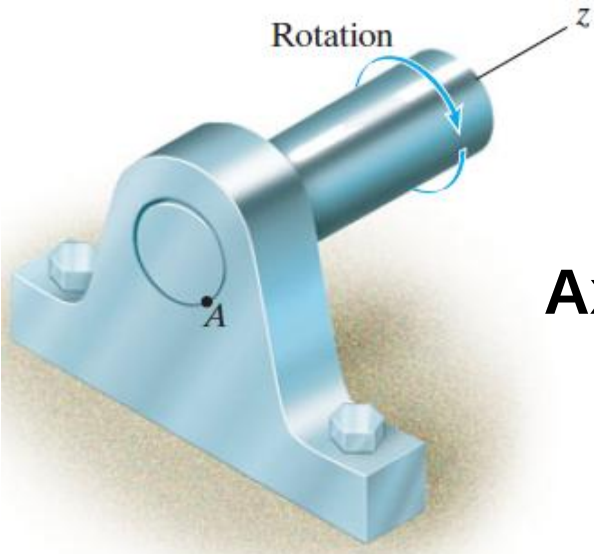
$\boldsymbol{Q = M'/d = 46.8/0.2 = 234}$ **N**

(Applied moment: M')

Friction in Machines :: Journal Bearing

- Rotating shaft

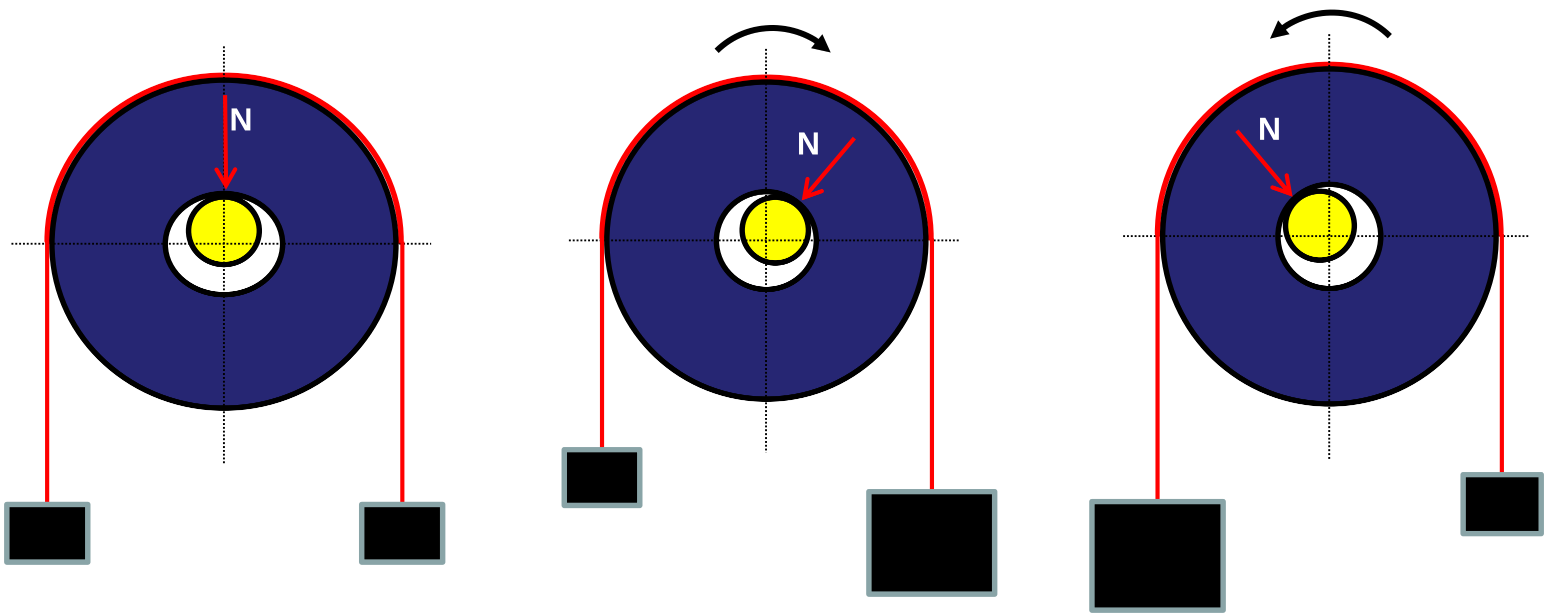




Axle Friction

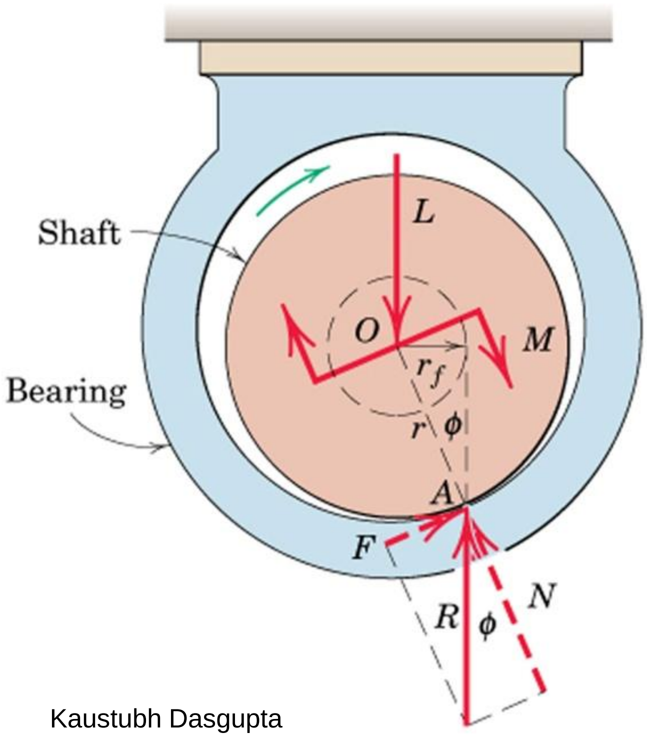
Friction in Machines :: Journal Bearing

- Normal reaction on bearing
  - Point of application
  - Friction tends to oppose the motion
  - Friction angle ϕ for the resultant force

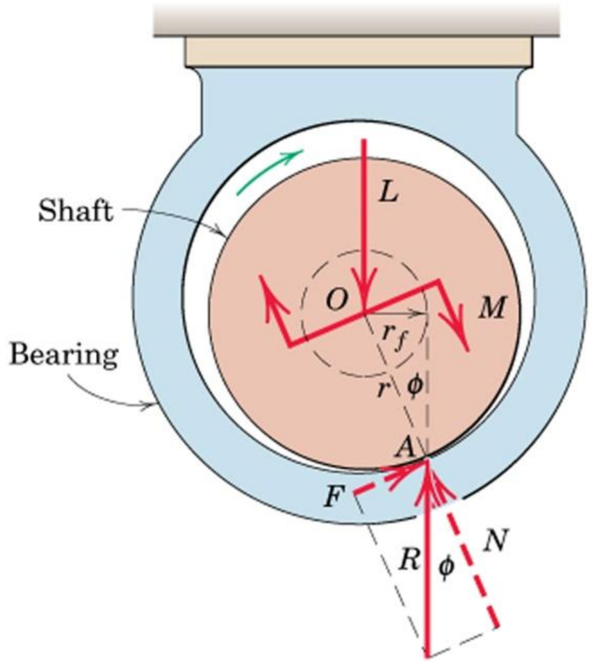


Friction in Machines :: Journal Bearing

- Lateral/Vertical load on shaft is L
- Partially lubricated bearing
  - *Direct contact along a line*
- Fully lubricated bearing
  - *Clearance, speed, lubricant viscosity*



Friction in Machines :: Journal Bearing



-*R* will be tangent to a small circle of radius r_f called the *friction circle*

$\sum M_A = 0 \rightarrow M = Lr_f = Lr \sin\phi$

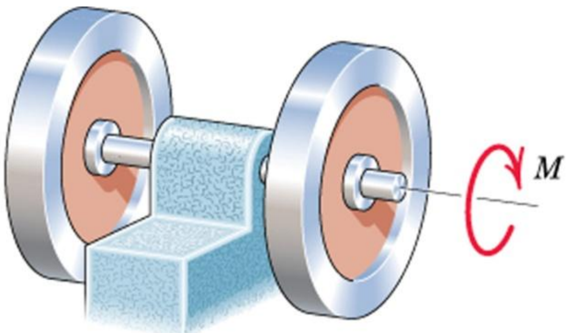
For a small coefficient of friction, ϕ is small → $\sin\phi \approx \tan\phi$

→ $M = \mu Lr$ (since $\mu = \tan\phi$) → Use equilibrium equations to solve a problem

→ Moment that must be applied to the shaft to overcome friction for a dry or partially lubricated journal bearing

Example on Journal Bearing

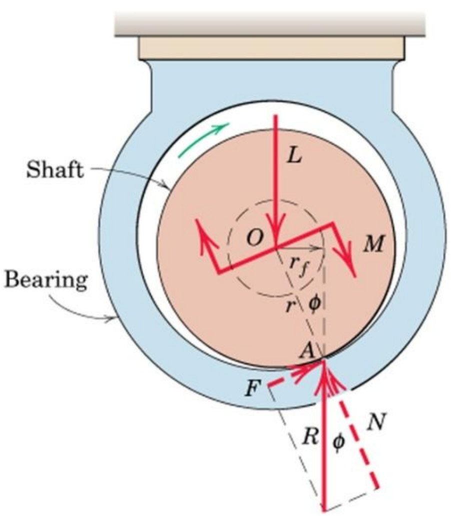
Two flywheels (each of mass 40 kg and diameter 40 mm) are mounted on a shaft, which is supported by a journal bearing. M = 3 Nm couple is reqd on the shaft to maintain rotation of the flywheels and shaft at a constant low speed.
Determine: (a) coeff of friction in the bearing, and (b) radius r_f of the friction circle.



Solution: Draw the FBD of the shaft and the bearing

(a) Moment equilibrium at O
$M = Rr_f = Rr\sin\phi$
M = 3 Nm, R = 2x40x9.81 = 784.8 N, r = 0.020 m
→ $\sin\phi$ = 0.1911 → $\phi = 11.02^o$

(b) $r_f = r\sin\phi$ = 3.82 mm



Friction in Machines :: Disc Friction

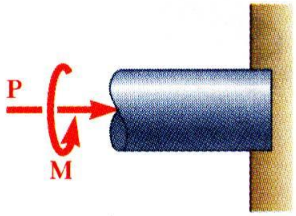
- Thrust Bearings

Sanding Machine

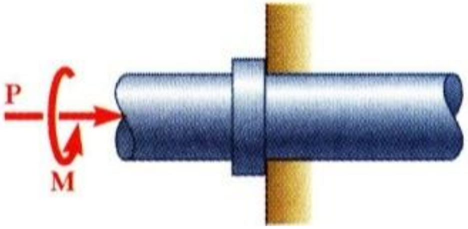
Friction in Machines :: Disk Friction

Thrust Bearings (Disk Friction)

- Thrust bearings provide axial support to rotating shafts.
- Axial load acting on the shaft is *P*.
- Friction between circular surfaces under distributed normal pressure (Ex: clutch plates, disc brakes)



End/Pivot Bearing



Collar Bearing

Consider two flat circular discs whose shafts are mounted in bearings: they can be brought under contact under *P*

Max torque that the clutch can transmit =

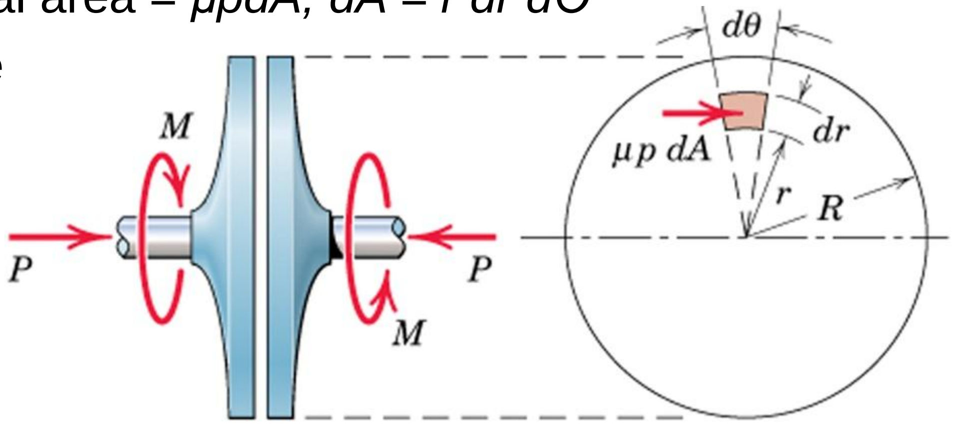
M required to slip one disc against the other

p is the normal pressure at any location between the plates

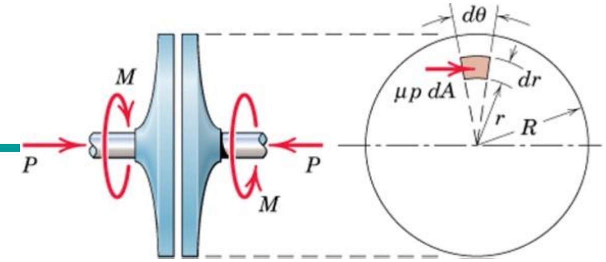
→ Frictional force acting on an elemental area = $\mu p dA$; $dA = r\, dr\, d\theta$

Moment of this elemental frictional force about the shaft axis = $\mu p r dA$

Total $M = \int \mu p r dA$ over the area of disc



Friction in Machines :: Disk Friction



Thrust Bearings (Disk Friction)

Assuming that μ and p are uniform over the entire surface → $P = \pi R^2 p$

→ Substituting the constant p in $M = \int \mu p r dA$ → $M = \frac{\mu P}{\pi R^2} \int_0^{2\pi} \int_0^R r^2 dr\, d\theta$

→ $M = \frac{2}{3} \mu P R$ ➔ Magnitude of moment reqd for impending rotation of shaft

≈ moment due to frictional force ***μp*** acting a distance ⅔ ***R*** from shaft center

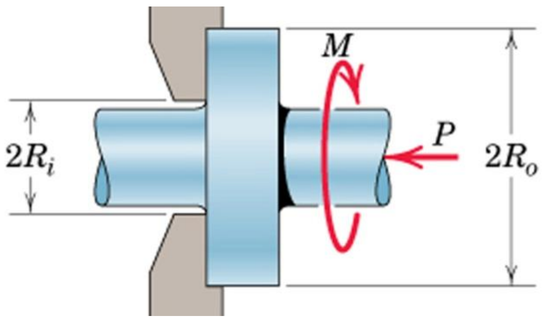
Frictional moment for worn-in plates is only about ¾ of that for the new surfaces

→ M for worn-in plates = ½(μPR)

If the friction discs are rings (Ex: Collar bearings) with outside and inside radius as R_o and R_i, respectively (limits of integration R_o and R_i) → $P = \pi(R_o^2 - R_i^2)p$

→ The frictional torque: $M = \frac{\mu P}{\pi\left(R_o^2 - R_i^2\right)} \int_0^{2\pi} \int_{R_i}^{R_o} r^2 dr\, d\theta$

→ $M = \frac{2}{3} \mu P \frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}$



Frictional moment for worn-in plates → $M = ½\, \mu P(R_o + R_i)$

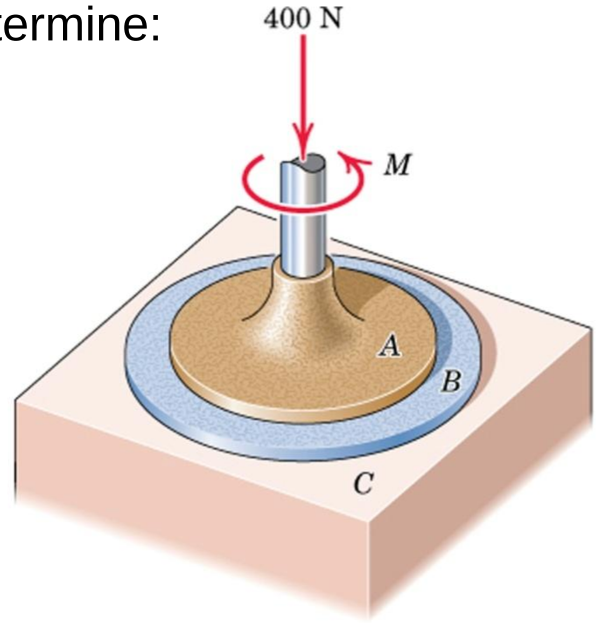
Applications of Friction in Machines

Examples: Disk Friction

Circular disk *A* (225 mm dia) is placed on top of disk *B* (300 mm dia) and is subjected to a compressive force of 400 N. Pressure under each disk is constant over its surface. Coeff of friction betn *A* and *B* = 0.4. Determine:

(a) the couple *M* which will cause *A* to slip on *B*.

(b) Min coeff of friction *μ* between *B* and supporting surface *C* which will prevent *B* from rotating.



Solution: $M = \frac{2}{3}\mu PR$

(a) Impending slip between *A* and *B*:

μ=0.4, P=400 N, R=225/2 mm

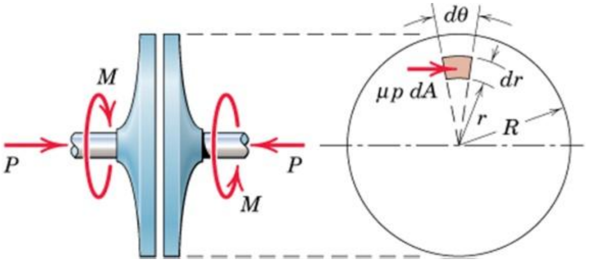
M = 2/3 x 0.4 x 400 x 0.225/2 → M = 12 Nm

(b) Impending slip between *B* and *C* :

Slip between *A* and *B* → M = 12 Nm

μ=? P=400 N, R=300/2 mm

12 = 2/3 x μ x 400 x 0.300/2 → μ = 0.3



Applications of Friction in Machines

Belt Friction

Impending slippage of flexible cables, belts, ropes over sheaves, wheels, drums

→ It is necessary to estimate the frictional forces developed between the belt and its contacting surface.

Consider a drum subjected to two belt tensions (T_1 and T_2)

M is the torque necessary to prevent rotation of the drum

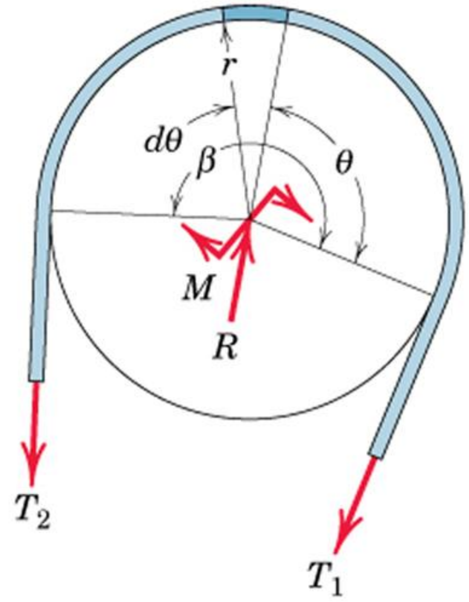
R is the bearing reaction

r is the radius of the drum

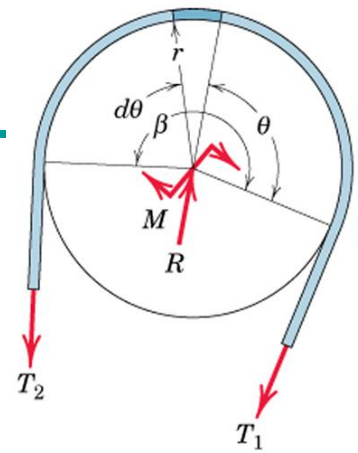
β is the total contact angle between belt and surface

(β in radians)

$T_2 > T_1$ since M is clockwise



Applications of Friction in Machines



Belt Friction: Relate T_1 and T_2 when belt is about to slide to left

Draw FBD of an element of the belt of length $r\,d\theta$

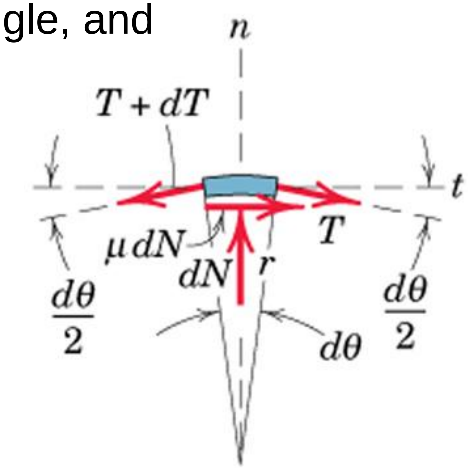
Frictional force for impending motion = $\mu\,dN$

Equilibrium in the t-direction: $T\cos\frac{d\theta}{2} + \mu\,dN = (T + dT)\cos\frac{d\theta}{2}$

→ $\mu dN = dT$ (cosine of a differential quantity is unity in the limit)

Equilibrium in the n-direction: $dN = (T + dT)\sin\frac{d\theta}{2} + T\sin\frac{d\theta}{2}$

→ $dN = 2Td\theta/2 = Td\theta$ (sine of a differential in the limit equals the angle, and product of two differentials can be neglected)



Combining two equations: $\frac{dT}{T} = \mu\,d\theta$

Integrating between corresponding limits: $\int_{T_1}^{T_2}\frac{dT}{T} = \int_0^{\beta}\mu\,d\theta$

→ $\ln\frac{T_2}{T_1} = \mu\beta$ → $\mathbf{T_2 = T_1\,e^{\mu\beta}}$ ($T_2 > T_1$; e = 2.718…; β in radians)

- Rope wrapped around a drum n times → $\beta = 2\pi n$ radians
- r not present in the above eqn → eqn valid for non-circular sections as well
- In belt drives, belt and pulley rotate at constant speed → the eqn describes condition of impending slippage.