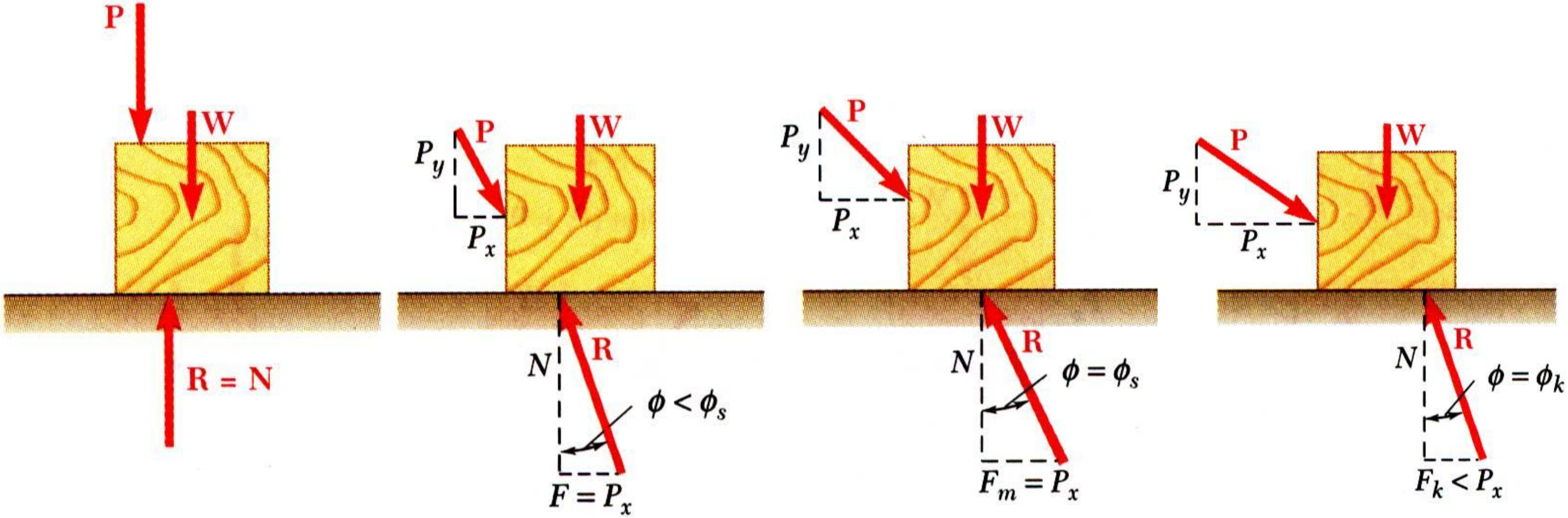
Mechanism of Dry Friction

Sometimes convenient to replace normal force *N* & friction force *F* by their resultant ***R***:



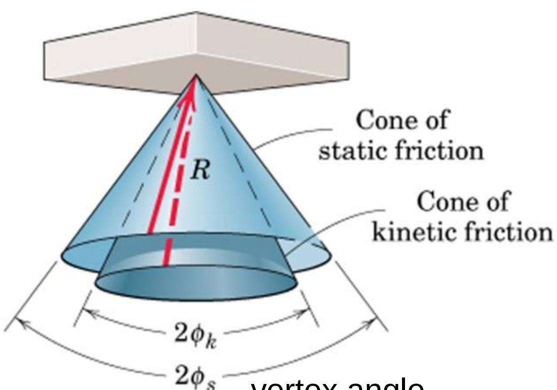
- No friction
- No motion
- Motion impending
- Motion

$$\tan\phi_s = \frac{F_m}{N} = \frac{\mu_s N}{N}$$

$$\tan\phi_s = \mu_s$$

$$\tan\phi_k = \frac{F_k}{N} = \frac{\mu_k N}{N}$$

$$\tan\phi_k = \mu_k$$



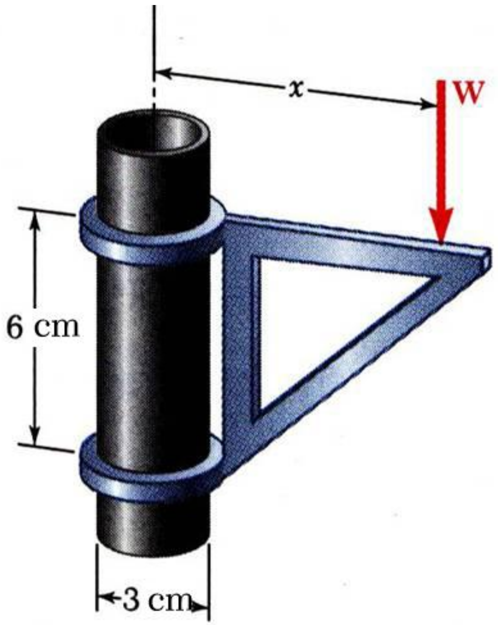
vertex angle

Friction Angles

ϕ_s = angle of static friction, ϕ_k = angle of kinetic friction

Dry Friction

Example

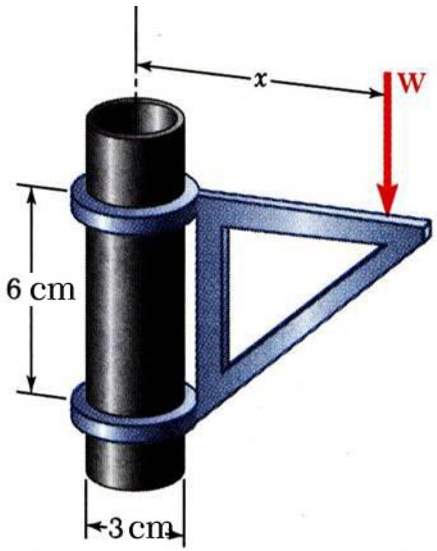


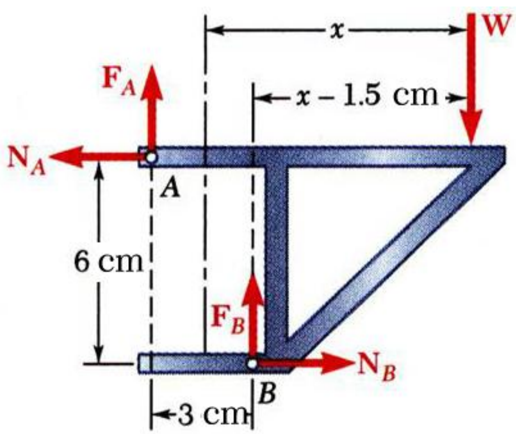
The moveable bracket shown may be placed at any height on the 3-cm diameter pipe. If the coefficient of friction between the pipe and bracket is 0.25, determine the minimum distance ***x*** at which the load can be supported. Neglect the weight of the bracket.

SOLUTION:

- When *W* is placed at minimum *x*, the bracket is about to slip and friction forces in upper and lower collars are at maximum value.
- Apply conditions for static equilibrium to find minimum *x*.

Dry Friction





<u>SOLUTION</u>:

- When W is placed at minimum x, the bracket is about to slip and friction forces in upper and lower collars are at maximum value.

$$F_A = \mu_s N_A = 0.25 N_A$$
$$F_B = \mu_s N_B = 0.25 N_B$$

- Apply conditions for static equilibrium to find minimum x.

$$\sum F_x = 0: \quad N_B - N_A = 0 \qquad N_B = N_A$$

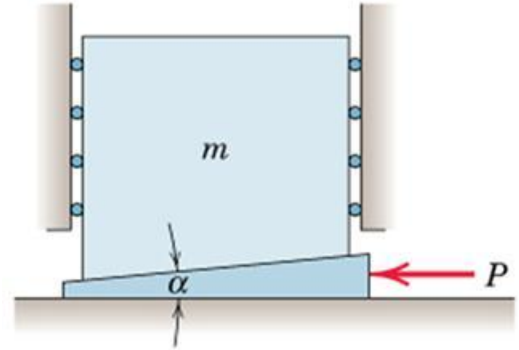
$$\sum F_y = 0: \quad F_A + F_B - W = 0$$
$$0.25 N_A + 0.25 N_B - W = 0$$
$$0.5 N_A = W \qquad N_A = N_B = 2W$$

$$\sum M_B = 0: N_A(6\text{ cm}) - F_A(3\text{ cm}) - W(x - 1.5\text{ cm}) = 0$$
$$6N_A - 3(0.25 N_A) - W(x - 1.5) = 0$$
$$6(2W) - 0.75(2W) - W(x - 1.5) = 0$$
$$\boxed{x = 12\text{ cm}}$$

Applications of Friction in Machines

Wedges

- Simple machines used to raise heavy loads
- Force required to lift block is significantly less than block weight
- Friction prevents wedge from sliding out
- **Minimum force *P*** required to raise block???



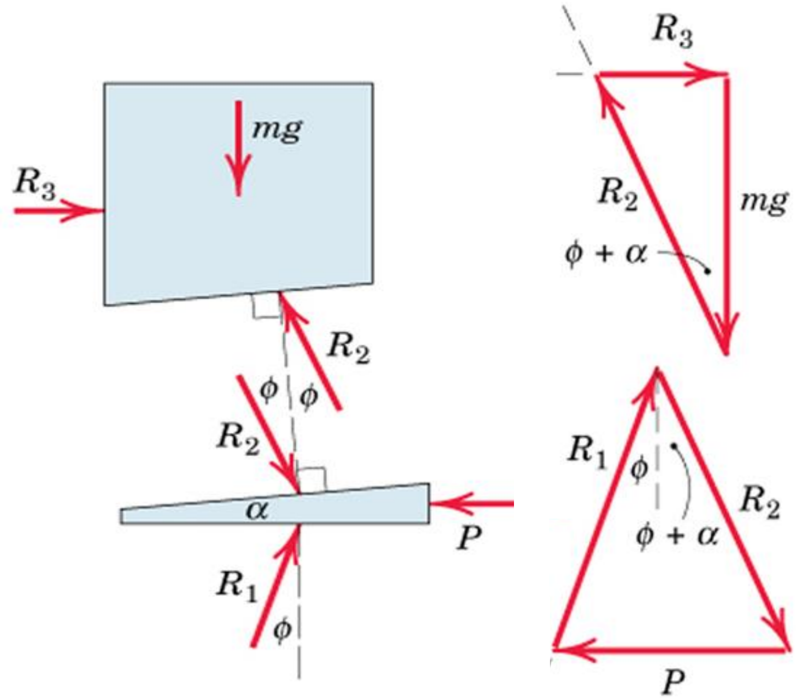
Coefficient of Friction for each pair of surfaces $\mu = \tan\phi$ (Static/Kinetic)

Free Body Diagrams
Reactions are inclined at an angle Φ from their respective normals and are in the direction opposite to the motion.

Force vectors acting on each body can also be shown.

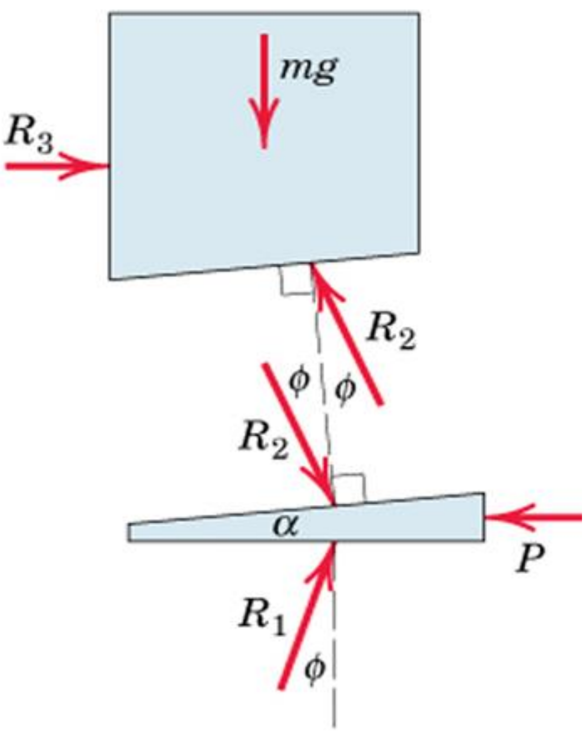
R_2 is first found from upper diagram since mg is known.

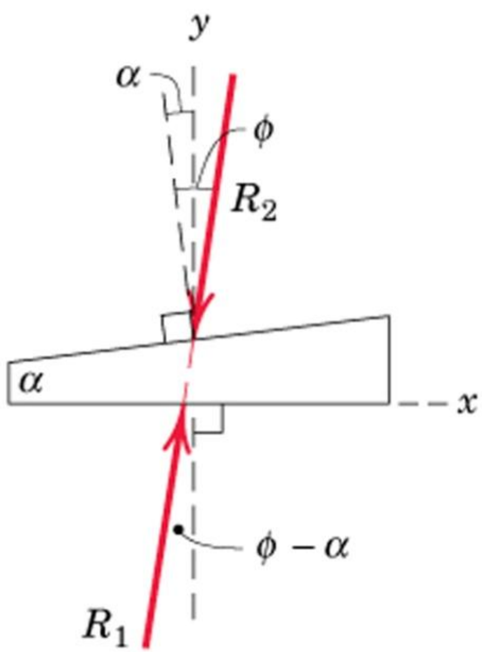
Then P can be found out from the lower diagra since R_2 is known.



Applications of Friction in Machines: Wedges

- Removal of force P
  - *Wedge remains in place*
  - *Equilibrium requirement :: Collinearity of R_1 and R_2*



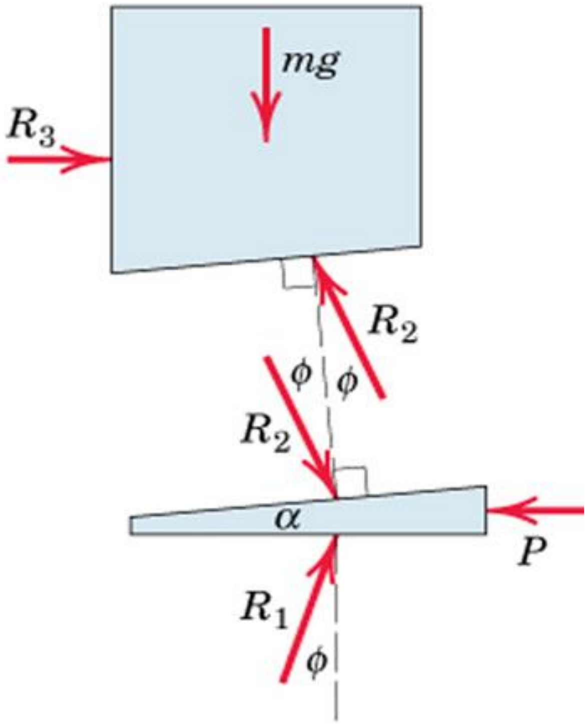


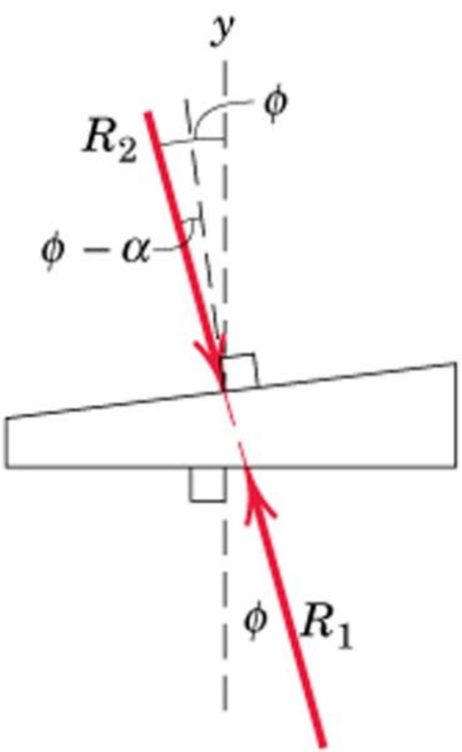
:: Impending slippage at upper surface

:: $\alpha < \phi$

Applications of Friction in Machines: Wedges

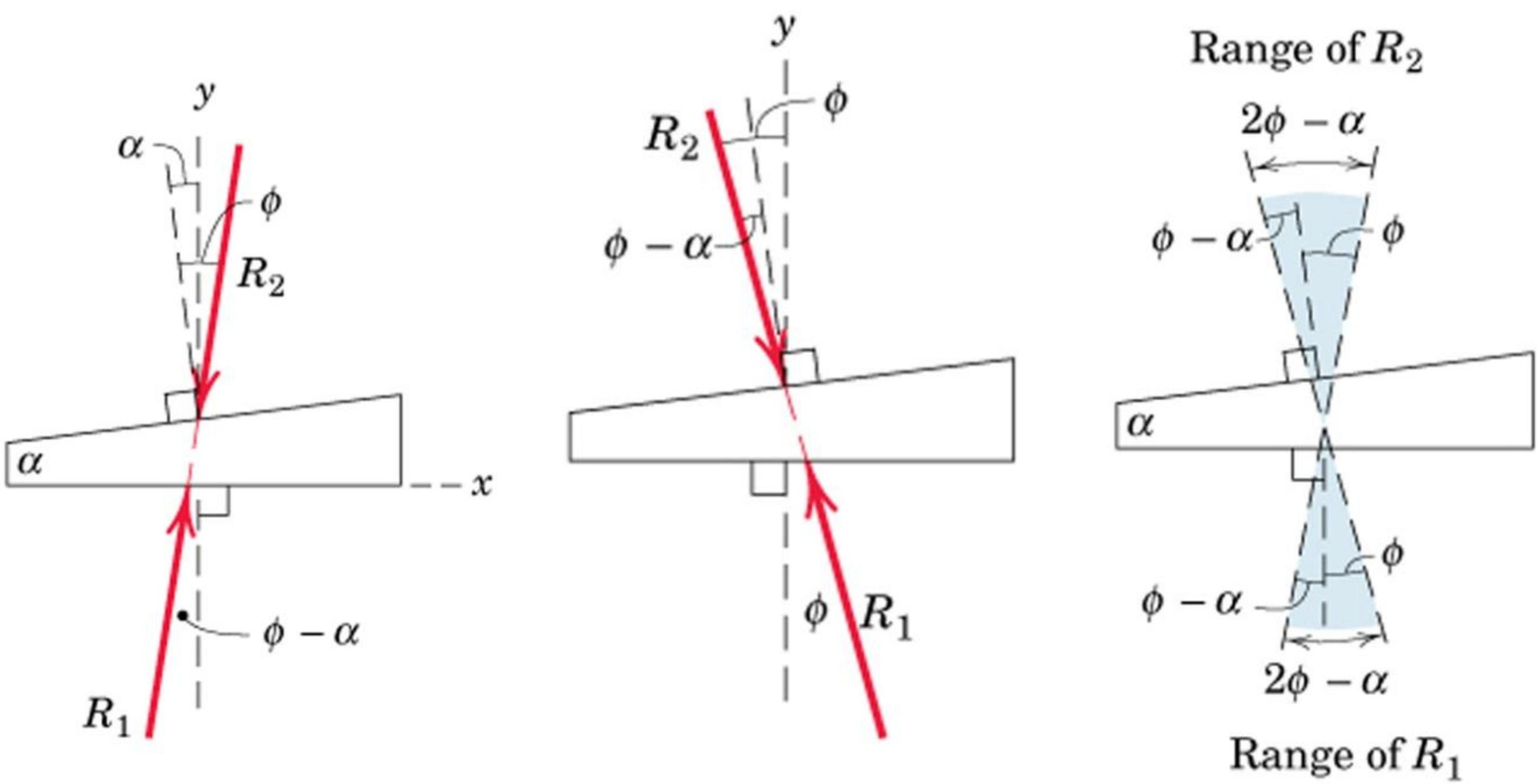
- Collinearity of reactions
  - *Impending slippage at lower surface* $\because \alpha < \phi$





Applications of Friction in Machines: Wedges

- **Simultaneous** occurring of **slippage** at both **upper** and **lower surfaces** (otherwise self-locking)
  - *Angular range for no slippage*
  - *Simultaneous slippage not possible for* $\alpha < 2\phi$



Applications of Friction in Machines: Wedges

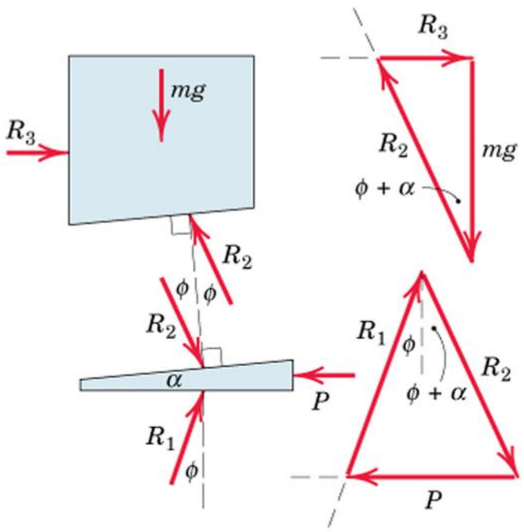
A pull P is required on the wedge for withdrawal of the wedge

→ **The reactions *R_1* and *R_2* must act on the opposite sides of their normal from those when the wedge was inserted**

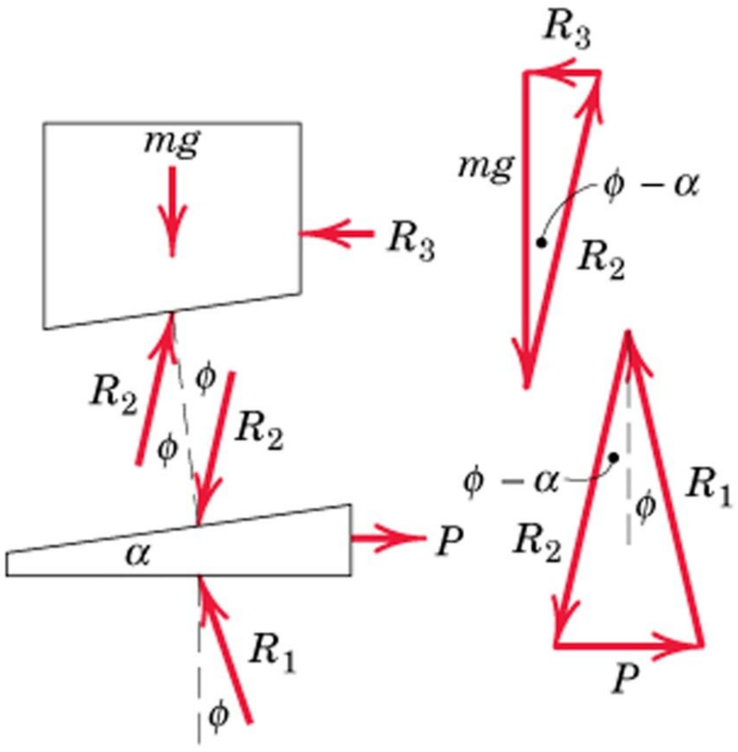
→ Solution by drawing FBDs and vector polygons

→ Graphical solution

→ Algebraic solutions from trigonometry



Forces to raise load

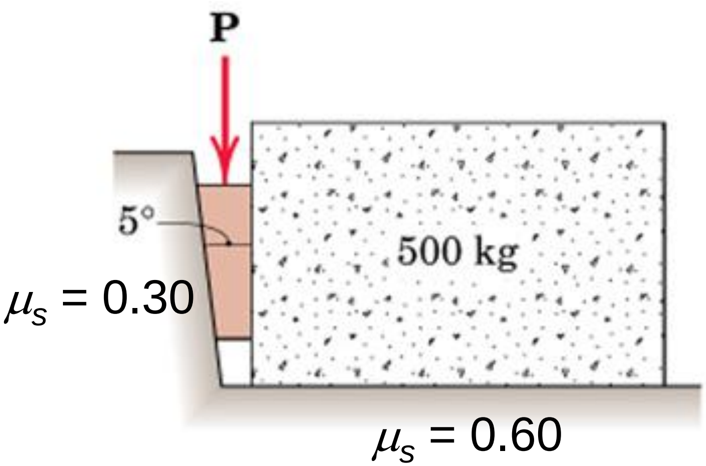


Forces to lower load

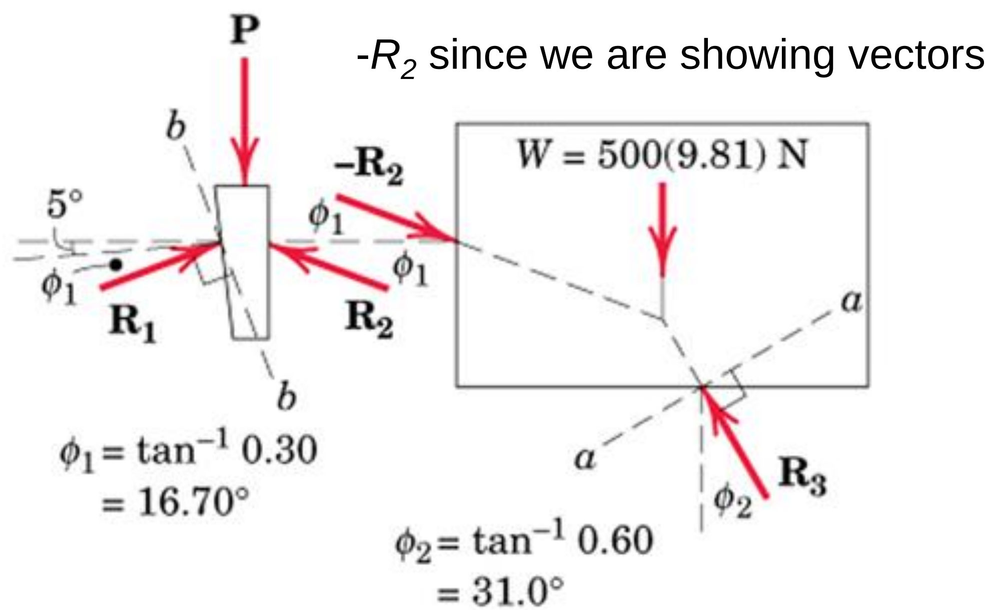
Applications of Friction in Machines

Example: Wedge

Coefficient of Static Friction for both pairs of wedge = 0.3
Coefficient of Static Friction between block and horizontal surface = 0.6
Find the least P required to move the block



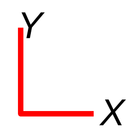
Solution: Draw FBDs

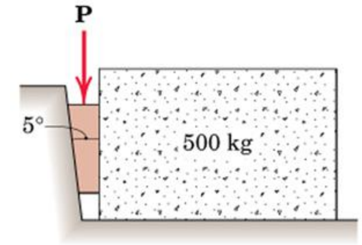


Applications of Friction in Machines

Solution: W = 500x9.81 = 4905 N

Three ways to solve





Method 1:

Equilibrium of FBD of the Block

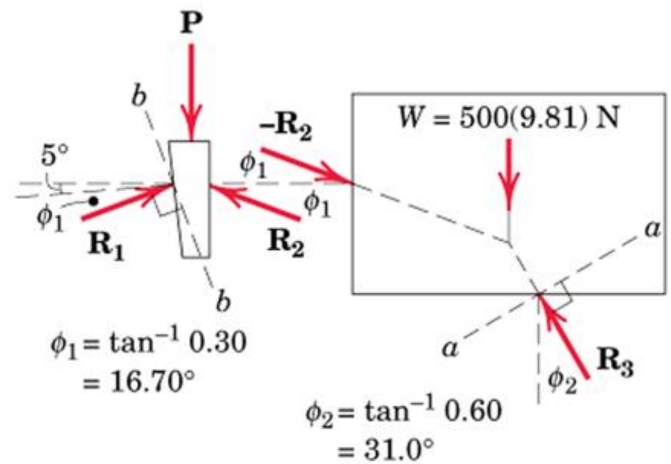
$\sum$**F$_X$ = 0**

$R_2 \cos \phi_1 = R_3 \sin \phi_2 \rightarrow R_2 = 0.538R_3$

$\sum$**F$_Y$ = 0**

$4905 + R_2 \sin \phi_1 = R_3 \cos \phi_2 \rightarrow R_3 = 6970$ N

$\rightarrow R_2 = 3750$ N



Equilibrium of FBD of the Wedge

$\sum$**F$_X$ = 0**

$R_2 \cos \phi_1 = R_1 \cos(\phi_1+5) \rightarrow R_1 = 3871$ N

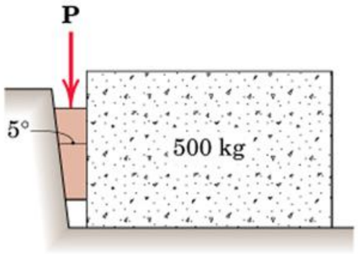
$\sum$**F$_Y$ = 0**

$R_1 \sin(\phi_1+5) + R_2 \sin \phi_1 = P$

$\rightarrow P = 2500$ N

Applications of Friction in Machines

Solution:

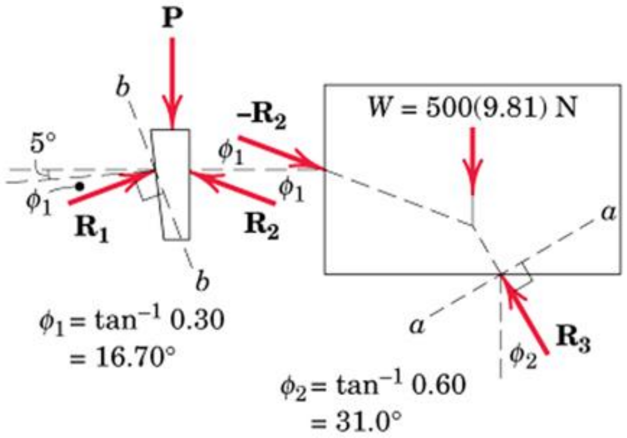


Method 2:

Using Equilibrium equations along reference axes *a-a* and *b-b*

→ No need to solve simultaneous equations

Angle between R_2 and *a-a* axis = 16.70+31.0 = 47.7°



Equilibrium of Block:

$$[\Sigma F_a = 0] \qquad 500(9.81)\sin 31.0° - R_2 \cos 47.7° = 0$$

$$R_2 = 3750 \text{ N}$$

Equilibrium of Wedge:

Angle between R_2 and *b-b* axis = 90-(2Φ_1+5) = 51.6°

Angle between *P* and *b-b* axis = Φ_1+5 = 21.7°

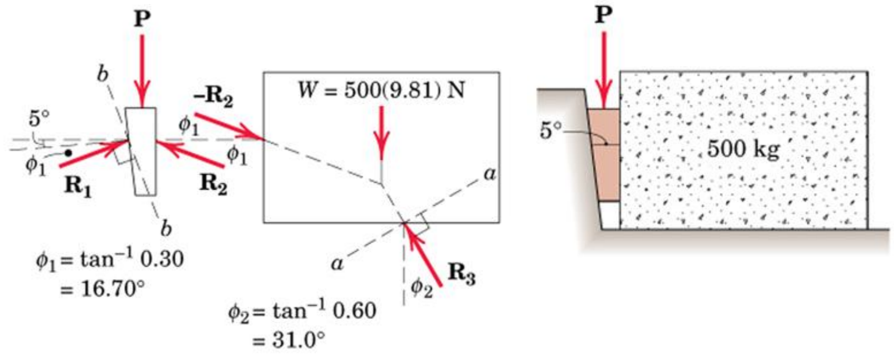
$$[\Sigma F_b = 0] \qquad 3750 \cos 51.6° - P \cos 21.7° = 0$$

$$P = 2500 \text{ N}$$

Applications of Friction in Machines

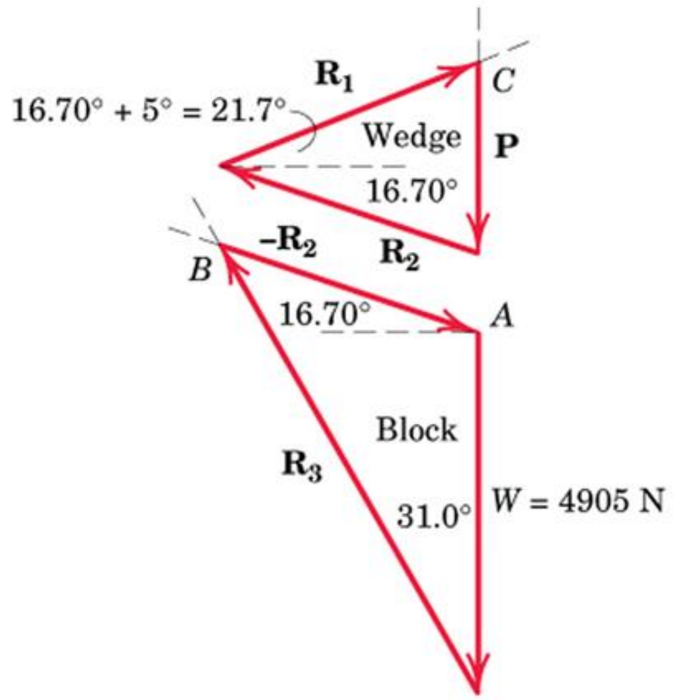
Solution:

Method 3:
Graphical solution using vector polygons



Starting with equilibrium of the block:
W is known, and directions of R_2 and R_3 are known
→ Magnitudes of R_2 and R_3 can be determined graphically

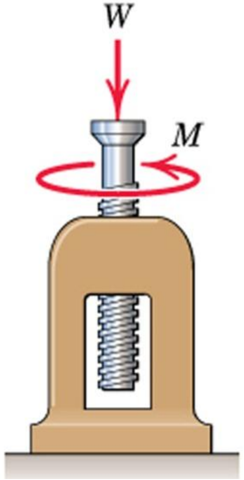
Similarly, construct vector polygon for the wedge from known magnitude of R_2, and known directions of R_2 , R_1, *and P.*
→ Find out the magnitude of *P* graphically



Applications of Friction in Machines

Square Threaded Screws

- Used for fastening and for transmitting power or motion
- Square threads are more efficient
- Friction developed in the threads largely determines the action of the screw



FBD of the Screw: R exerted by the thread of the jack frame on a small portion of the screw thread is shown

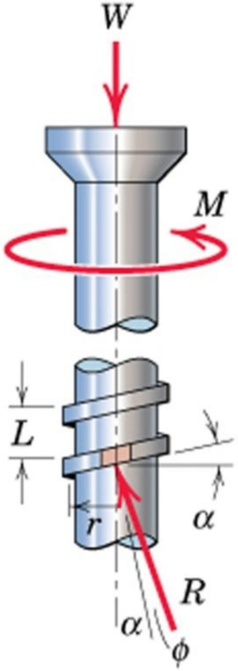
Lead = L = advancement per revolution

L = Pitch – for single threaded screw

L = 2xPitch – for double threaded screw (twice advancement per revolution)

Pitch = axial distance between adjacent threads on a helix or screw

Mean Radius = r ; α = Helix Angle

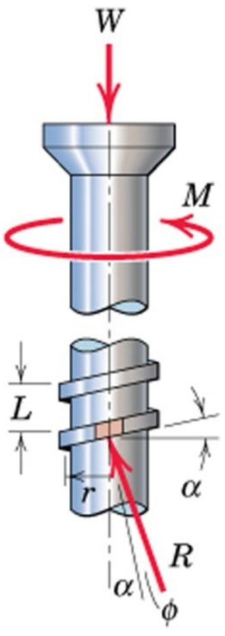


Similar reactions exist on all segments of the screw threads

Analysis similar to block on inclined plane since friction force does not depend on area of contact.

- Thread of base can be “unwrapped” and shown as straight line. Slope is $2\pi r$ horizontally and lead L vertically.

Applications of Friction in Machines: Screws



If M is just sufficient to turn the screw → Motion Impending
Angle of friction = ϕ (made by R with the axis normal to the thread)
→ $\tan\phi = \mu$

Moment of R @ vertical axis of screw = $R\sin(\alpha+\phi)r$
→ Total moment due to all reactions on the thread = $\sum R\sin(\alpha+\phi)r$
→ Moment Equilibrium Equation for the screw:

→ $M = [r\sin(\alpha+\phi)]\sum R$

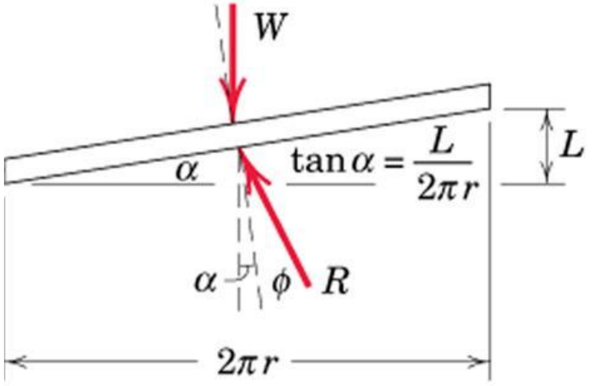
Equilibrium of forces in the axial direction: $W = \sum R\cos(\alpha+\phi)$

→ $W = [\cos(\alpha+\phi)]\sum R$

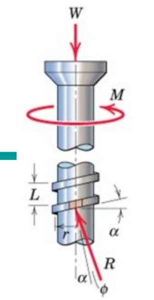
Finally → $\boldsymbol{M = W\, r \tan(\alpha+\phi)}$

Helix angle α can be determined by unwrapping the thread of the screw for one complete turn

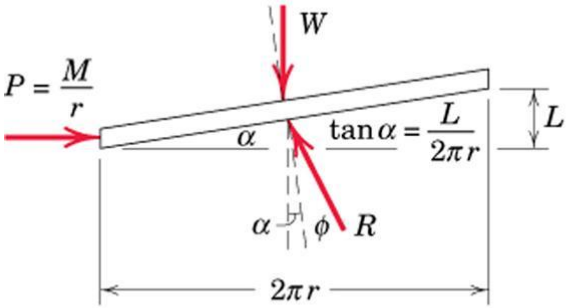
$\boldsymbol{\alpha = \tan^{-1}(L/2\pi r)}$

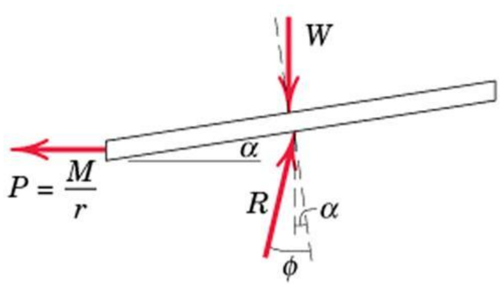


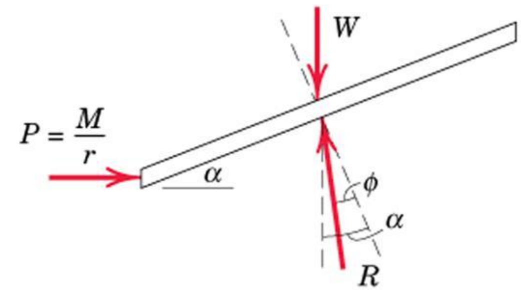
Applications of Friction in Machines: Screws



Alternatively, action of the entire screw can be simulated using unwrapped thread of the screw







To Raise Load

Equivalent force required to push the movable thread up the fixed incline is:
$P = M/r$

From Equilibrium:

$$M = W\,r\tan(\alpha + \phi)$$

If M is removed: the screw will remain in place and be self-locking provided $\alpha<\phi$ and will be on the verge of unwinding if $\alpha=\phi$

To Lower Load ($\alpha<\phi$)

To lower the load by unwinding the screw, We must reverse the direction of M as long as $\alpha<\phi$

From Equilibrium:

$$M = W\,r\tan(\phi - \alpha)$$

→ This is the moment required to unwind the screw

To Lower Load ($\alpha>\phi$)

If $\alpha > \phi$, the screw will unwind by itself. Moment required to prevent unwinding:

From Equilibrium:

$$M = W\,r\tan(\alpha - \phi)$$