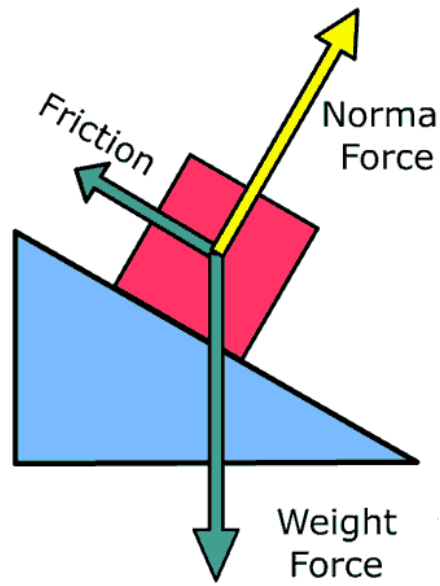
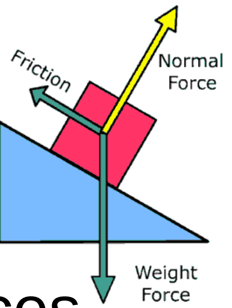
# Friction

The effective design of a brake system, such as the one for this bicycle, requires an efficient capacity for the mechanism to resist frictional forces. In this chapter, we will study the nature of friction and show how these forces are considered in engineering analysis and design.



# Friction




Usual Assumption till now:

Forces of action and reaction between contacting surfaces act normal to the surface

→ valid for interaction between smooth surfaces

→ often involves only a relatively small error in solution

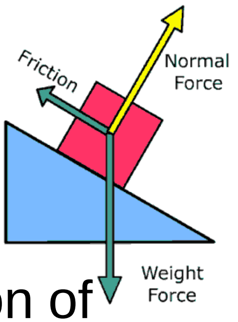
→ in many cases ability of contacting surfaces to support tangential forces is very important (Ex: Figure above)

## Frictional Forces

Tangential forces generated between contacting surfaces

- occur in the interaction between all real surfaces
- always act in a direction opposite to the direction of motion

# Friction




## Frictional forces are Not Desired in some cases:

- Bearings, power screws, gears, flow of fluids in pipes, propulsion of aircraft and missiles through the atmosphere, etc
  - Friction often results in a loss of energy, which is dissipated in the form of heat
  - Friction causes Wear

## Frictional forces are Desired in some cases:

- Brakes, clutches, belt drives, wedges
- walking depends on friction between the shoe and the ground

Ideal Machine/Process: Friction small enough to be neglected

Real Machine/Process: Friction must be taken into account

# Types of Friction

## Dry Friction (Coulomb Friction)

occurs between unlubricated surfaces of two solids

Effects of dry friction acting on exterior surfaces of rigid bodies → ME101

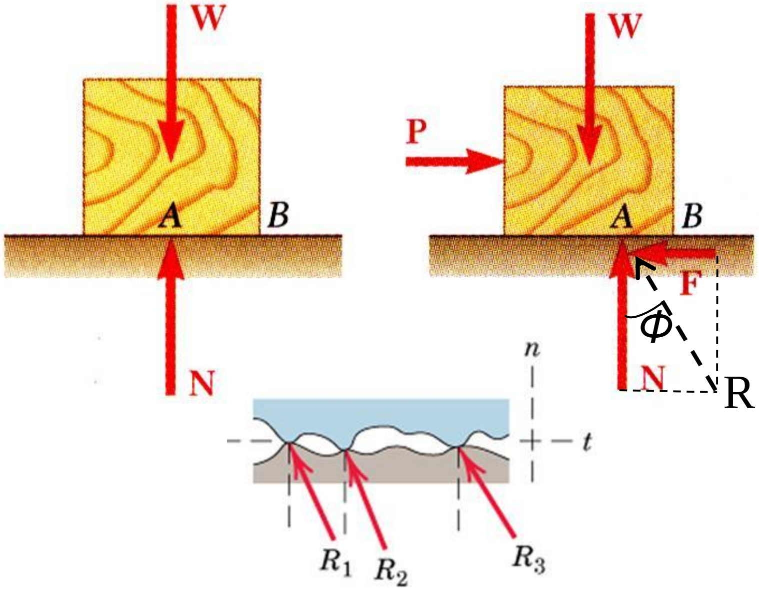
## Fluid Friction

occurs when adjacent layers in a fluid (liquid or gas) move at a different velocities. Fluid friction also depends on viscosity of the fluid. → Fluid Mechanics

## Internal Friction

occurs in all solid materials subjected to cyclic loading, especially in those materials, which have low limits of elasticity → Material Science

# Mechanism of Dry Friction

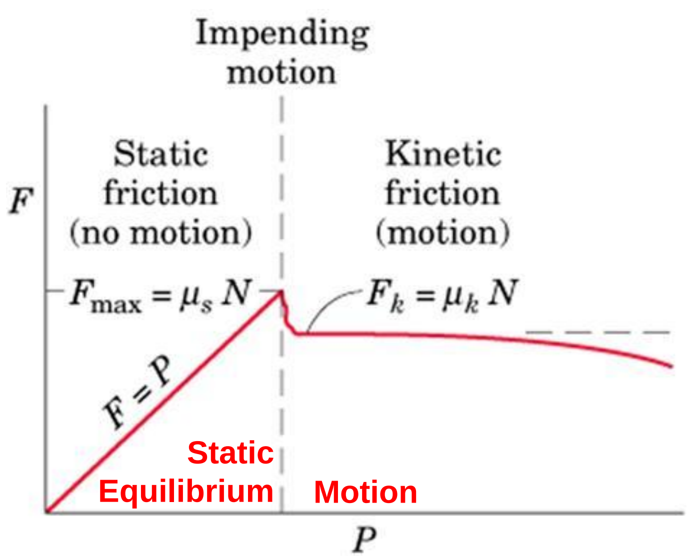



- Block of weight *W* placed on horizontal surface. Forces acting on block are its weight and reaction of surface *N*.
- Small horizontal force *P* applied to block. For block to remain stationary, in equilibrium, a horizontal component *F* of the surface reaction is required. *F* is a *Static-Friction force*.
- As *P* increases, static-friction force *F* increases as well until it reaches a maximum value $F_m$.

$$F_m = \mu_s N$$

- Further increase in P causes the block to begin to move as *F* drops to a smaller *Kinetic-Friction force* $F_k$.

$$F_k = \mu_k N$$




$\mu_s$ is the Coefficient of Static Friction
$\mu_k$ is the Coefficient of Kinetic Friction

# Mechanism of Dry Friction

**Table 8.1. Approximate Values of Coefficient of Static Friction for Dry Surfaces**

| | |
|---|---|
| Metal on metal | 0.15–0.60 |
| Metal on wood | 0.20–0.60 |
| Metal on stone | 0.30–0.70 |
| Metal on leather | 0.30–0.60 |
| Wood on wood | 0.25–0.50 |
| Wood on leather | 0.25–0.50 |
| Stone on stone | 0.40–0.70 |
| Earth on earth | 0.20–1.00 |
| Rubber on concrete | 0.60–0.90 |

*A friction coefficient reflects roughness, which is a geometric property of surfaces*

- Maximum static-friction force:

$$F_m = \mu_s N$$

- Kinetic-friction force:

$$F_k = \mu_k N$$

$$\mu_k \cong 0.75\mu_s$$

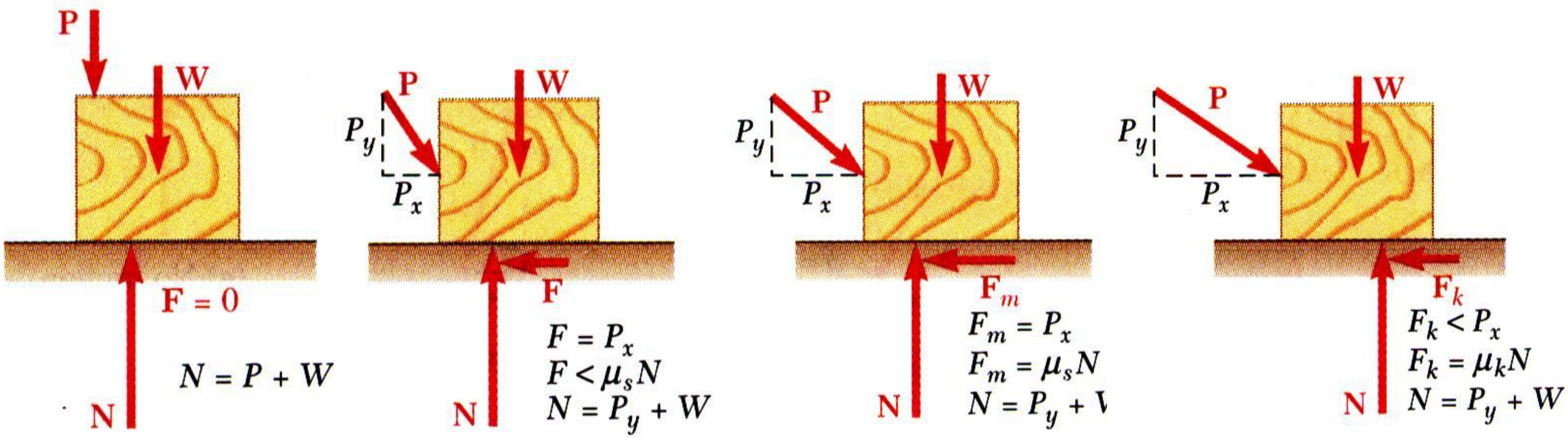
- Maximum static-friction force and kinetic-friction force are:
  - proportional to normal force
  - dependent on type and condition of contact surfaces
  - independent of contact area

Surfaces in relative motion :: the contacts are more nearly along the tops of the humps
:: t-components of the *R*'s are smaller than the "at rest" condition

→ Force necessary to maintain motion is generally less than that required to start the block when the surface irregularities are more nearly in mesh → $F_m > F_k$

# Mechanism of Dry Friction

- **Four possible situations** for a rigid body in contact with a horizontal surface:




- No friction, ($P_x = 0$)

Equations of Equilibrium Valid

- No motion, ($P_x < F_m$)

Equations of Equilibrium Valid

- Motion impending, ($P_x = F_m$)

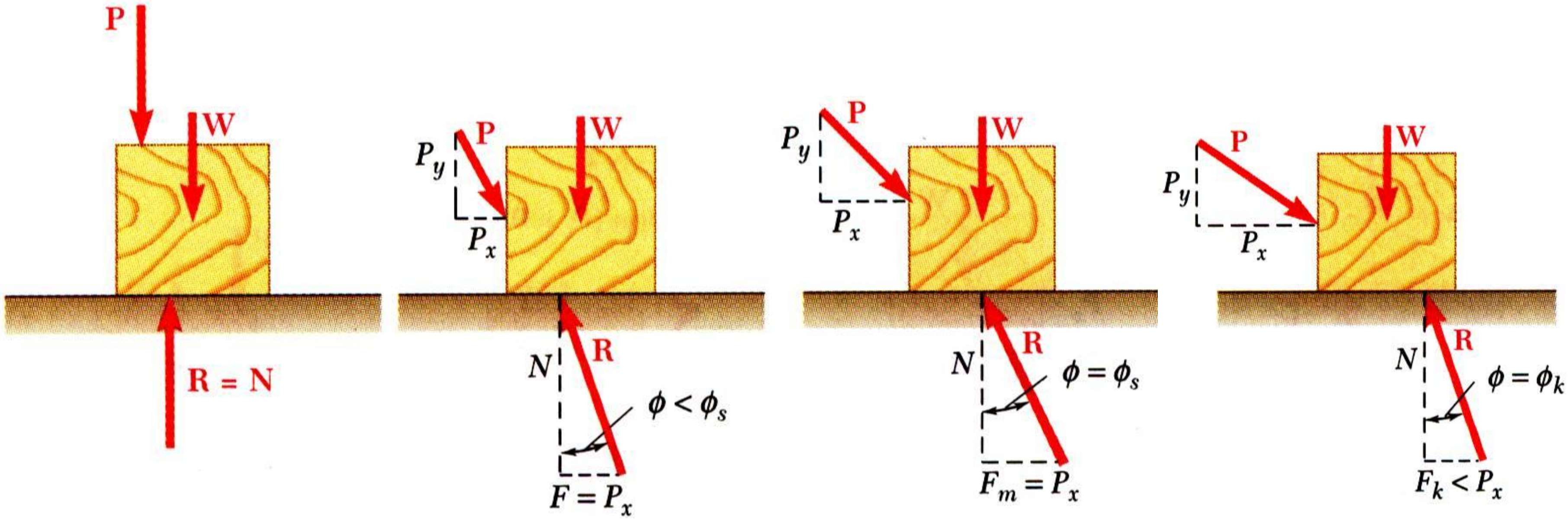
Equations of Equilibrium Valid

- Motion, ($P_x > F_m$)

Equations of Equilibrium Not Valid

# Mechanism of Dry Friction

Sometimes convenient to replace normal force *N* & friction force *F* by their resultant ***R***:



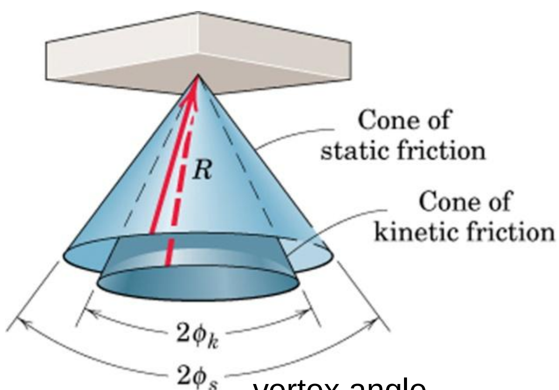

- No friction
- No motion
- Motion impending
- Motion

$$\tan\phi_s = \frac{F_m}{N} = \frac{\mu_s N}{N}$$

$$\tan\phi_s = \mu_s$$

$$\tan\phi_k = \frac{F_k}{N} = \frac{\mu_k N}{N}$$

$$\tan\phi_k = \mu_k$$



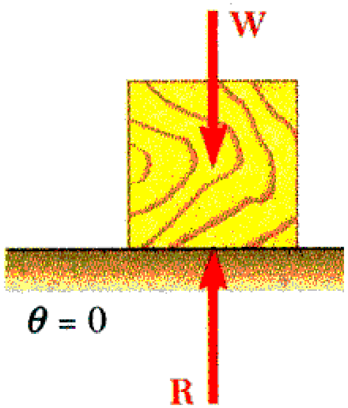

vertex angle

**Friction Angles**

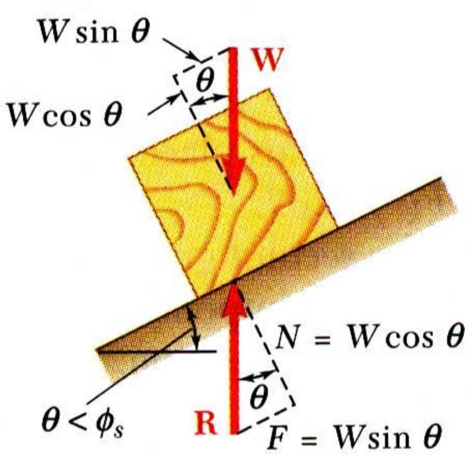
$\phi_s$ = angle of static friction, $\phi_k$ = angle of kinetic friction

# Mechanism of Dry Friction

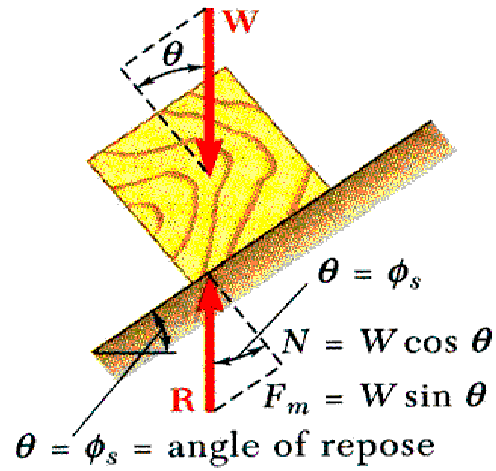
- Consider block of weight $W$ resting on board with variable inclination angle $\theta$.




- No friction

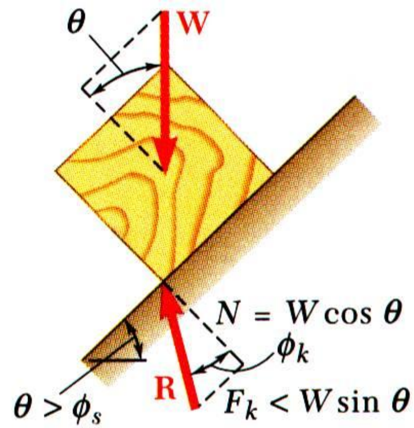



- No motion




- Motion impending

Angle of Repose = Angle of Static Friction




- Motion

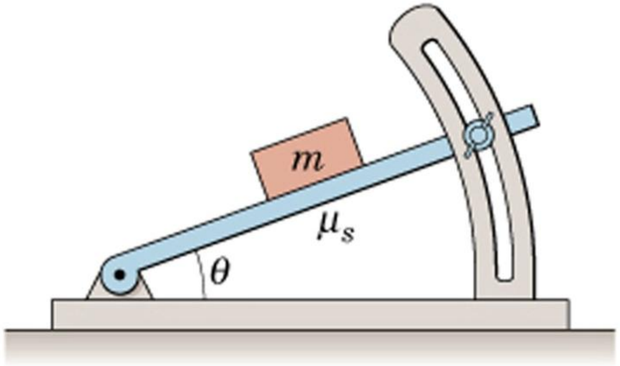
The reaction **R is not vertical** anymore, and the forces acting on the block are not balanced

# Dry Friction

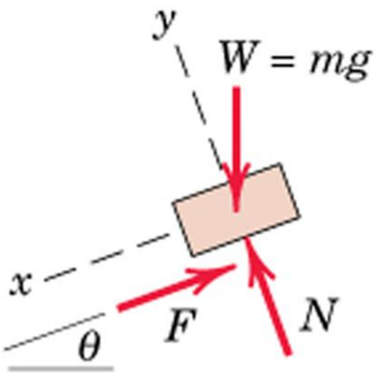
## Example

Determine the maximum angle $\theta$ before the block begins to slip.

$\mu_s$ = Coefficient of static friction between the block and the inclined surface




Solution: Draw the FBD of the block




$$[\Sigma F_x = 0] \qquad mg \sin\theta - F = 0 \qquad F = mg \sin\theta$$

$$[\Sigma F_y = 0] \qquad -mg \cos\theta + N = 0 \qquad N = mg \cos\theta$$

$$F/N = \tan\theta$$

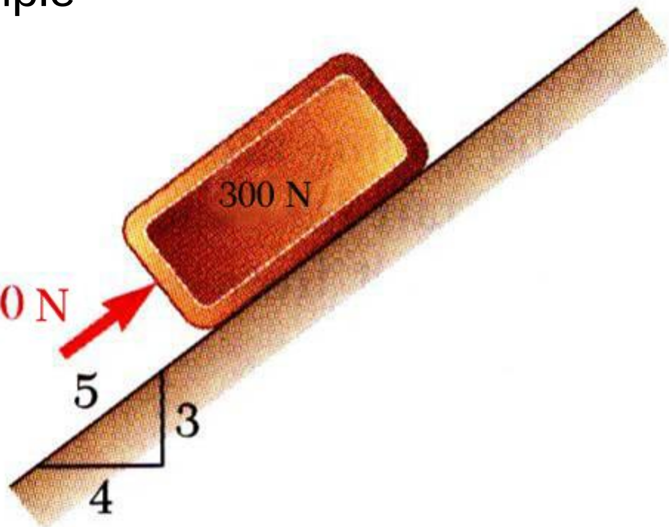
Max angle occurs when $\boldsymbol{F = F_{max} = \mu_s N}$

Therefore, for impending motion: $\mu_s = \tan\theta_{max} \quad \text{or} \quad \theta_{max} = \tan^{-1}\mu_s$

The maximum value of $\theta$ is known as *Angle of Repose*

# Dry Friction

Example

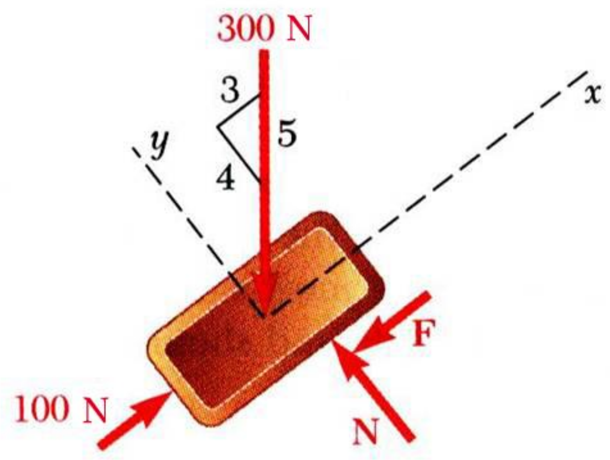



A 100 N force acts as shown on a 300 N block placed on an inclined plane. The coefficients of friction between the block and plane are $\mu_s = 0.25$ and $\mu_k = 0.20$. Determine whether the block is in equilibrium and find the value of the friction force.

SOLUTION:

- Determine values of friction force and normal reaction force from plane required to maintain equilibrium.
- Calculate maximum friction force and compare with friction force required for equilibrium. If it is greater, block will not slide.
- If maximum friction force is less than friction force required for equilibrium, block will slide. Calculate kinetic-friction force.

# Dry Friction




SOLUTION:

- Determine values of friction force and normal reaction force from plane required to maintain equilibrium.

$$\sum F_x = 0: \qquad 100\text{ N} - \tfrac{3}{5}(300\text{ N}) - F = 0$$

$$F = -80\text{ N} \qquad \rightarrow F \text{ acting upwards}$$

$$\sum F_y = 0: \qquad N - \tfrac{4}{5}(300\text{ N}) = 0$$

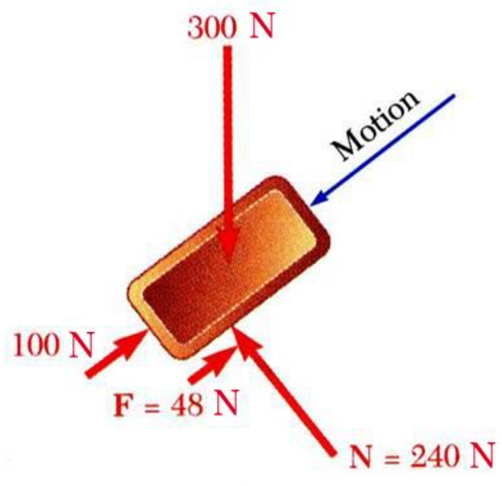
$$N = 240\text{ N}$$

- Calculate maximum friction force and compare with friction force required for equilibrium. If it is greater, block will not slide.

$$F_m = \mu_s N \qquad F_m = 0.25(240\text{ N}) = 60\text{ N}$$

*The block will slide down the plane along F.*

# Dry Friction



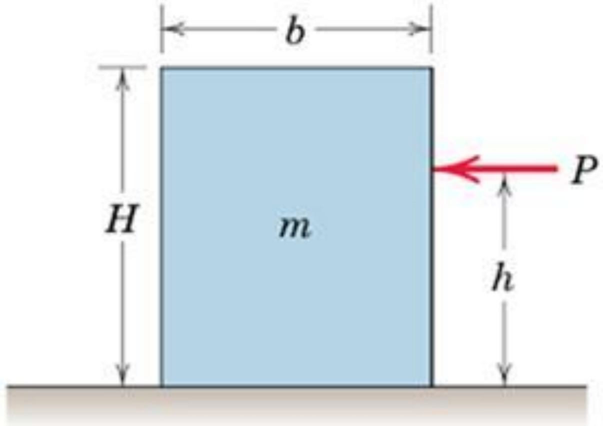

- If maximum friction force is less than friction force required for equilibrium, block will slide. Calculate kinetic-friction force.

$$F_{actual} = F_k = \mu_k \mathrm{N}$$
$$= 0.20(240\ \mathrm{N})$$

$$\boxed{F_{actual} = 48\ \mathrm{N}}$$

# Dry Friction

Example

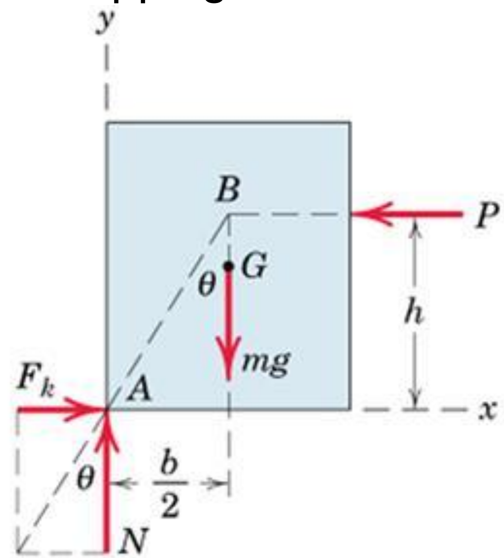



The block moves with constant velocity under the action of *P*. $\mu_k$ is the Coefficient of Kinetic Friction. Determine:

(a) Maximum value of *h* such that the block slides without tipping over

(b) Location of a point C on the bottom face of the block through which resultant of the friction and normal forces must pass if *h*=*H*/2

**Solution**: (a) FBD for the block on the verge of tipping:



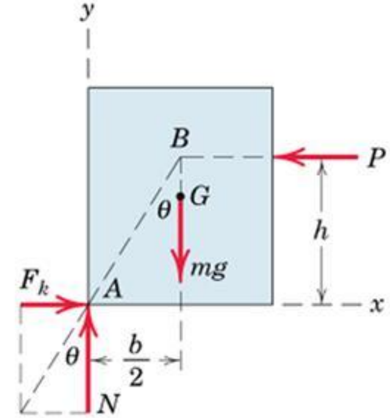

The resultant of $F_k$ and *N* passes through point B through which P must also pass, since three coplanar forces in equilibrium are concurrent.

Friction Force:

$F_k = \mu_k N$ since slipping occurs

$\Theta = \tan^{-1}\mu_k$

# Dry Friction




**Solution (a)** Apply Equilibrium Conditions (constant velocity!)

$$[\Sigma F_y = 0] \qquad N - mg = 0 \qquad N = mg$$

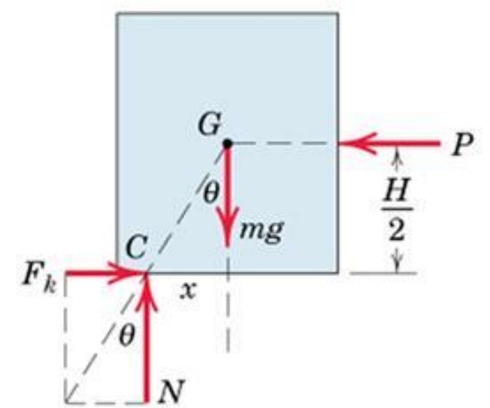
$$[\Sigma F_x = 0] \qquad F_k - P = 0 \qquad P = F_k = \mu_k N = \mu_k mg$$

$$[\Sigma M_A = 0] \qquad Ph - mg\frac{b}{2} = 0 \qquad h = \frac{mgb}{2P} = \frac{mgb}{2\mu_k mg} = \frac{b}{2\mu_k}$$

Alternatively, we can directly write from the geometry of the FBD:

$$\tan\theta = \mu_k = \frac{b/2}{h} \qquad h = \frac{b}{2\mu_k}$$

If *h* were greater than this value, moment equilibrium at A would not be satisfied and the block would tip over.




**Solution (b)** Draw FBD

$\Theta = \tan^{-1}\boldsymbol{\mu_k}$ since the block is slipping. From geometry of FBD:

$$\frac{x}{H/2} = \tan\theta = \mu_k \qquad \text{so} \qquad x = \mu_k H/2$$

Alternatively use equilibrium equations