Beams – SFD and BMD

Shear and Moment Relationships

$$w = -\frac{dV}{dx}$$

Slope of the shear diagram = - Value of applied loading

$$V = \frac{dM}{dx}$$

Slope of the moment curve = Shear Force

Both equations not applicable at the point of loading because of discontinuity produced by the abrupt change in shear.

Beams – SFD and BMD

$$w = -\frac{dV}{dx}$$ Degree of *V* in *x* is one higher than that of *w*

$$V = \frac{dM}{dx}$$ Degree of *M* in *x* is one higher than that of *V*

→ Degree of *M* in *x* is two higher than that of *w*

Combining the two equations → $\boxed{\dfrac{d^2M}{dx^2} = -w}$

→ M :: obtained by integrating this equation twice

- → **Method is usable** only if ***w* is a continuous function of *x*** (other cases not part of this course)

Beams – SFD and BMD

Shear and Moment Relationships

Expressing *V* in terms of w by integrating $w = -\dfrac{dV}{dx}$

$$\int_{V_0}^{V} dV = -\int_{x_0}^{x} w\,dx$$

OR **$V = V_0$ + (the negative of the area under the loading curve from x_0 to x)**

V_0 is the shear force at x_0 and V is the shear force at x

Expressing *M* in terms of V by integrating $V = \dfrac{dM}{dx}$

$$\int_{M_0}^{M} dM = \int_{x_0}^{x} V\,dx$$

OR **$M = M_0$ + (area under the shear diagram from x_0 to x)**

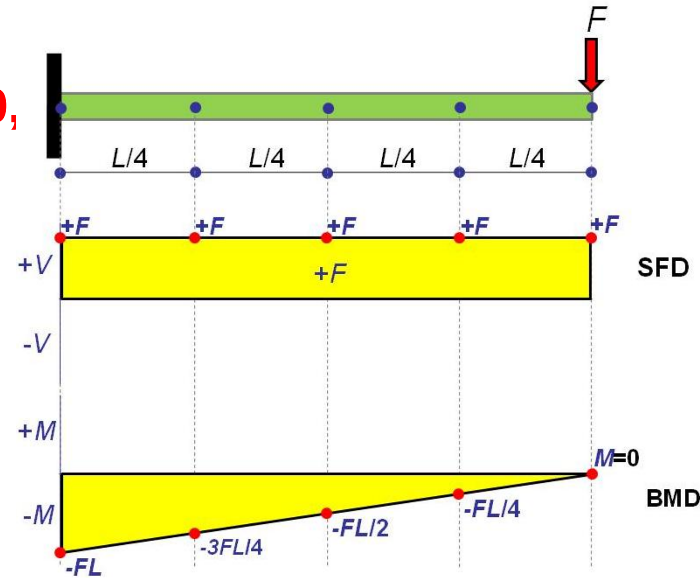
M_0 is the BM at x_0 and M is the BM at x

Beams – SFD and BMD

$V = V_0$ + (negative of area under the loading curve from x_0 to x)

$M = M_0$ + (area under the shear diagram from x_0 to x)

If there is no externally applied moment M_0 at $x_0 = 0$, total moment at any section equals the area under the shear diagram up to that section



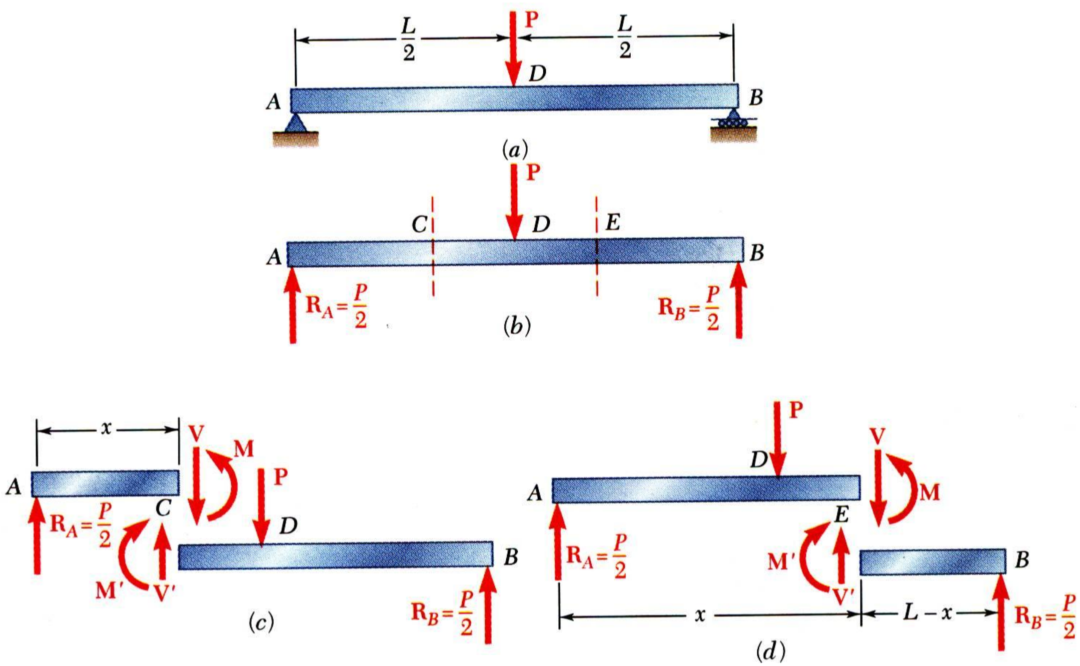
When ***V* passes through zero** and is a **continuous function of *x*** with ***dV/dx* ≠ 0** (i.e., nonzero loading)

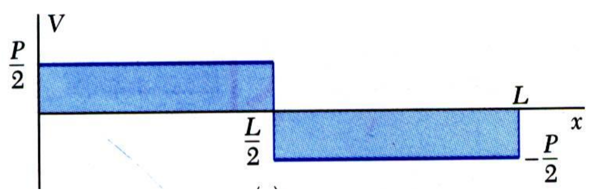
$$\frac{dM}{dx} = 0$$

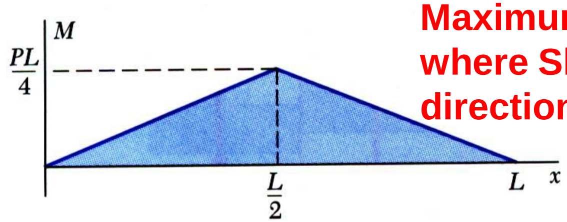
→ **BM** will be a **maximum** or **minimum** at this point

Critical values of BM also occur when **SF crosses the zero axis discontinuously** (*e.g.*, Beams under concentrated loads)

Beams – SFD and BMD: Example (1)







Maximum BM occurs where Shear changes the direction

- Draw the SFD and BMD.
- Determine reactions at supports.
- Cut beam at *C* and consider member *AC*,

$$V = +P/2 \quad M = +Px/2$$

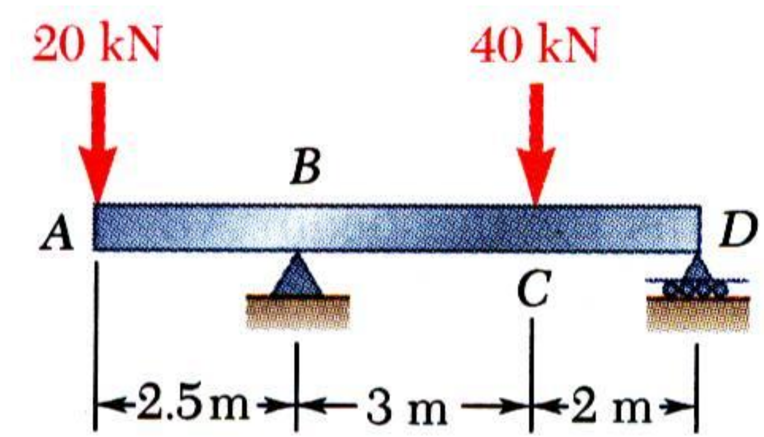
- Cut beam at *E* and consider member *EB*,

$$V = -P/2 \quad M = +P(L-x)/2$$

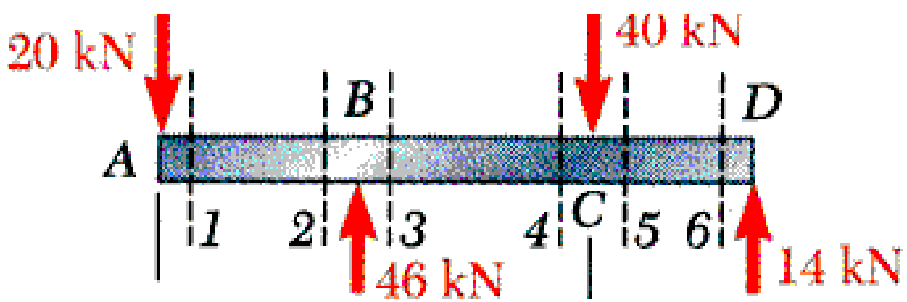
- For a **beam** subjected to **<u>concentrated loads</u>**, **shear** is **constant** between **loading points** and **moment varies linearly**

Beams – SFD and BMD: Example (2)

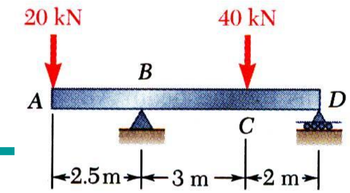
Draw the shear and bending moment diagrams for the beam and loading shown.

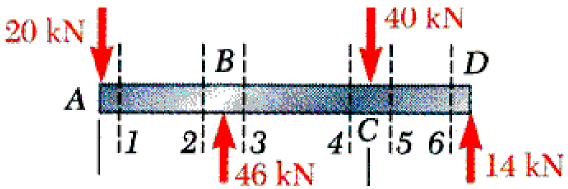


Solution: Draw FBD and find out the support reactions using equilibrium equations



SFD and BMD: Example (2)





Use equilibrium conditions at all sections to get the unknown SF and BM

$$\sum F_y = 0: \quad -20\,\text{kN} - V_1 = 0 \quad \boxed{V_1 = -20\,\text{kN}}$$

$$\sum M_1 = 0: \quad (20\,\text{kN})(0\,\text{m}) + M_1 = 0 \quad \boxed{M_1 = 0}$$

V_2 = -20 kN; M_2 = -20x kNm

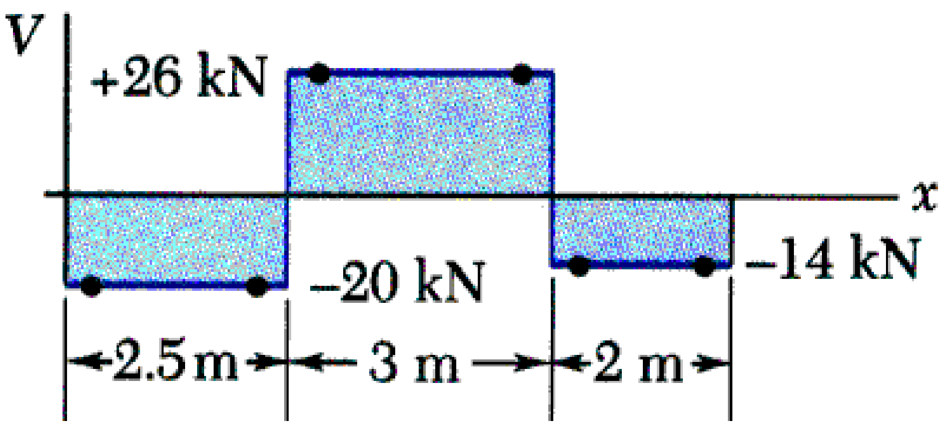
V_3 = +26 kN; M_3 = -20x+46×0 = -20x → M_B= -50 kNm

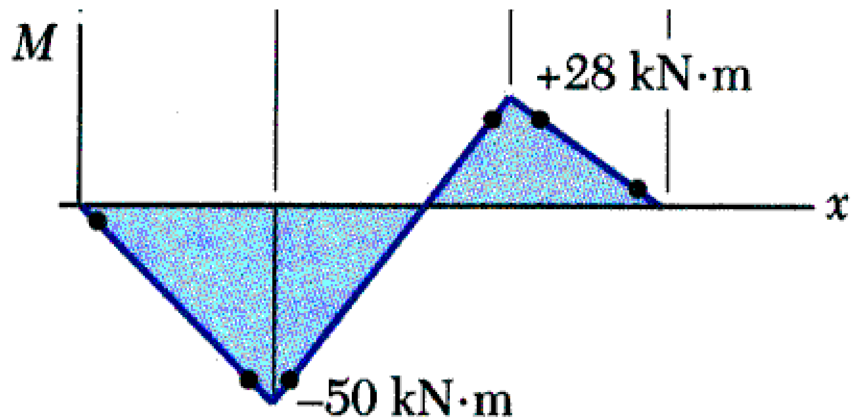
V_4 = +26 kN; M_4 = -20x+46×(x-2.5) → M_C= +28 kNm

V_5 = -14 kN; M_5 = -20x+46×(x-2.5)-40×0 → M_C= +28 kNm

V_6 = -14 kN; M_6 = -20x+46×(x-2.5)-40×(x-5.5)

→ M_D= 0 kNm



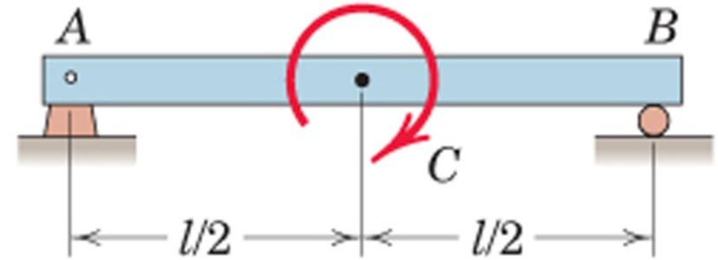


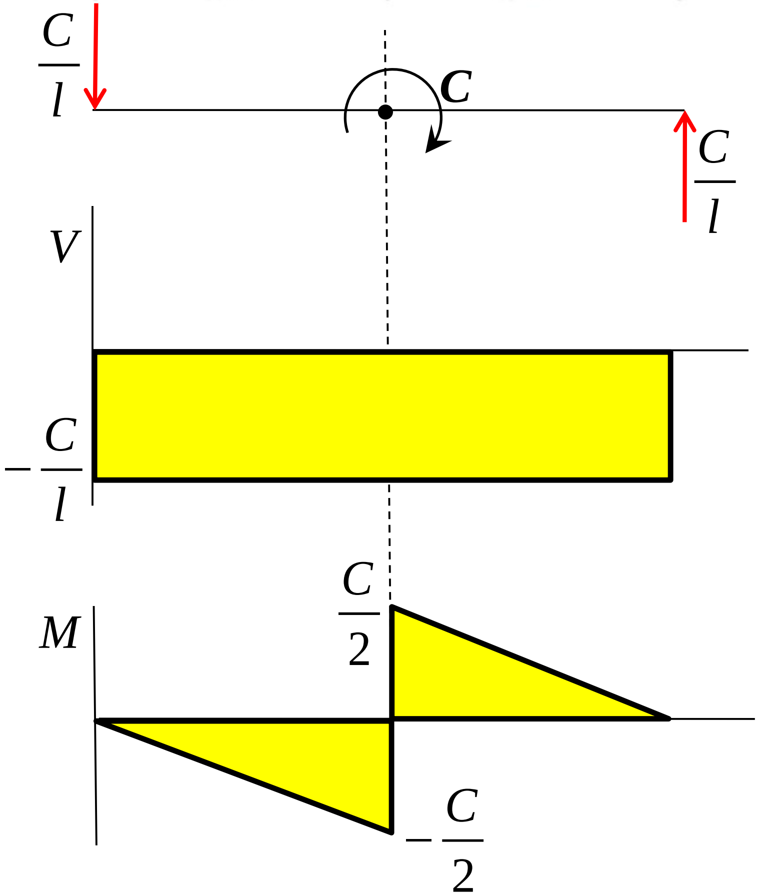
Beams – SFD and BMD: Example (3)

Draw the SFD and BMD for the beam acted upon by a clockwise couple at mid point

Solution: Draw FBD of the beam and Calculate the support reactions

Draw the SFD and the BMD starting From any one end





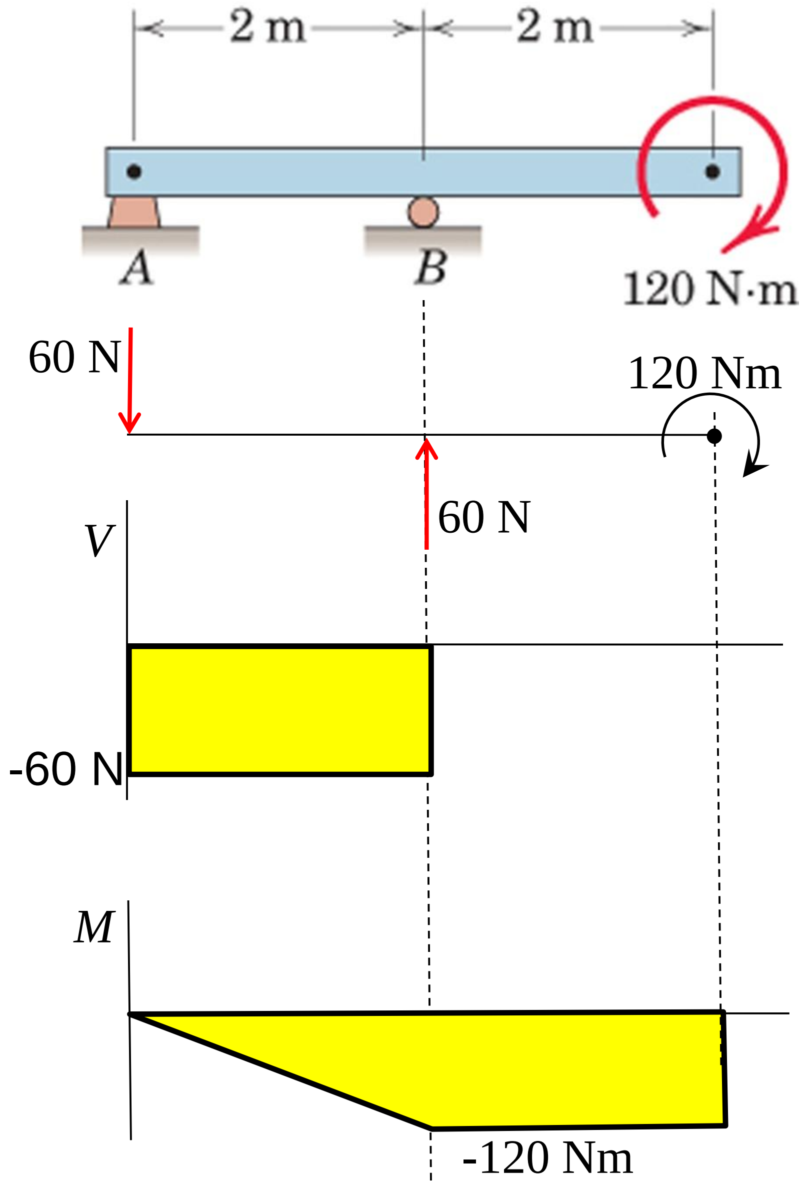
Beams – SFD and BMD: Example (3)

Draw the SFD and BMD for the beam

Solution: Draw FBD of the beam and Calculate the support reactions

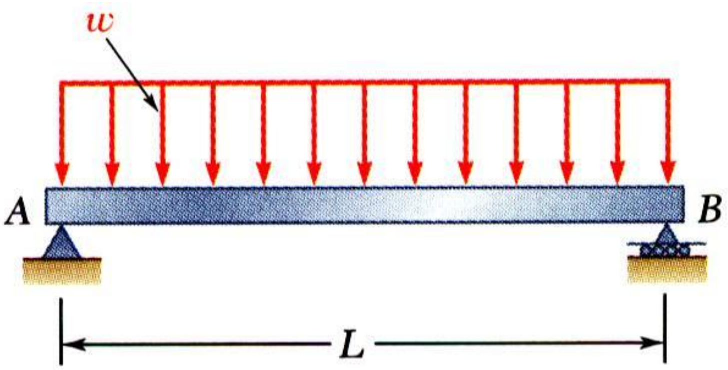
$\sum M_A = 0 \rightarrow R_A = 60$ N↓
$\sum M_B = 0 \rightarrow R_B = 60$ N↑

Draw the SFD and the BMD starting from any one end



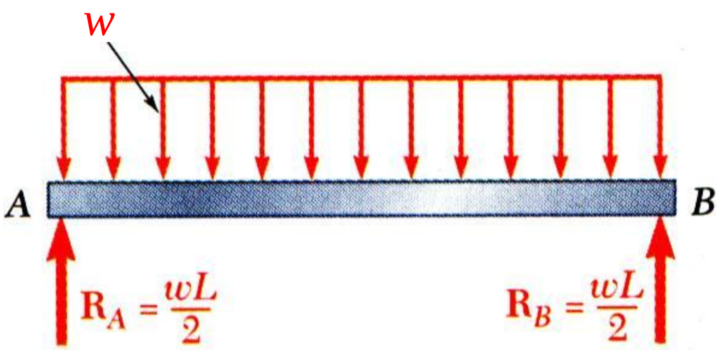
Beams – SFD and BMD: Example (4)

Draw the SFD and BMD for the beam



Solution:

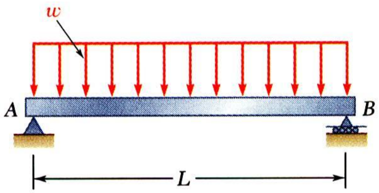
Draw FBD of the entire beam and calculate support reactions using equilibrium equations



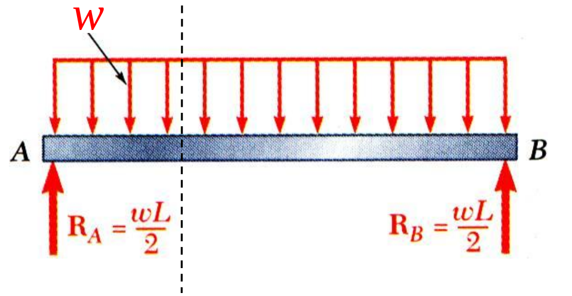
Reactions at supports: $R_A = R_B = \dfrac{wL}{2}$

Develop the relations between loading, shear force, and bending moment and plot the SFD and BMD

Beams – SFD and BMD: Example (4)

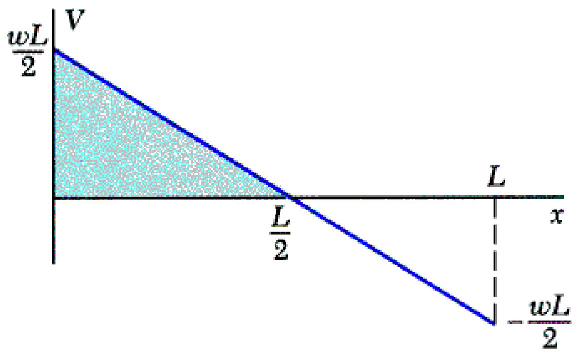


Shear Force at any section: $V = \dfrac{wL}{2} - wx = w\left(\dfrac{L}{2} - x\right)$



Alternatively, $V - V_A = -\int_0^x w\,dx = -wx$

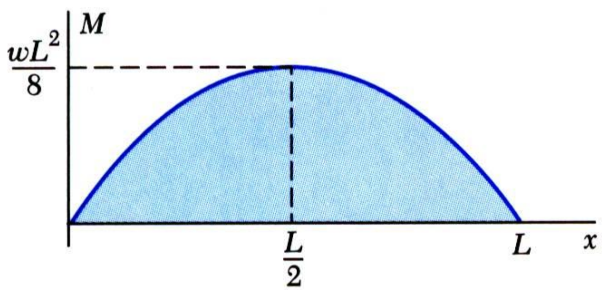
$$V = V_A - wx = \frac{wL}{2} - wx = w\left(\frac{L}{2} - x\right)$$



BM at any section: $M = \dfrac{wL}{2}x - wx\dfrac{x}{2} = \dfrac{w}{2}\left(Lx - x^2\right)$

Alternatively, $M - M_A = \int_0^x V dx$

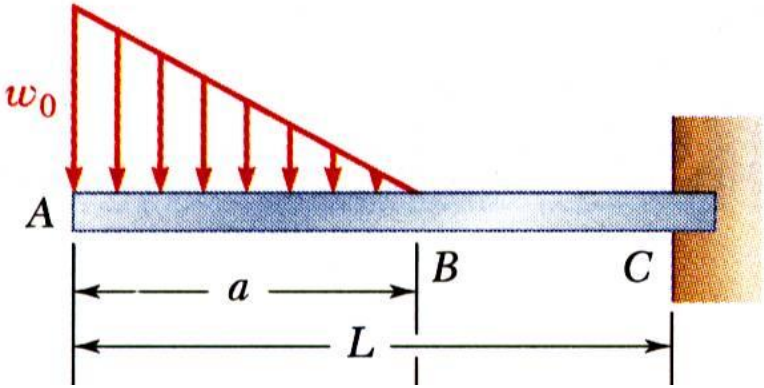
$$M = \int_0^x w\left(\frac{L}{2} - x\right)dx = \frac{w}{2}\left(Lx - x^2\right)$$

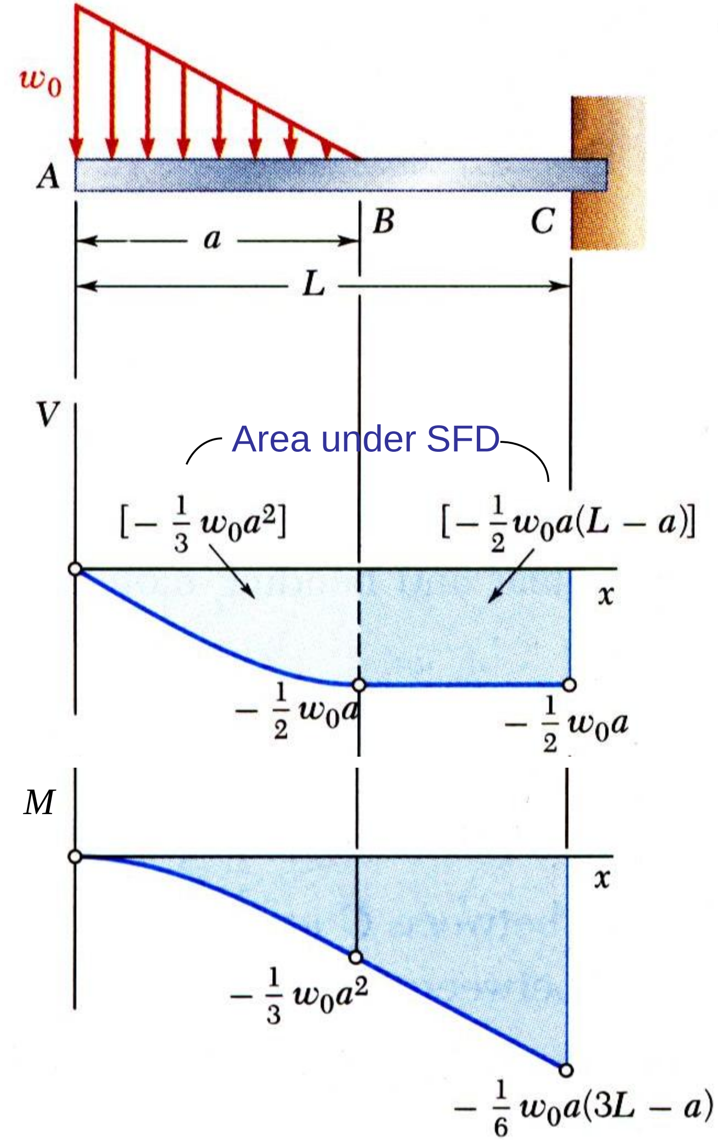


$$M_{\max} = \frac{wL^2}{8} \quad \because \left(M \text{ at } \frac{dM}{dx} = V = 0\right)$$

Beams – SFD and BMD: Example (5)

Draw the SFD and BMD for the Beam





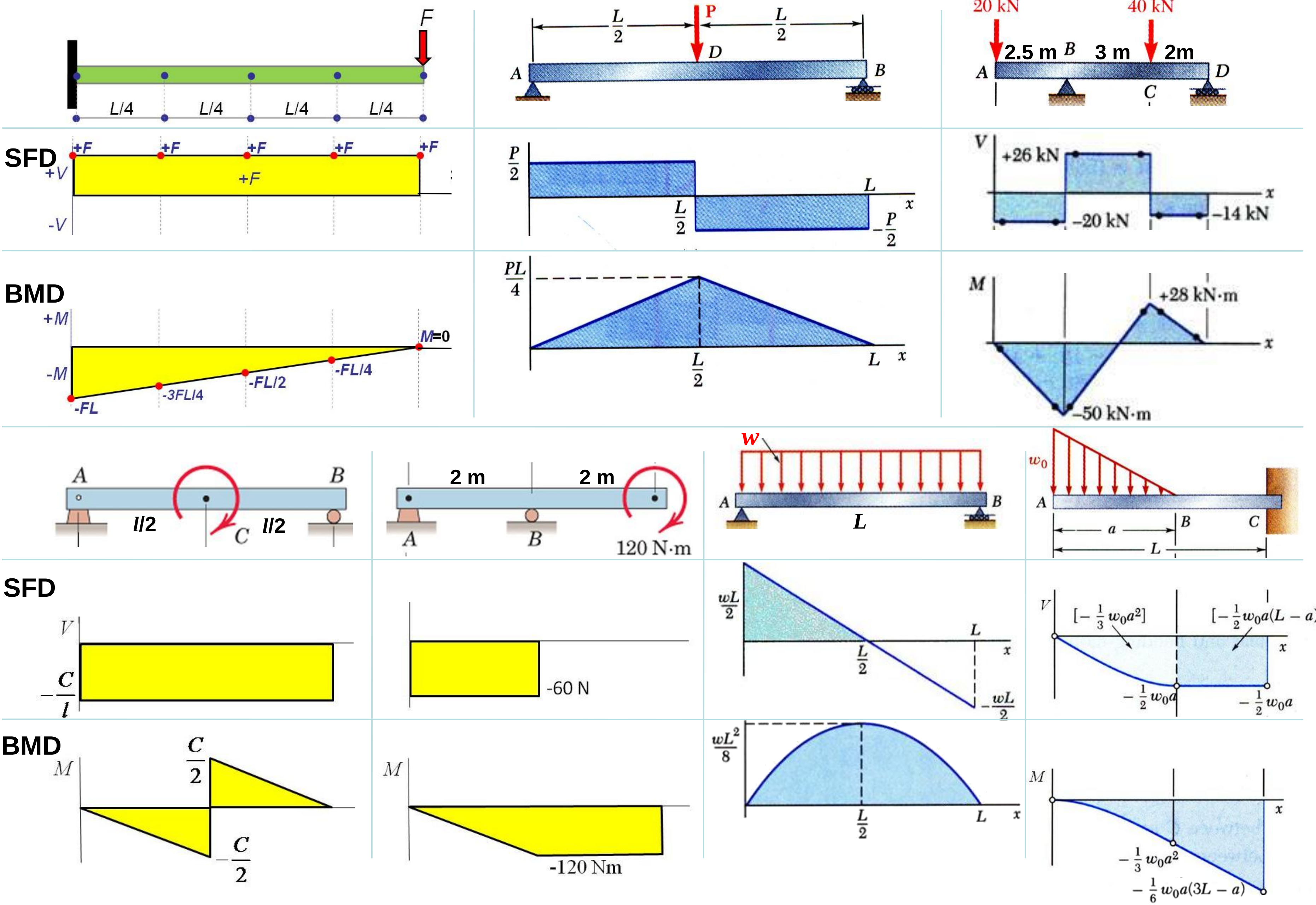
Solution:

SFD and BMD can be plotted without determining support reactions since it is a cantilever beam.

However, values of SF and BM can be verified at the support if support reactions are known.

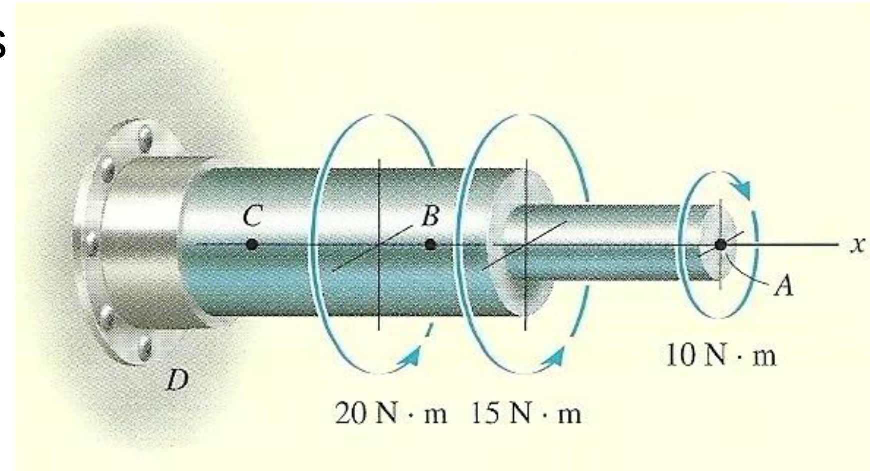
$$R_C = \frac{w_0 a}{2} \uparrow; \qquad M_C = \frac{w_0 a}{2}\left(L - \frac{a}{3}\right) = \frac{w_0 a}{6}(3L - a)$$

Beams – SFD and BMD: Summary

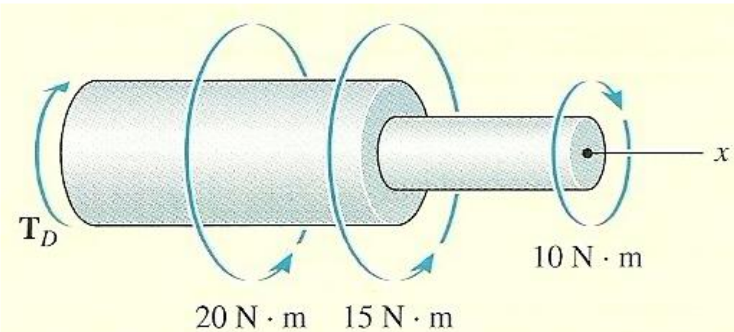


Beams – Internal Effects

Example: Find the internal torques at points B and C of the circular shaft subjected to three concentrated torques

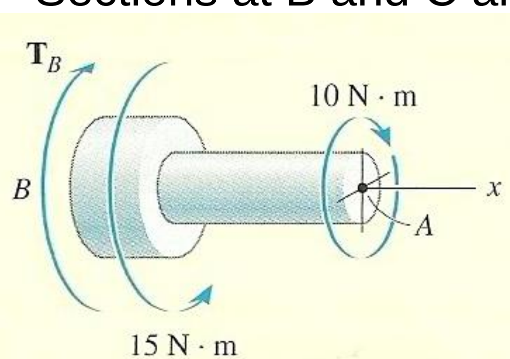


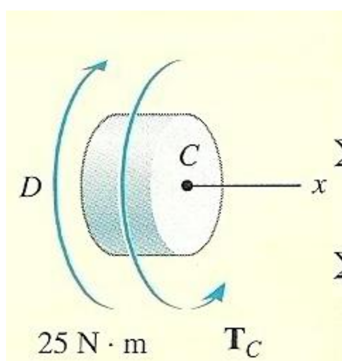
Solution: FBD of entire shaft



$$\Sigma M_x = 0; \quad -10\ \mathrm{N \cdot m} + 15\ \mathrm{N \cdot m} + 20\ \mathrm{N \cdot m} - T_D = 0$$
$$T_D = 25\ \mathrm{N \cdot m}$$

Sections at B and C and FBDs of shaft segments AB and CD





Segment AB

$$\Sigma M_x = 0; \quad -10\ \mathrm{N \cdot m} + 15\ \mathrm{N \cdot m} - T_B = 0 \qquad T_B = 5\ \mathrm{N \cdot m}$$

Segment CD

$$\Sigma M_x = 0; \quad T_C - 25\ \mathrm{N \cdot m} = 0 \qquad T_C = 25\ \mathrm{N \cdot m}$$

Cables

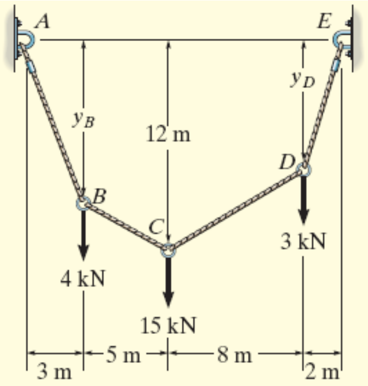
Flexible and Inextensible Cables

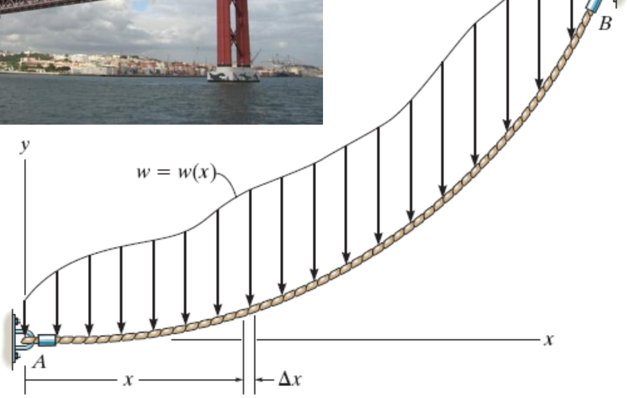
Important Design Parameters

Tension
Span
Sag
Length

Cables

- Relations involving Tension, Span, Sag, and Length are reqd
  - Obtained by examining the cable as a body in equilibrium
- It is assumed that any resistance offered to bending is negligible → Force in cable is always along the direction of the cable.
- Flexible cables may be subjected to concentrated loads or distributed loads





Cables

- In some cases, weight of the cable is negligible compared with the loads it supports.
- In other cases, weight of the cable may be significant or may be the only load acting → weight cannot be neglected.

Three primary cases of analysis: Cables subjected to

1. concentrated load, **2.** distributed load, **3.** self weight

→ Requirements for equilibrium are formulated in identical way **provided Loading is coplanar with the cable**

Cables

Primary Assumption in Analysis:

The cable is perfectly Flexible and Inextensible

Flexible → cable offers no resistance to bending

→ tensile force acting in the cable is always tangent to the cable at points along its length

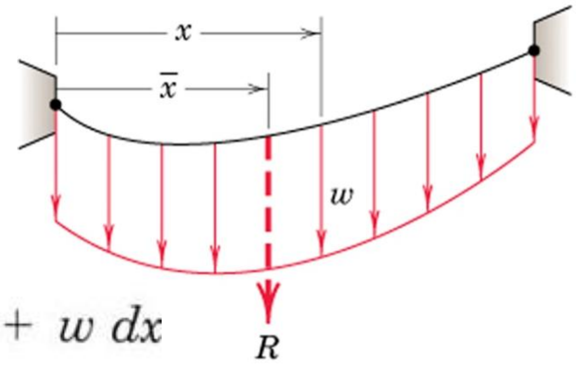
Inextensible → cable has a constant length both before and after the load is applied

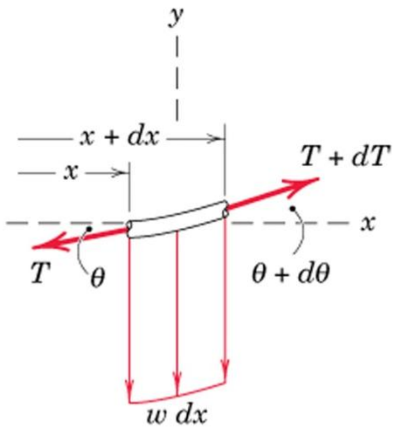
→ once the load is applied, geometry of the cable remains fixed

→ cable or a segment of it can be treated as a rigid body

Cables

General Relationships





Equilibrium of the element:

$$(T + dT)\sin(\theta + d\theta) = T\sin\theta + w\,dx$$

$$(T + dT)\cos(\theta + d\theta) = T\cos\theta$$

Simplifying using trigonometric expansions and dropping second order terms

$$T\cos\theta\,d\theta + dT\sin\theta = w\,dx$$

$$-T\sin\theta\,d\theta + dT\cos\theta = 0$$

Further simplifying: → $d(T\sin\theta) = w\,dx$

→ $d(T\cos\theta) = 0$

Finally: $$\boxed{\frac{d^2y}{dx^2} = \frac{w}{T_0}}$$

where $T_0 = T\cos\theta$

→Equilibrium Equation
→Differential Equation for the Flexible Cable
-Defines the shape of the cable
-Can be used to solve two limiting cases of cable loading

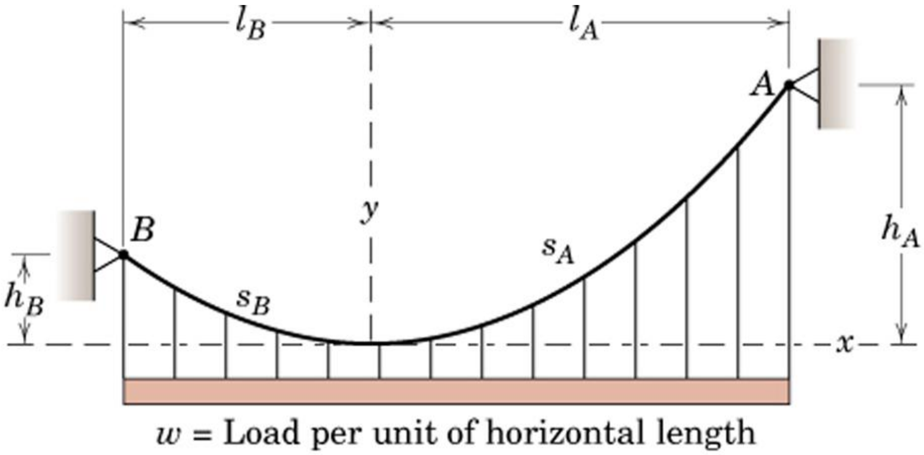
Cables: Parabolic Cable

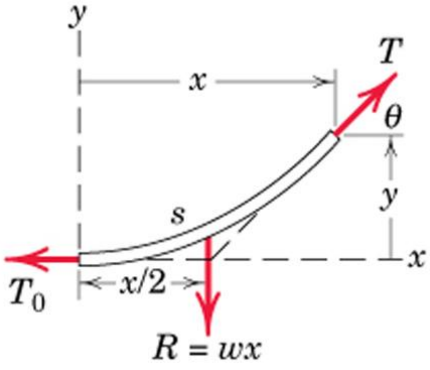
Parabolic Cable

Intensity of vertical loading is constant

→ It can be proved that the cable hangs in a **Parabolic Arc**

→ Differential Equation can be used to analyse





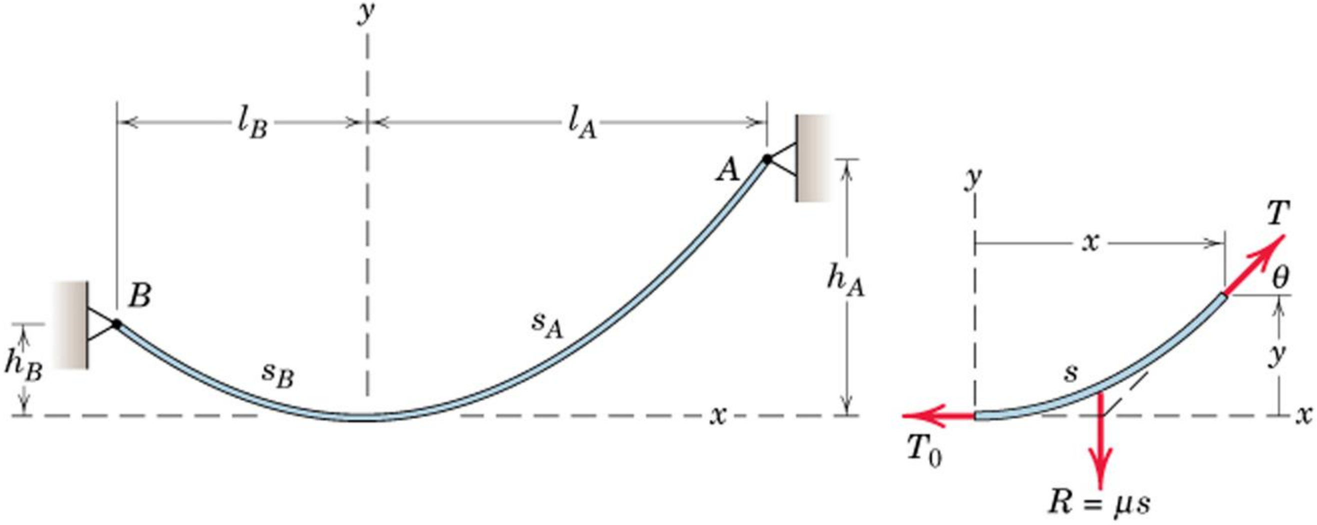
Cables: Catenary Cable

Catenary Cable

Hanging under the action of its own weight

→ It can be proved that the cable hangs in a curved shape called **Catenary**

→ Differential Equation can be used to analyse



μ is the self weight per unit length