CE 601 NUMERICAL METHODS

TUTORIAL – 10

No Marks **Date: 16-November-2012**

This tutorial is not required to be submitted for evaluation. This is a practice tutorial and you may solve the problems on your own. You may also use computational programs like Matlab, Mathematica, Fortran, C, C++, etc. or any other convenient programming language (maybe even MS-Excel) to evaluate operations like additions, multiplications, matrix operations, etc.

1. The one-dimensional advection-dispersion equation $\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} = D\frac{\partial^2 C}{\partial x^2}$ can be used to describe pollutant transport in rivers. (u is advection velocity, D is dispersion coefficient, C is concentration). Use Crank-Nicolson scheme to solve the PDE in the domain $0 \le x \le 10.0$ for the following boundary and initial conditions $C(0, x) = 0.0$; $C(t, 0.0) = 0.2$ and $\frac{\partial C}{\partial x}(t, 10.0) = 0$. Use appropriate grid and time-step sizes for solving. The parameters are u = 3.5×10^{-5} m/s, D = 1×10^{-6} m^2/s.
2. Derive the first-order upwind approximation of the unsteady one-dimensional convection equation $\frac{\partial f}{\partial t} + u\frac{\partial C}{\partial x} = 0$ for $u > 0.0$.
3. Consider the two-dimensional diffusion equation: $\frac{\partial f}{\partial t} = \alpha\left(\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}\right)$ for any arbitrary domain *xy*-plane. Derive the FTCS approximation. Perform consistency and stability analyses.