Solved Example Set-2

SOLVED EXAMPLE SET- 2 (10-AUG-2012)

1. Solve using Doolittle's algorithm the following systems of linear equations

$$\begin{bmatrix} 2 & 7 & 5 \\ 6 & 20 & 10 \\ 4 & 3 & 0 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 14 \\ 36 \\ 7 \end{Bmatrix} \quad \text{and} \quad \begin{bmatrix} 2 & 7 & 5 \\ 6 & 20 & 10 \\ 4 & 3 & 0 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} -4 \\ -16 \\ 7 \end{Bmatrix}$$

SOLN.

Doolittle's algorithm suggest that

$k = 1, 2, 3, 4, \ldots, (n-1)$

$l_{kk} = 1$

$l_{ik} = 0$ for $i < k$

$u_{1j} = a_{1j}$

$u_{ij} = a_{ij}^{(k)} = a_{ij}^{(k-1)} - l_{ik}\, a_{kj}^{(k-1)}$;

$i = k+1, k+2, \ldots, n$

$l_{ik} = \dfrac{a_{ik}^{(k-1)}}{a_{kk}^{(k-1)}}$ $j = k, k+1, k+2, \ldots, n$

$l_{nn} = 1$

Therefore, for the given problem $n = 3$

$k = 1, 2$, $i = k+1, \ldots, 3$, $j = k, k+1, \ldots, 3$

Step	l_{kk}	$l_{ik} = 0$, $i<k$	l_{ik}, $i>k$	$a_{ij}^{(k)}$
$k=1$	$l_{11} = 1$	N.A	$l_{21} = \frac{a_{21}}{a_{11}} = 3$	$a_{22}^{(1)} = 20 - 3\times 7 = -1$
				$a_{23}^{(1)} = 10 - 3\times 5 = -5$
			$l_{31} = \frac{a_{31}}{a_{11}} = 2$	$a_{32}^{(1)} = 3 - 2\times 7 = -11$
				$a_{33}^{(1)} = 0 - 2\times 5 = -10$
$k=2$	$l_{22} = 1$	$l_{12} = 0$	$l_{32} = \frac{a_{32}^{(1)}}{a_{22}^{(1)}} = 11$	$a_{33}^{(2)} = -10 - 11\times(-5) = 45$

$$l_{nn} = l_{33} = 1$$

$$\therefore [L] = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 11 & 1 \end{bmatrix}$$

$$[U] = \begin{bmatrix} 2 & 7 & 5 \\ 0 & -1 & -5 \\ 0 & 0 & 45 \end{bmatrix}$$

To solve the given systems:

$$[L]\{c\} = \{b\} \quad \text{and} \quad [U]\{x\} = \{c\}$$

$\therefore$ The forward and backward substitution required

$$\left.\begin{aligned} c_1 &= b_1 \\ c_i &= b_i - \sum_{j=1}^{i-1} l_{ij}\, x_j \quad ; \quad i = 2, 3 \end{aligned}\right\} \rightarrow \text{Forward substitution}$$

$$\left.\begin{aligned} x_3 &= \frac{c_3}{u_{33}} \\ x_i &= \frac{c_i - \sum_{j=i+1}^{3} u_{ij}\, x_j}{u_{ii}} \quad ; \quad i = 2, 1 \end{aligned}\right\} \rightarrow \text{Backward Substitution}$$

b_i	c_i	x_i
$b_1 = 14$	$c_1 = 14$	$x_1 = 14 - (5\times 1) - (7\times -1) = 16$
$b_2 = 36$	$c_2 = 36 - 3\times 14 = -6$	$x_2 = -6 - (-5\times 1) = -1$
$b_3 = 7$	$c_3 = 7 - 2\times 14 - 11\times(-6) = 45$	$x_3 = 1$
$b_1 = -4$	$c_1 = -4$	$x_1 = -4 - (5\times 1.311) - (7\times 2.555) = -28.444$
$b_2 = -16$	$c_2 = -16 - 3\times(-4) = -4$	$x_2 = -4 - (-5\times 1.311) = 2.555$
$b_3 = 7$	$c_3 = 7 - 2\times(-4) - 11\times(-4) = 59$	$x_3 = 1.311$