Solved Example Set-1

CE 601 NUM. METHODS

SOLVED EXAMPLES - 1 02-AUG-2012

1. Solve the system of linear equations

$$x_1 + 3x_2 + 2x_3 - x_4 = 9$$
$$4x_1 + 2x_2 + 5x_3 + x_4 = 27$$
$$3x_1 - 3x_2 + 2x_3 + 4x_4 = 19$$
$$-x_1 + 2x_2 - 3x_3 + 5x_4 = 14$$

using Gauss-elimination algorithm.

Soln. The given system can be represented as:

$$\begin{bmatrix} 1 & 3 & 2 & -1 \\ 4 & 2 & 5 & 1 \\ 3 & -3 & 2 & 4 \\ -1 & 2 & -3 & 5 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{Bmatrix} 9 \\ 27 \\ 19 \\ 14 \end{Bmatrix}$$

The matrix $[A]$ is 4×4 size.

No. of steps for elimination $= n-1 = 4-1 = 3$

Step 1 $(k=1)$

$$l_{21} = \frac{a_{21}}{a_{11}} = 4, \quad l_{31} = \frac{a_{31}}{a_{11}} = 3, \quad l_{41} = \frac{a_{41}}{a_{11}} = -1$$

$$\left.\begin{aligned} a_{ij}^{(1)} &= a_{ij} - a_{1j} l_{i1} \\ b_i^{(1)} &= b_i - l_{i1} b_1 \end{aligned}\right\} \quad \begin{aligned} i &= 2, 3, 4 \\ j &= 1, 2, 3, 4 \end{aligned}$$

$\therefore$ The augmented matrix:

$$\left[\begin{array}{cccc|c} 1 & 3 & 2 & -1 & 9 \\ 4 & 2 & 5 & 1 & 27 \\ 3 & -3 & 2 & 4 & 19 \\ -1 & 2 & -3 & 5 & 14 \end{array}\right] \begin{array}{l} \\ R_2 = R_2 - 4R_1 \\ R_3 = R_3 - 3R_1 \\ R_4 = R_4 - (-1)R_1 \end{array}$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 3 & 2 & -1 & 9 \\ 0 & -10 & -3 & 5 & -9 \\ 0 & -12 & -4 & 7 & -8 \\ 0 & 5 & 1 & 4 & 23 \end{array}\right]$$

Step 2 $(k=2)$

$$\ell_{i2} = \frac{a_{i2}^{(1)}}{a_{22}^{(1)}} \; ; \quad \left.\begin{aligned} a_{ij}^{(2)} &= a_{ij}^{(1)} - \ell_{i2}\, a_{2j}^{(1)} \\ b_i^{(2)} &= b_i^{(1)} - \ell_{i2}\, b_2^{(1)} \end{aligned}\right\} \begin{aligned} i &= 3,4 \\ j &= 2,3,4 \end{aligned}$$

$$\therefore \left.\begin{aligned} R_3 &= R_3 - \left(\tfrac{-12}{-10}\right) R_2 \\ R_4 &= R_4 - \left(\tfrac{5}{-10}\right) R_2 \end{aligned}\right\} \Rightarrow \left[\begin{array}{cccc|c} 1 & 3 & 2 & -1 & 9 \\ 0 & -10 & -3 & 5 & -9 \\ 0 & 0 & -0.4 & 1 & 2.8 \\ 0 & 0 & -0.5 & 6.5 & 18.5 \end{array}\right]$$

Step 3 $(k=3)$

$$\ell_{i3} = \frac{a_{i3}^{(2)}}{a_{33}^{(2)}} \; ; \quad \left.\begin{aligned} a_{ij}^{(3)} &= a_{ij}^{(2)} - \ell_{i3}\, a_{3j}^{(2)} \\ b_i^{(3)} &= b_i^{(2)} - \ell_{i3}\, b_3^{(2)} \end{aligned}\right\} \begin{aligned} i &= 4 \\ j &= 3,4 \end{aligned}$$

$$\therefore R_4 = R_4 - \left(\frac{-0.5}{-0.4}\right) R_3 \Rightarrow \left[\begin{array}{cccc|c} 1 & 3 & 2 & -1 & 9 \\ 0 & -10 & -3 & 5 & -9 \\ 0 & 0 & -0.4 & 1 & 2.8 \\ 0 & 0 & 0 & 5.25 & 15 \end{array}\right]$$

Backsubstitution

$$x_4 = \frac{b_n^{(3)}}{a_{nn}^{(3)}} = \frac{15}{5.25} = 2.857$$

$$x_3 = \frac{b_3^{(2)} - \sum_{j=4}^{4} a_{3j}^{(2)} x_j}{a_{33}^{(2)}} = \frac{2.8 - 1 \times 2.857}{-0.4} = \underline{\underline{0.143}}$$

$$x_2 = \frac{b_2^{(1)} - \sum_{j=3}^{4} a_{2j}^{(1)} x_j}{a_{22}^{(1)}} = \frac{-9 - [-3 \times 0.143 + 5 \times 2.857]}{-10} = \underline{\underline{2.286}}$$

$$x_1 = \underline{\underline{4.713}}$$