A Computer Program development for solving linear system by LU Decomposition using Doolittle's algorithm.

1. Solve the following linear systems $A\vec{x} = \vec{b}$, where $\vec{x} = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix}$; using Doolittle's algorithm.

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ -1 & 1 & 2 & 3 \\ 1 & -1 & 1 & 2 \\ -1 & 1 & -1 & 5 \end{bmatrix}; \ \vec{b}_a = \begin{Bmatrix} 10 \\ 5 \\ 3 \\ 4 \end{Bmatrix}; \ \vec{b}_b = \begin{Bmatrix} -4 \\ -5 \\ -3 \\ -4 \end{Bmatrix}; \ \vec{b}_c = \begin{Bmatrix} -2 \\ -3 \\ 1 \\ -8 \end{Bmatrix}$$

We will discuss how to develop a Matlab program using the algorithm discussed in the class.

Doolittle's algorithm suggest that

$k = 1, 2, 3, 4, \ldots, (n-1)$

$l_{kk} = 1$

$l_{ik} = 0$ for $i < k$

$u_{1j} = a_{1j}$

$u_{ij} = a_{ij}^{(k)} = a_{ij}^{(k-1)} - l_{ik}\, a_{kj}^{(k-1)}$;

$i = k+1, k+2, \ldots, n$

$l_{ik} = \dfrac{a_{ik}^{(k-1)}}{a_{kk}^{(k-1)}}$ $\quad j = k, k+1, k+2, \ldots, n$

$l_{nn} = 1$

To solve the given systems:

$[L]\{c\} = \{b\}$ and $[U]\{x\} = \{c\}$

∴ The forward and Backward substitution required

$$\left.\begin{aligned} c_1 &= b_1 \\ c_i &= b_i - \sum_{j=1}^{i-1} l_{ij}\, x_j \quad ; \ i = 2, 3, 4 \end{aligned}\right\} \rightarrow \text{Forward substitution}$$

$$\left.\begin{aligned} x_4 &= \frac{c_4}{u_{44}} \\ x_i &= \frac{c_i - \sum_{j=i+1}^{3} u_{ij}\, x_j}{u_{ii}} \quad ; \ i = 3, 2, 1 \end{aligned}\right\} \rightarrow \text{Backward Substitution}$$

```
% Doolittle's algorithm for LU Decomposition and solution 16-August-2012
```

```
% Given A, ba, bb, bc. Their dimensions nxn. To find xa, xb, and xc
% For that first L and U matrices need to be found.
%
AA = [1 2 3 4
   -1 1 2 3
   1 -1 1 2
   -1 1 -1 5];
A = AA;
ba = [10
   5
   3
   4];
bb = [-4
   -5
   -3
   -4];
bc = [-2
   -3
   1
   -8];
nn=length(A);
% Define the existence of L and U matrices
L(1:nn,1:nn)=0;
U=A;
% Start Do loop for LU Factorisation
for k = 1:nn-1
   for i = k+1:nn
   L(k,k)=1.0;
   lik = A(i,k)/A(k,k);
   for j=k:nn
      A(i,j)=A(i,j)-lik*A(k,j);
      U(i,j)=A(i,j);
   end
   L(i,k)=lik;
   end
end
L(nn,nn)=1
%
%Forward substitution for vector c.
%
c(1:nn)=0;
x(1:nn)=0;
%xa=x;
%xb=x;
%xc=x;
for p = 1:3
   if p==1
      b(1:nn)=ba(1:nn)
   elseif p==2
      b(1:nn)=bb(1:nn)
   elseif p==3
      b(1:nn)=bc(1:nn)
   end
   c(1)= b(1)
   for i=2:nn
      c(i)=b(i);
      for j=1:i-1
         c(i)=c(i)-L(i,j)*c(j)
      end
   end
   x(nn)= c(nn)/U(nn,nn);
   for i=nn-1:-1:1
      x(i)=c(i);
```

```
        for j=nn:-1:i+1
            x(i)=x(i)-U(i,j)*x(j);
        end
        x(i)=x(i)/U(i,i)
    end
    if p==1
        xa=x'
    elseif p==2
        xb=x'
    elseif p==3
        xc=x'
    end
end
fid=fopen('out.txt','w');
fprintf(fid,'L = ')
for i=1:nn
    fprintf(fid,'\n')
    for j= 1:nn
        fprintf(fid,'%10.5f', L(i,j));
    end
end
fprintf(fid,'\n')
fprintf(fid,'U = ')
for i=1:nn
    fprintf(fid,'\n')
    for j= 1:nn
        fprintf(fid,'%10.5f', U(i,j));
    end
end
fprintf(fid,'\3n')
fprintf(fid,'xa = ')
fprintf(fid,'%10.5f',xa(1:nn));
fprintf(fid,'\n')
fprintf(fid,'xb = ')
fprintf(fid,'%10.5f',xb(1:nn));
fprintf(fid,'\n')
fprintf(fid,'xc = ')
fprintf(fid,'%10.5f',xc(1:nn));
fclose(fid)
```

The output fi8le is given as below:

```
L =
   1.00000   0.00000   0.00000   0.00000
  -1.00000   1.00000   0.00000   0.00000
   1.00000  -1.00000   1.00000   0.00000
  -1.00000   1.00000  -1.00000   1.00000
U =
   1.00000   2.00000   3.00000   4.00000
   0.00000   3.00000   5.00000   7.00000
   0.00000   0.00000   3.00000   5.00000
   0.00000   0.00000   0.00000   7.00000
xa =    1.00000   1.00000   1.00000   1.00000
xb =    1.00000   1.00000  -1.00000  -1.00000
xc =    1.00000  -1.00000   1.00000  -1.00000
```