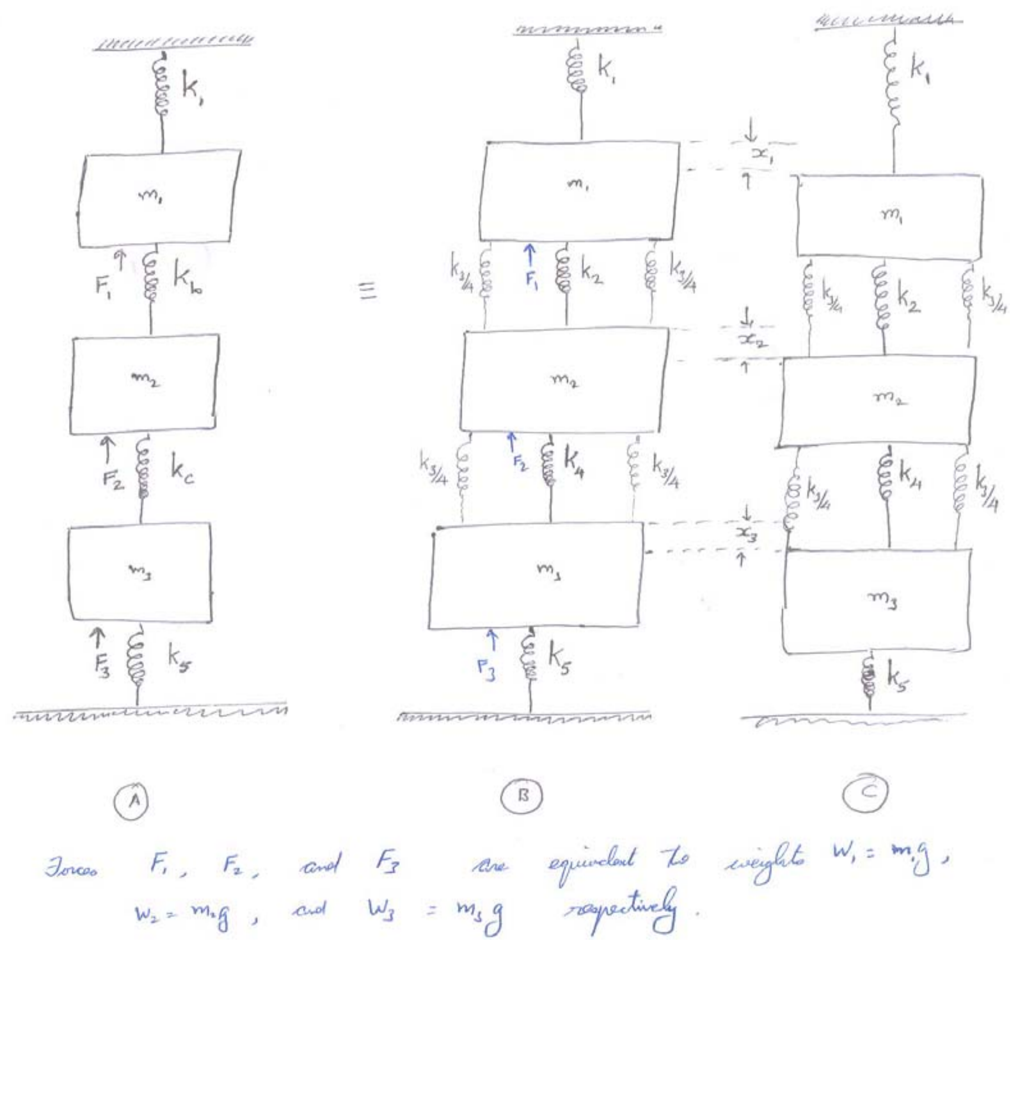
SYSTEM OF LINEAR EQUATIONS

As discussed yesterday many of the engineering problems are required to solve the system of linear equations.

How do you formulate a system of equations for a given problem?

Consider an engineering mechanics problem

→ The problem is from Statics.



Forces F_1, F_2, and F_3 are equivalent to weights $W_1 = m_1 g$, $W_2 = m_2 g$, and $W_3 = m_3 g$ respectively.

Figure (A) represents the actual system

Figure (B) is obtained by dissociating the equivalent spring stiffness between mass m_1 & m_3, and m_1 & m_2, into m_2 & m_3 as k_3, k_2, and k_4

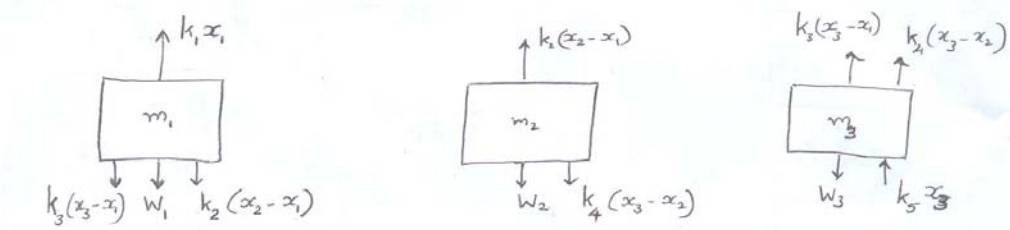
Figure (C) represents the static situation when the forces F_1, F_2, and F_3 are removed.

Due to removal of F_1, F_2, F_3 → there will be deflection in the system marked as x_1, x_2, and x_3.

We have studied that in statics $\Sigma F = 0$.

We can ~~consider~~ take each of the mass blocks m_1, m_2, and m_3 and consider them to be in equilibrium.

Draw the free-body diagrams for m_1, m_2, m_3.



$\therefore$ $\Sigma F = 0$ equation for each block.

$$(k_1 + k_2 + k_3)x_1 - k_2 x_2 - k_3 x_3 = W_1$$

$$-k_2 x_1 + (k_2 + k_4)x_2 - k_4 x_3 = W_2$$

$$-k_3 x_1 - k_4 x_2 + (k_3 + k_4 + k_5)x_3 = W_3$$

This can be represented as a system of equation and also in matrix form.

$$\begin{bmatrix} (k_1 + k_2 + k_3) & -k_2 & -k_3 \\ -k_2 & (k_2 + k_4) & -k_4 \\ -k_3 & -k_4 & (k_3 + k_4 + k_5) \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} W_1 \\ W_2 \\ W_3 \end{Bmatrix}$$

For example, if ~~k_1~~ $k_1 = 0.35$ N/m

$k_4 = k_2 = 0.30$ N/m

$k_3 = 0.25$ N/m

$k_5 = 0.80$ N/m

$W_1 = W_2 = W_3 = 20$ N

Then.

$$\begin{bmatrix} 0.90 & -0.30 & -0.25 \\ -0.30 & 0.60 & -0.30 \\ -0.25 & -0.30 & 1.35 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 20 \\ 20 \\ 20 \end{Bmatrix}$$

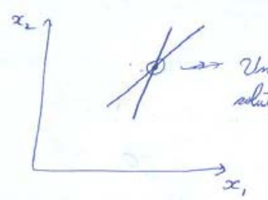
You can solve this to get values of x_1, x_2, and x_3.

Q: What are the types or possible solutions for any system of linear equations?

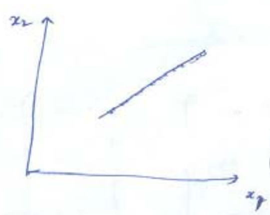
We can discuss this for a simple system of two linear algebraic equations.

$$a_{11}\, x_1 + a_{12}\, x_2 = b_1$$
$$a_{21}\, x_1 + a_{22}\, x_2 = b_2$$

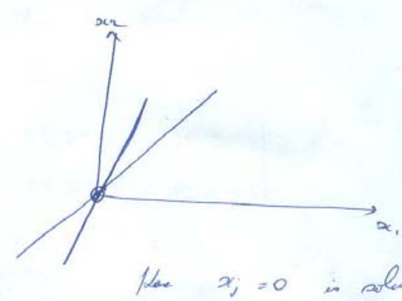
The above system represents two line segments in (x_1, x_2) co-ordinate system



No solution because two lines are parallel



Say all the points inside are solution. (i.e. Infinite)



Here $x_j = 0$ is solution. Homogeneous case trivial solution.

Q: Again how will you solve the system of linear equations?

Soln.: There are two approaches

→ Direct Elimination method

→ Iterative methods

Direct Elimination Methods

→ Gauss elimination
→ Gauss - Jordan method
→ Matrix - Inverse method
→ Doolittle LU Factorisation

Iterative methods

→ Jacobi iteration
→ Gauss - Seidel iteration
→ Successive - over relaxation (SOR)

Matrix Properties

You have seen that system of linear equations ~~are~~ can be represented by matrix methods.
Therefore, you need to revise the basic matrix algebra for this course.

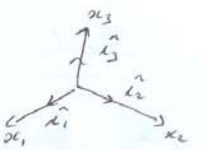
$$\text{Matrix } [A] = \underset{\substack{n \times m \\ \text{matrix}}}{[a_{i,j}]} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & & & \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix}$$

Vectors

→ Column $\{x\} = [x_i] = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix}$

→ Row $[y] = [y_i] = \begin{bmatrix} y_1 & y_2 & \cdots & y_m \end{bmatrix}$

* Unit Vectors $\hat{i}$



$$\|\hat{i}\| = 1$$

Reading exercise
Associative
Commutative
Distributive

* Square Matrix

* Diagonal Matrix, Identity Matrix

* Triangular

→ Upper

→ Lower

$$U = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & a_{22} & a_{23} & a_{24} \\ 0 & 0 & a_{33} & a_{34} \\ 0 & 0 & 0 & a_{44} \end{bmatrix}$$

* Tridiagonal matrix

$$T = \begin{pmatrix} a_{11} & a_{12} & a_{13} & 0 & 0 \\ a_{21} & a_{22} & a_{23} & a & 0 \\ 0 & a_{32} & a_{33} & a_{34} & 0 \\ 0 & 0 & a_{43} & a_{44} & a_{45} \\ 0 & 0 & 0 & a_{54} & a_{55} \end{pmatrix}$$

* Matrix Addition $\quad [A] + [B] = [a_{ij}] + [b_{ij}] = (c_{ij}) = [C]$

$$= [a_{ij} + b_{ij}]$$

* Multiplication

$$[AB] = [a_{ij}][b_{ij}] = \sum_{k=1}^{m} a_{ik} b_{kj} \; ; \quad i = 1, 2, \ldots n, \quad j = 1, 2, \ldots r$$

$$= [c_{ij}] = [C]$$

$[A] \rightarrow n \times m$

$[B] \rightarrow m \times r$

QUIZ - 1

The total expenditure in rupees incurred for a person is based on his spending money for two products. The cost for the first product is x_1 (₹) per unit and x_2 (₹) per unit. On a partial day person A purchased 30 units of Product 1 and 20 units of Product 2 for a total expenditure of Rs. 300. Person B purchased 25 unit of PI and 35 units of PII for Rs. 320. Formulate the system and find x_1, x_2?