CE 601: Numerical Methods
Lecture 2

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- In this course of numerical methods, we will see over the period of time:

- To solve systems of linear algebraic equations
- To solve eigen-value problems
- To solve non-linear equations
- To use polynomial approximations and interpolation
- Numerical differentiation, difference formulas
- Numerical integration
- Numerical solutions of ODE – Initial and Boundary value ODE
- Numerical solution of PDE:1) Elliptic PDE, 2) Parabolic PDE, 3) Hyperbolic PDE
- Introduction to FEM

- Numerical methods often require computational tools or devices to compute the numerical solutions of the mathematical expressions.
- With the advent of high speed computers, numerical methods are now highly applied in all fields of engineering and science.
- To implement numerical method techniques using computers one needs to develop: Algorithms and Flow charts for computational sequence etc.
- Any programming language or software can be used for applying numerical methods.

- While using computers you may come across certain properties like:
  - Significant digits (or figures)
  - Accuracy
  - Precision
- Q. What are these terms?
- We will explain briefly through some examples:

186.57 -> has five significant digits

2893.8745 -> has eight significant digits

- You are asking a variable in a computer program to store a number that has specified significant digits.
- Now if computer is capable of storing two significant digits, then for a number like 8.23 will be stored in that computer as 8.2.
- If some computer display a number say 8.40, then it has capability of storing three significant digits. The second digit after decimal is exactly 0 and therefore significant.

- 00289.6 -> has four significant digits

  0.00682 -> has threes significant digits

  45000 -> has two significant digits

- The *accuracy* of the approximate number in the computer is based on number of significant digits.

e.g. In 0.00683 there are three significant digits. So, its accuracy is based on three significant digits.

- *Precision* -> describes the position of the rightmost significant digit. With respect to decimal points, it refers the position of decimal places.

  0.00682 -> has an accuracy of 3 significant digits, precision of two decimal places

  389.27 -> accuracy of 5 significant digits, precision of tw0 decimal places.

  3.8927 -> accuracy of 5 significant digits, precision of four decimal places

  3892700 -> accuracy of 5 significant digits, precision of hundred

  0.0000004 -> accuracy of 1 significant digit, precision of 6 decimal places

- You can have high accuracy in your approximate number, but less precise.
- Similarly you may have less accurate but high precision numbers
- Due to approximations in the numbers you may come up with *errors*. Common errors in numerical methods are:
    - Errors in the parameters of the problem
    - Algebraic errors
    - Iteration errors
    - Approximation errors
    - Roundoff errors

- Number Representation
  - in computers we have to represent numbers based on number system. You have number systems like -> decimal, binary, octal, hexadecimal etc.
  - In binary you have two digits -> 0,1

Decimal 10 digits -> 0,1,2,3,4,5,6,7,8,9

Octal 8 digits -> 0,1,2,3,4,5,6,7

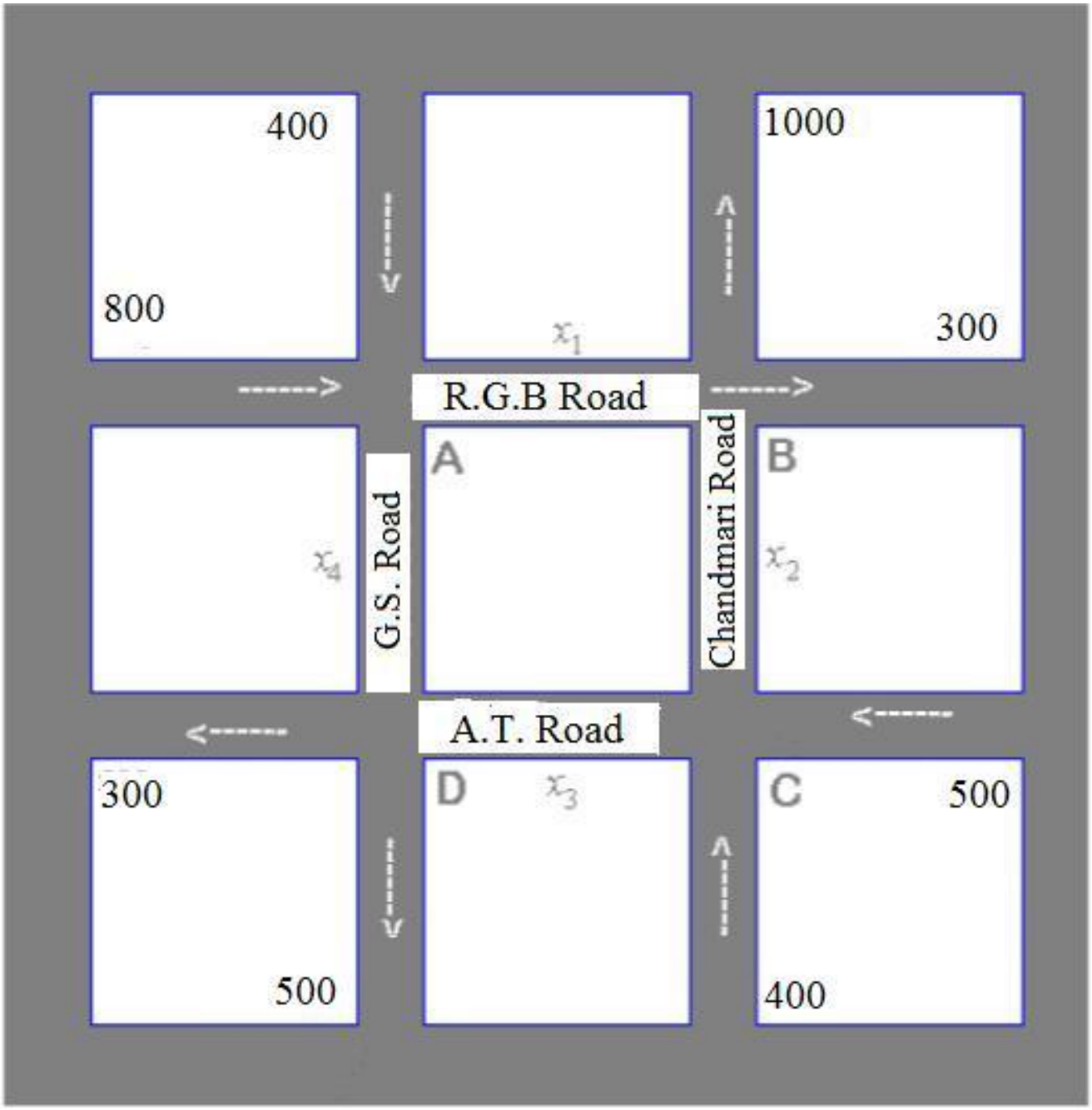
  - Digital computers use binary system
  - The most common of bianry number is 32-bit number.

System of Linear Equations

- First portion we are going to study is to find the solution of system of linear equations
- As an engineer why do we need to study to solve system of linear equations, that we studied in school levels?
- We should be aware how to formulate a system of linear equations from a given engineering problem.

- E.g. Traffic congestion is encountered in a metro city. The traffic officer and administrator together sat on a city street map to suggest improvements by incorporating traffic signals at intersections. All streets in the city map are one-way and directions are given.
- We have to design time at intersections based on number of vehicles.

(Concept adopted from Dr. Mousa Hussein's lecture notes)


400
800
x_1
1000
300
R.G.B Road
x_4
G.S. Road
A
Chandmari Road
B
x_2
A.T. Road
300
500
D
x_3
C
500
400

- So objective is to find the number vehicles passing intersections:
- Given at

✓ Corner A: 800 cars/hour come from R.G. B road

400 cars/hour arrive through G.S. road

✓ Corner B: 300 cars/hour leave B through RGB road

1000 cars/hour leave B through Chandmari road

✓ Corner C: 500 cars/hour enter A.T. road to C

400 cars/hour arrive C through Chandmari road

✓ Corner D: 300 cars/hour leave D through A.T. road

500 cars/hour leave D through G.S. road

- Let

✓ x_1 -> number of vehicles leaving junction A through RGB road towards junction B

✓ x_2 -> number of vehicles arriving junction B through Chandmari road from junction C

✓ x_3 -> number of vehicles leaving junction C through A.T. road towards junction D

✓ x_4 -> number of vehicles arriving junction D through G.S. road from junction A

- Let us suggest assumptions:

(i) the number of cars leaving a junction = number of cars arriving at a junction

(ii) Streets are one-way.

(iii) x_1, x_2, x_3, x_4 can only be positive.

At junction A, $x_1 + 0.x_2 + 0.x_3 + x_4 = 400 + 800$

At junction B, $x_1 + x_2 + 0.x_3 + 0.x_4 = 1000 + 300$

At junction C, $0.x_1 + x_3 + x_3 + 0.x_4 = 400 + 500$

At junction D, $0.x_1 + 0.x_3 + x_3 + x_4 = 300 + 500$

Form the system of linear equations to solve.

i.e., $$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{Bmatrix} 1200 \\ 1300 \\ 900 \\ 800 \end{Bmatrix}$$