LECTURE 32

RADIAL FLOWS TO WELLS

31-MAR-2014

Last day we discussed about radial flows to wells that penetrate confined aquifers.

→ We suggested that the wells fully penetrate the confined aquifer.

→ For steady state, radial flow to a well

$$Q = 2\pi K b \frac{h - h_w}{\ln(r/r_w)}$$

where K → istropic hydraulic conductivity
b → thickness of aquifer
h_w → depth to water surface ~~from~~ in piezometer or pumping well for steady discharge Q
r_w → radius of well.

→ Subsequently we also discussed about equi-potential lines and flow lines (streamlines).

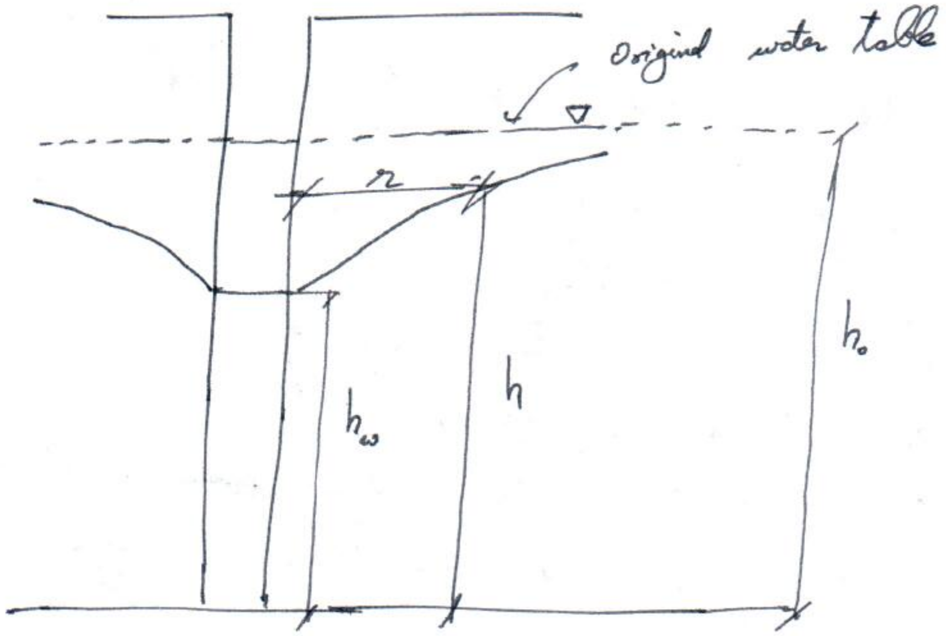
∴ If you have at two radial distances r_1 and r_2 where drawdowns are s_1 and s_2

Then $$Q = \frac{2\pi K b (s_1 - s_2)}{\ln(r_2/r_1)}$$

Steady Radial flow in unconfined aquifers

Again we are assuming, the radial flows to the wells in unconfined aquifers are pre-dominantly in horizontal direction.

* The well completely penetrates the unconfined aquifer.



→ The well discharge is :

$$Q = -2\pi r K h \frac{dh}{dr} \quad \rightarrow (1)$$

The boundary conditions are: At $r = r_o$, $h = h_o$
At $r = r_w$, $h = h_w$

Solving (1), we get:

$$Q = \frac{-\pi K (h_o^2 - h_w^2)}{\ln (r_o / r_w)}$$

If ~~so~~ we consider two radial distances r_1 and r_2 from the centre line in the well, then we can write

$$Q = -\pi K \frac{h_2^2 - h_1^2}{\ln (r_2/r_1)}$$

From this relationship, we can infer the hydraulic conductivity

$$K = \frac{Q \ln (r_2/r_1)}{\pi (h_2^2 - h_1^2)}$$

⇒ While dealing with unconfined flow, we need to interpret that the Dupuit's assumptions may get violated near the well due to large components of flow in vertical directions.

However for overall well flow analyses, we can still persist with above equation.

⇒ Also note that we consider aquifer phreatic surface to be horizontal initially before pumping started.

Example as taken from TODD & MAYS (2005)

A well penetrates completely an unconfined aquifer. Before pumping $h_o = 25$ m. After long time steady state conditions are reached and the following observations are made. At $r_1 = 50$ m, $s_1 = 3$ m and at $r_2 = 150$ m, $s_2 = 1.2$ m. Compute hydraulic conductivity and radius of influence for this pumping discharge $Q = 0.05$ m³/s.

Answer.

We have to use $K = \dfrac{Q \ln (r_2/r_1)}{\pi (h_2^2 - h_1^2)}$

As at $r_1 = 50$ m, $s_1 = 3$.

$$\therefore \quad h_1 = h_0 - s_1$$
$$= 25 - 3 = 22 \text{ m}$$

At $r_2 = 150$ m, $h_2 = 25 - 1.2 = 23.8$ m

$Q = 0.05$ m³/s $= 4320$ m³/d

$$\therefore K = \frac{Q}{\pi\left(h_2^2 - h_1^2\right)} \ln\left(\frac{r_2}{r_1}\right)$$

$$= \frac{4320}{\pi\left(23.8^2 - 22^2\right)} \ln\left(\frac{150}{50}\right)$$

$$= \underline{18.3 \text{ m/d}}$$

Radius of influence is evaluated by finding $r = r_0$ where $h = h_0$ still in steady conditions:

$$Q = \frac{\pi K \left(h_0^2 - h_1^2\right)}{\ln\left(r_0/r_1\right)}$$

i.e. $$\ln(r_0) - \ln(r_1) = \frac{\pi K}{Q}\left(h_0^2 - h_1^2\right)$$

$$\ln(r_0) = \ln(r_1) + \frac{\pi K}{Q}\left(h_0^2 - h_1^2\right)$$

or $$\ln\left(\frac{r_0}{r_1}\right) = \frac{\pi K}{Q}\left(h_0^2 - h_1^2\right)$$

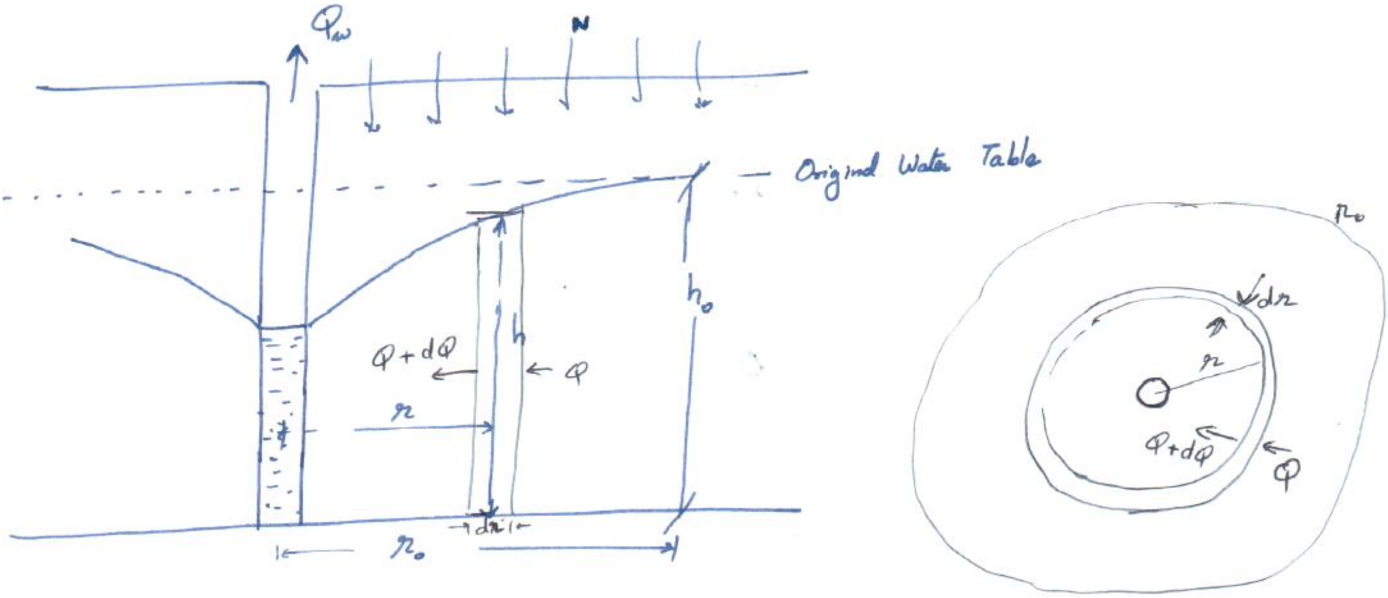
$$\frac{r_0}{r_1} = \exp\left[\frac{\pi K}{Q}\left(h_0^2 - h_1^2\right)\right]$$

$$r_0 = r_1 \exp\left[\frac{\pi K}{Q}\left(h_0^2 - h_1^2\right)\right]$$

$$= 50 \exp\left[\frac{\pi\, 18.3}{4320}\left(25^2 - 22^2\right)\right]$$

$$= 326.5 \approx \underline{327 \text{ m}}$$

Steady radial flow in Unconfined aquifer well with uniform recharge



⇒ For a steady discharge pumping of Q_w from an unconfined well, to analyse the radial flow into the well.
The aquifer region is recharged at a rate N (say $-N$ m/d).

⇒ Due to recharge, the discharge at any radial distance may not be same as Q_w, although steady state conditions prevail.

⇒ Let us assume a hollow cylinder of radius 'r' and thickness dr.
The recharge or addition of water in this cylinder concentric thickness 'dr' occurs.

∴ Addition of discharge $d\varphi$ occurs through this cylindrical strip. (Recharge ~~is~~ is given as $-N$)

$$d\varphi = -2\pi r\,dr(-N)$$

or $\varphi = \pi r^2 N + c$

At the well $r \to r_w$, $\varphi = \varphi_w$

$$\cancel{\varphi = -\pi r^2 N + \varphi_w}$$

$\varphi_w = \pi r_w^2 N + c$

$\varphi_w - \pi r_w^2 N = c$

Similarly at $r \to 0$, $\varphi = \varphi_w$ along the centre line of the well.

$$\varphi_w = \pi \times 0 + c \quad \text{or} \quad c = \varphi_w$$

$$\therefore \quad \varphi = \pi r^2 N + \varphi_w \quad \to (1)$$

or Equation for flow to the well:

$$\varphi = -2\pi r K h \frac{dh}{dr} \quad \to (2)$$

Comparing (1) & (2):

$$-2\pi r K h \frac{dh}{dr} = \pi r^2 N + \varphi_w$$

Integrating in the range h from h to h_0 and r from r to r_0, we get:

$$\cancel{h_0^2 - h_w^2 = \frac{-N}{2K}(r^2 - r_0^2)}$$

$$h\frac{dh}{dr} = \frac{-\pi r^2 N}{2\pi r K} - \frac{\varphi_w}{2\pi r K}$$

i.e. $$h \frac{dh}{dr} = -\frac{rN}{2K} - \frac{Q_w}{2\pi r K}$$

$$\text{or} \quad \int_{h}^{h_0} h\,dh = -\frac{1}{2K}\left[\int_{r}^{r_0} rN\,dr + \int_{r}^{r_0} \frac{Q_w}{\pi r}\,dr\right]$$

$$\text{or} \quad \frac{1}{2}\left[h_0^2 - h^2\right] = -\frac{1}{2K}\left[\frac{N}{2}\left(r_0^2 - r^2\right) + \frac{Q_w}{\pi} \ln\left(\frac{r_0}{r}\right)\right]$$

$$\text{or} \quad h_0^2 - h^2 = -\frac{N}{2K}\left(r_0^2 - r^2\right) - \frac{Q_w}{\pi K} \ln\left(\frac{r_0}{r}\right)$$

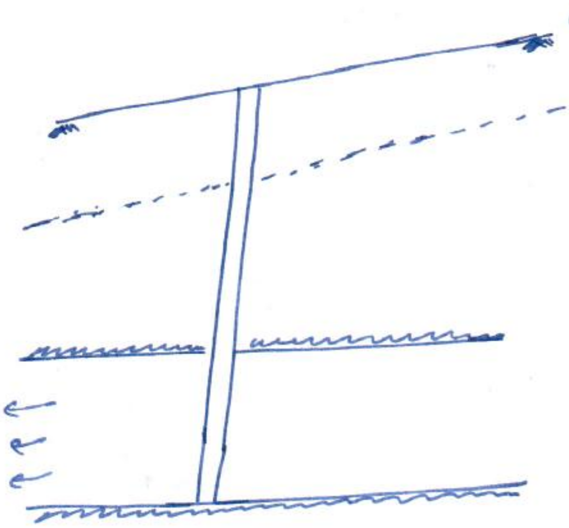
WELLS IN UNIFORM FLOW FIELD

Till now, the aquifer phreatic surface (for unconfined aquifers) and imaginary piezometric surface (in confined aquifers) were assumed as horizontal surfaces before the commencement of pumping.

That means that practically there is negligible flow in the aquifers (This is not possible).

The aquifers naturally have some sort of flow in subsurface according to the prevalent hydraulic gradient.

∴ Let us consider a confined aquifer that has uniform flow.



→ The imaginary piezometric surface has a gradient and it is assumed that this gradient is constant.
Therefore there will be uniform flow in the confined aquifer.