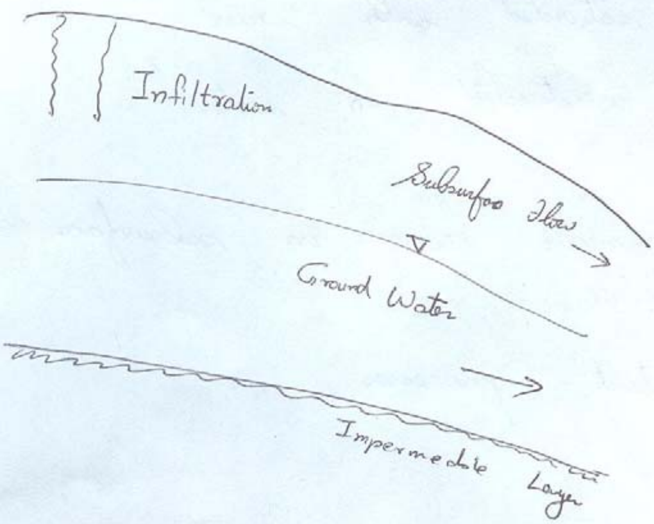
Lecture 9 **24-January-2013**

The Subsurface Water System



As told earlier, water that precipitates on surface of earth — part of that infiltrates into sub-surface of earth.

→ They further go deep to form groundwater.

The sub-surface soil are in general considered as porous media.

• → A porous media is a solid volume in space that consists of inter connected voids inside.

$$\text{Porosity}, \quad \varepsilon = \frac{\text{Volume of voids}}{\text{Total volume of porous media}}$$

Water Content, $\theta = \dfrac{\text{Volume of water}}{\text{Total volume of media}}$

→ Flow ~~is~~ can be saturated with water

→ Flow ~~is~~ can be unsaturated in water

Q: What are the main processes in subsurface water system?

A: Three important processes

INFILTRATION

↓

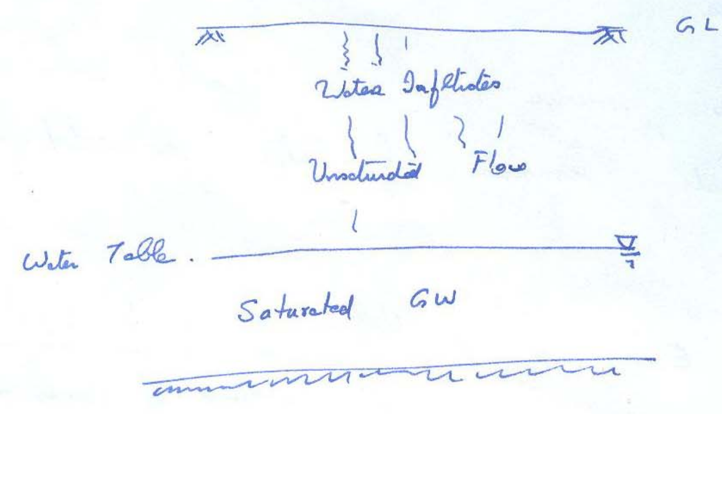
UNSATURATED FLOW

↓

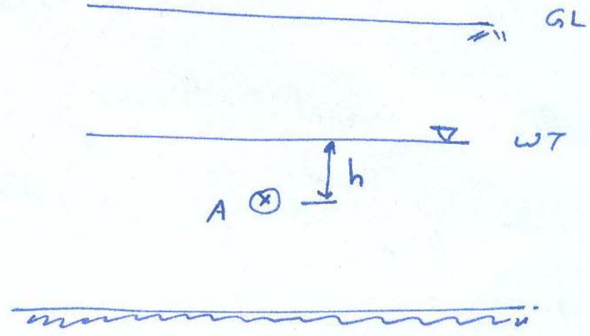
GROUNDWATER FLOW

Q: What is Water Table?

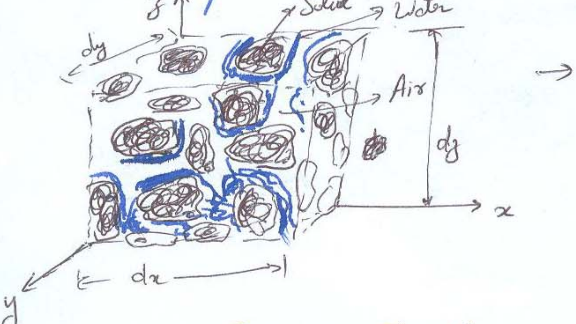
The surface or line where the water in a saturated porous medium is at atmospheric pressure.



Below water table pressure > atmospheric
(Usually hydrostatic i.e. $p = \rho g h$)



Q: Above water table pressure ? What happens?
→ The pressure will be less than atmospheric.



Let us apply continuity equation.

→ The control volume

Sides dx, dy, dz

$\therefore$ Volume $= dx\,dy\,dz$

→ Let moisture content $= \theta$

$\therefore$ Volume of water in porous media (CV)

$= \theta\, dx\,dy\,dz$

Darcy velocity is assumed to be obeyed for water flow in pores.

Darcy velocity, $q = \frac{\text{Discharge}}{\text{Area}}$ = Darcy Flux.

Using Reynold's transport theorem:

B = Mass of liquid water in porous soil

$\beta = 1$

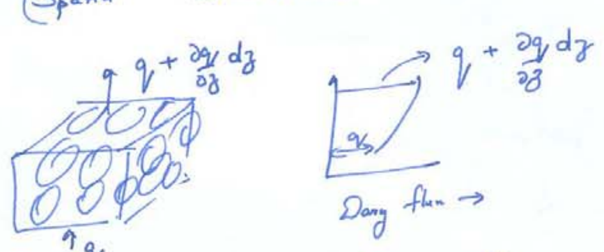
$$\frac{DB}{Dt} = 0 = \frac{d}{dt}\iiint_{cv} \rho_w \, dV + \iint_{cs} \rho_w \, \vec{v}\cdot\hat{n} \, dA$$

The time rate of change of mass of water stored in the control volume

$$\frac{d}{dt}\iiint_{cv} \rho_w \, dV = \frac{d}{dt}\left(\rho_w \, \theta \, dx\,dy\,dz\right)$$

$$= \rho_w \, dx\,dy\,dz \, \frac{\partial \theta}{\partial t}$$

(Spatial Dimensions fixed).



Let us consider that Darcy flux change w.r.t elevation (Horizontal flow let us neglect here).

At bottom $= q$

At top $= q + \frac{\partial q}{\partial z} dz$

$$\therefore \iint_{cs} \rho_w \, \vec{v}\cdot\hat{n} \, dA = \rho_w \left(q + \frac{\partial q}{\partial z} dz\right) dx\,dy - \rho_w \, q \, dx\,dy$$

$$= \rho_w \, dx\,dy\,dz \, \frac{\partial q}{\partial z}$$

$\therefore$ RTT becomes

$$0 = \rho_w \, dx\,dy\,dz \frac{\partial \theta}{\partial t} + \rho_w \, dx\,dy\,dz \frac{\partial q}{\partial z}$$

or $$\boxed{\frac{\partial \theta}{\partial t} + \frac{\partial q}{\partial z} = 0}$$

for incompressible water.

From Darcy's law you know

q = Darcy velocity $= -K i$

where i = hydraulic gradient

Total energy head, h = press head + velocity head + datum head

i.e. $$h = \frac{p}{\rho g} + \frac{v^2}{2} + z$$

$$\therefore i = \frac{\partial h}{\partial z}$$

or $$q = -K \frac{\partial h}{\partial z}$$; K = Hydraulic conductivity.

Q: Can you tell what is hydraulic head at water table.

Neglecting velocity head

$$h = \frac{p}{\rho g} + z$$

As $p = 0$, $\therefore$ $\underline{\underline{h = z}}$

Above water table → the soil particles such water or has potential to vigorously attract water. This will cause negative pressure (or less than atmospheric).

This suction quantity can be represented in terms of head ψ.

$$\therefore \quad h = \frac{p}{\rho g} + z$$

p → becomes negative $= \psi$

$$\text{or} \quad \psi = \frac{p}{\rho g}$$

$$\text{i.e.} \quad h = \psi + z$$

$$q = -K \frac{\partial}{\partial z}(\psi + z)$$

$$= -K \frac{\partial \psi}{\partial z} - K$$

$$= -K \frac{d\psi}{d\theta}\frac{\partial \theta}{\partial z} - K$$

$$= -\left(D \frac{\partial \theta}{\partial z} + K\right)$$

$$\therefore \quad \frac{\partial \theta}{\partial t} + \frac{\partial q}{\partial z} = 0$$

$$\equiv \quad \boxed{\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left(D \frac{\partial \theta}{\partial z} + K\right)}$$

→ This is 1 Dimensional Richards Equation

where D = soil water diffusivity.

(24/01/2013)

INFILTRATION

→ The process of water penetrating from the ground surface into the soil.

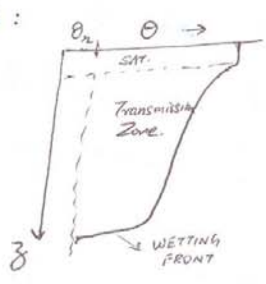
→ So this penetration (infiltration) can be assessed w.r.t time (How much water penetrates per unit time)

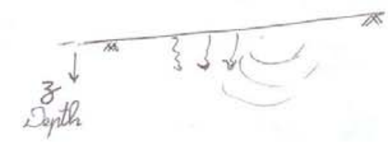
→ It is called infiltration rate.

→ Depends on
  ↳ Soil surface
  ↳ Vegetation
  ↳ Soil properties like porosity, permeability.

→ Due to variations in soil and soil properties with respect to horizontal and vertical dimensions, infiltration is highly complex.

Consider the distribution of water (or moisture) within a soil profile:

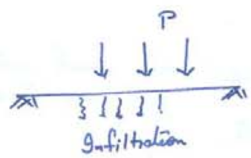




→ So the moisture content if you plot w.r.t depth, you will see
- ★ The topmost portion of soil is almost saturated.
- ★ Then there is transmission zone
- ★ There is a wetting front below. This wetting front advances below w.r.t time.

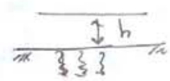
→ Infiltration rate f $[L\,T^{-1}]$
↖ Units

Q: So what happens when rain falls on surface of earth ?



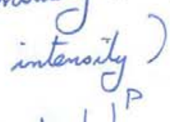
Q: How much quantity will be infiltrated ? How do you know ?

If the water that falls on surface gets collected and ponds to a certain depth



→ This means that the infiltration of water into the soil can occur at its maximum. Because there is enough water on the top that can be infiltrated. The rate at which it can infiltrate in such situation is <u>Potential Infiltration Rate</u>.

If the soil is some what dry initially. A meagre rainfall occurs (maybe with less intensity)
All the rain that falls can fully infiltrate.



That is the soil can infiltrate still more if rain fall intensity is more. In such circumstances infiltration occurs at less than potential infiltration rate.

Cumulative Infiltration (F)

→ The total accumulated depth of water infiltrated during a given time period.

$$F = \int_0^t f(\tau)\, d\tau \quad ; \quad f(\tau) \rightarrow \text{Infiltration rate.}$$

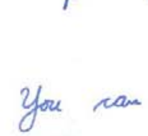
You can also say $f(t) = \frac{dF}{dt}$

→ The infiltration rate 'f' can change w.r.t time.

Horton's Infiltration

One of the preliminary expressions for infiltration.

→ Consider a dry soil. That is the voids in the soil porous media are devoid of water.

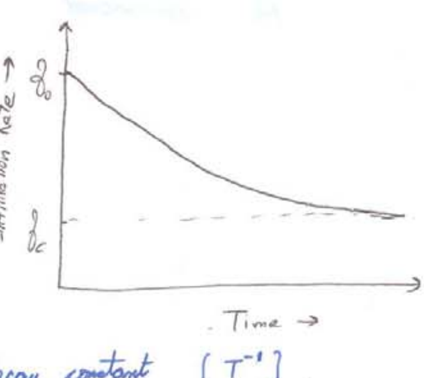


→ If you pour some volume of water, all of them may infiltrate. You can measure the time for that quantity to infiltrate

→ Again if you pour same volume of water on the same soil, it may take more time to infiltrate. That is infiltration rate decreases with time.

Horton suggested that:

If an infiltration begins at a rate f_0, it may exponentially decrease till it reach a constant f_c.



$$f(t) = f_c + \left(f_0 - f_c\right) e^{-kt}$$

$k \rightarrow$ decay constant $[T^{-1}]$.

If you recall Richards equation $\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left(D\frac{\partial \theta}{\partial z} + K\right)$

→ if we assume D and K are constants, then

$$\frac{\partial \theta}{\partial t} = D\frac{\partial^2 \theta}{\partial z^2}$$ (the standard diffusion equation).

→ Solving this you get $\theta(z,t)$.

→ Recall moisture diffusion $D\frac{\partial \theta}{\partial z}$. From $\theta(z,t)$ if you ~~obtain~~ solve $D\frac{\partial \theta}{\partial z}$ then it is as good as the Horton's equation. $D \rightarrow [L^2T^{-1}]$

Philip's Equation

Philip also solved Richard's equation to get expression for infiltration rate.

$$F(t) = St^{1/2} + Kt$$

$S \rightarrow$ sorptivity (a function of soil suction potential).

$K \rightarrow$ hydraulic conductivity.

$$\frac{dF}{dt} = f(t) = \frac{1}{2}St^{-1/2} + K$$

He considered K and D vary w.r.t θ.