CE 311

LECTURE 8

23-JAN-2013

The Evaporation Process

In the last class we suggested that there are two factors that influence evaporation.

(i) Supply of energy to provide latent heat of vaporisation

(ii) Ability to transport the formed vapor away from water surface.

⇒ The first factor we analysed using energy-balance method.

* Used continuity equation for water in liquid phase
* Continuity equation in vapor phase
* The energy conservation equation

$$\frac{dH}{dt} - \frac{dW}{dt} = \frac{d}{dt}\iiint_{cv}\left(e_u + \frac{v^2}{2} + gz\right)\rho_w\, dU + \iint_{cs}\left(e_u + \frac{v^2}{2} + gz\right)\rho_w\, \vec{v}\cdot\hat{n}\, dA$$

→ For the evaporation pan control volume

↳ $\frac{dW}{dt} = 0$ Rate of work done by the system

↳ The second term in RHS $\iint_{cs}\left(e_u + \frac{v^2}{2} + gz\right)\rho\, \vec{v}\cdot\hat{n}\, dA = 0$

↳ The rate of change of elevation $\frac{dz}{dt}$ is very small

$$\therefore \frac{d}{dt}\iiint_{cv}\left(e_u + \frac{v^2}{2} + gz\right)\rho_w\, dU \approx \frac{d}{dt}\iiint_{cv} e_u\, \rho_w\, dU$$

Let us consider a unit area of water surface

Net Radiation flux $= R_n$ (W/m^2)

Water supplies sensible heat flux to air stream $= H_s$

Water supplies ground heat flux to ground surface $= G$

$$\therefore \frac{dH}{dt} = R_n - H_s - G$$

⇒ At any instant of time let us assume temperature of water within the control volume → constant.

∴ Only change in heat stored within the control volume = Change in internal energy of water evaporated.

$$= l_v \dot{m}_v$$

l_v → Latent heat of vaporisation

$$\therefore E = \frac{1}{l_v \rho_w}\left(R_n - H_s - G\right)$$

⇒ The second factor for evaporation — the ability to transport vapor away from the surface.

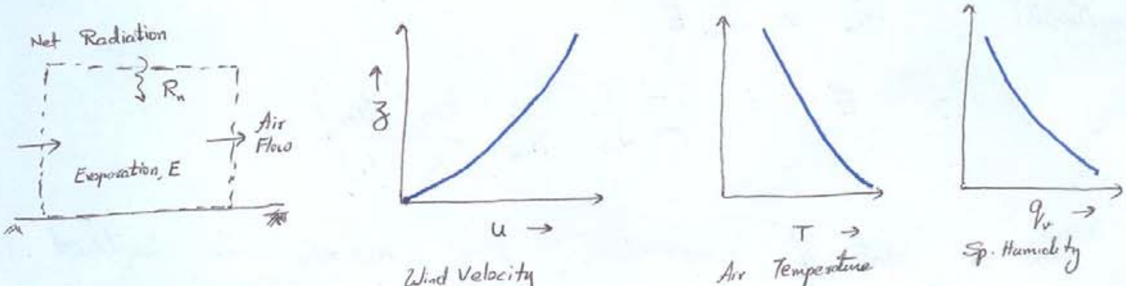
→ This has to be analyzed using aerodynamic method.

The Aerodynamic Method

→ The ability to transport vapor away from the surface depends on:

↳ Humidity gradient
↳ Wind speed

→ The transport equations for mass and momentum are to be included in the analyses here.



For a unit horizontal area:

The vapor mass flux moving upward by convection, $\dot{m}_v = -\rho_a K_w \frac{dq_v}{dz}$

K_w → vapor eddy diffusivity

→ The momentum flux through this horizontal area

$$\tau = \rho_a K_m \frac{du}{dz}$$

K_m → momentum diffusivity.

If the wind velocities and specific humidities are measured at elevations z_1 and z_2 then:

$$\frac{dq_v}{dz} \approx \frac{q_{v_2} - q_{v_1}}{z_2 - z_1} \quad \text{and} \quad \frac{du}{dz} \approx \frac{u_2 - u_1}{z_2 - z_1}$$

Now you can write

$$\dot{m}_v = - \tau \frac{K_w}{K_m} \frac{(q_{v_2} - q_{v_1})}{(u_2 - u_1)}$$

Recall $\dot{m}_v = \rho_w E$

$$\therefore \quad E = - \frac{\tau}{\rho_w} \frac{K_w}{K_m} \frac{(q_{v_2} - q_{v_1})}{(u_2 - u_1)}$$

This is rate of evaporation by aerodynamic method.

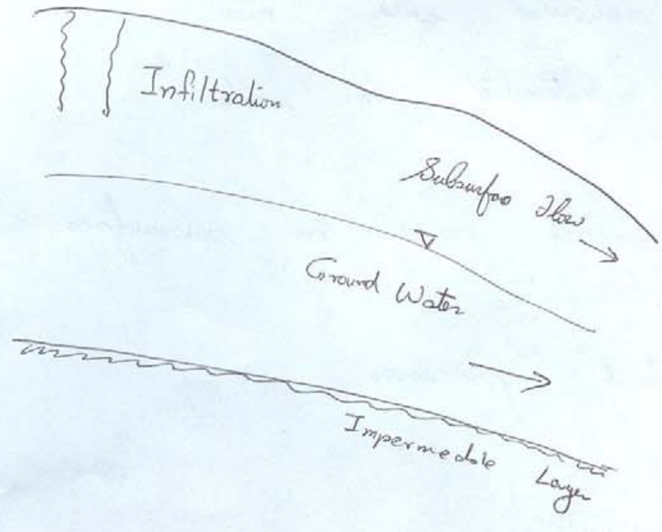
As both processes are involved you can consider $E_r \rightarrow$ due to radiation, $E_a \rightarrow$ due to transport

Combinedly $E = f(E_r) + g(E_a)$

Evapo-Transpiration

- Combination of evaporation from soil surface and beneath (to a certain depth) and transpiration from vegetation.
- The factors are similar as explained for evaporation.

SUB-SURFACE WATER



As told earlier, water that precipitates on surface of earth – part of that infiltrates into sub-surface of earth.

→ They further go deep to form groundwater.

The sub-surface soil are in general considered as porous media.

* → A porous media is a solid volume in space that consists of inter connected voids inside.

$$\text{Porosity}, \quad \varepsilon = \frac{\text{Volume of voids}}{\text{Total volume of porous media}}$$

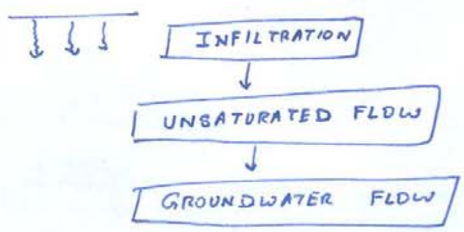
Water Content, $\theta = \dfrac{\text{Volume of water}}{\text{Total volume of media}}$

→ Flow can be saturated with water

→ Flow can be unsaturated in water

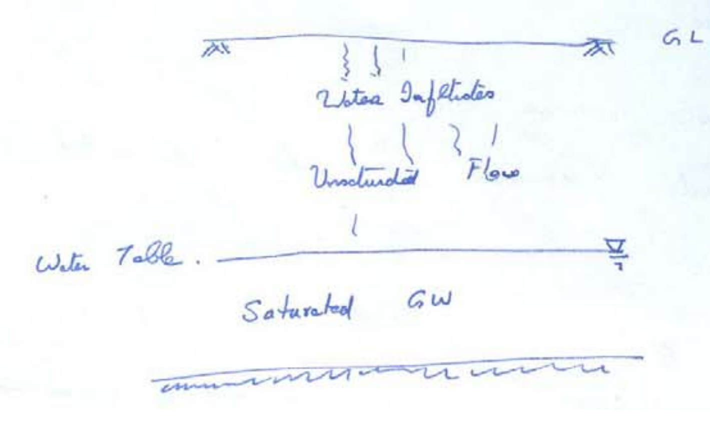
Q: What are the main processes in subsurface water system?

A: Three important processes

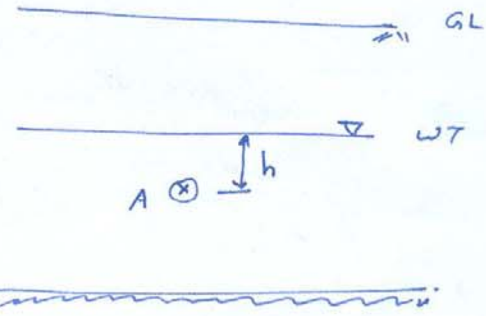


Q: What is Water Table?

The surface or line where the water in a saturated porous medium is at atmospheric pressure.



Below water table pressure > atmospheric
(Usually hydrostatic i.e. $p = \rho g h$)



Q: Above water table pressure ? What happens?
→ The pressure will be less than atmospheric.

Let us apply continuity equation.

→ The control volume

Sides dx, dy, dz

$\therefore$ Volume $= dx\, dy\, dz$

→ Let moisture content $= \theta$

$\therefore$ Volume of water in porous media (CV)

$= \theta\, dx\, dy\, dz$

Darcy velocity is assumed to be Beyond for water flow in pores.

Darcy velocity, $q = \dfrac{\text{Discharge}}{\text{Area}}$ = Darcy Flux.

Using Reynold's transport theorem:

B = Mass of liquid water in porous soil

$\beta = 1$

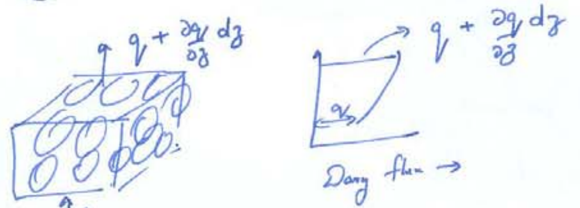
$$\frac{DB}{Dt} = 0 = \frac{d}{dt}\iiint_{cv} \rho_w \, dU + \iint_{cs} \rho_w \, \vec{v}\cdot\hat{n} \, dA$$

The time rate of change of mass of water stored in the control volume

$$\frac{d}{dt}\iiint_{cv} \rho_w \, dU = \frac{d}{dt}\left(\rho_w \, \theta \, dx\,dy\,dz\right)$$

$$= \rho_w \, dx\,dy\,dz \, \frac{\partial \theta}{\partial t}$$

(Spatial Dimensions fixed).



Let us consider that Darcy flux change w.r.t elevation (Horizontal flow let us neglect here).

At bottom $= q$

At top $= q + \dfrac{\partial q}{\partial z} dz$

$$\therefore \iint_{cs} \rho_w \, \vec{v}\cdot\hat{n} \, dA = \rho_w\left(q + \frac{\partial q}{\partial z} dz\right) dx\,dy - \rho_w \, q \, dx\,dy$$

$$= \rho_w \, dx\,dy\,dz \, \frac{\partial q}{\partial z}$$

$\therefore$ RTT Becomes

$$0 = \rho_w \, dx\,dy\,dz \, \frac{\partial \theta}{\partial t} + \rho_w \, dx\,dy\,dz \, \frac{\partial q}{\partial z}$$

or $$\boxed{\frac{\partial \theta}{\partial t} + \frac{\partial q}{\partial z} = 0}$$

for incompressible water.

From Darcy's law you know

q = Darcy velocity $= -K\,i$

where i = hydraulic gradient

Total energy head, h = press head + velocity head + datum head

i.e. $$h = \frac{p}{\rho g} + \frac{v^2}{2} + z$$

$$\therefore i = \frac{\partial h}{\partial z}$$

or $$q = -K \frac{\partial h}{\partial z}$$; K = Hydraulic conductivity.

Q: Can you tell what is hydraulic head at water table.

Neglecting velocity head

$$h = \frac{p}{\rho g} + z$$

As $p = 0$, $\therefore$ $\underline{\underline{h = z}}$

Above water table → the soil particles such water or has potential to vigorously attract water. This will cause negative pressures (or less than atmospheric).

This suction quantity can be represented in terms of head ψ.

$$\therefore \quad h = \frac{p}{\rho g} + z$$

$p \rightarrow$ becomes negative $= \psi$

$$\text{or} \quad \psi = \frac{p}{\rho g}$$

$$\text{i.e.} \quad h = \psi + z$$

$$q = -K \frac{\partial}{\partial z}\left(\psi + z\right)$$

$$= -K \frac{\partial \psi}{\partial z} - K$$

$$= -K \frac{d\psi}{d\theta}\frac{\partial \theta}{\partial z} - K$$

$$= -\left(D \frac{\partial \theta}{\partial z} + K\right)$$

$$\therefore \quad \frac{\partial \theta}{\partial t} + \frac{\partial q}{\partial z} = 0$$

$$\equiv \boxed{\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left(D \frac{\partial \theta}{\partial z} + K\right)}$$

→ This is 1 Dimensional Richards Equation

where D = soil water diffusivity.