CE 311 HWRE

LECTURE 7
16 - JAN - 2013

EVAPORATION

In the last few classes, we discussed on atmospheric water system.

The next important atmospheric process for hydrology is Evaporation.

In an open water surface, there are two factors that influence evaporation.

(i) Supply of energy to provide latent heat of vaporisation.

(ii) Ability to transport the formed vapor away from water surface.

Q: What is the source of energy?

→ Solar radiation

Q. What are the factors for transport of vapor?

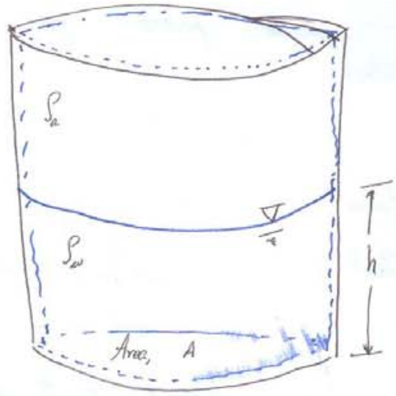
→ The wind velocity in the region

→ Specific humidity gradient in the air

* If someone is interested to analyse evaporation, the foremost objective is to determine the evaporation rate E. (How?)

Energy Balance Method

Consider an evaporation pan here for analysis.



→ An evaporation pan is a circular tank containing water.

→ Rate of evaporation is inferred from rate of fall of water surface.

Consider the continuity equation for water in liquid phase :

→ The extensive property $= B =$ Mass of liquid water

$$= \rho_w A h$$

Now the material derivative $\frac{DB}{Dt} = -\dot{m}_v$

(Can you guess why ?)

$$\therefore \quad -\dot{m}_v = \frac{d}{dt}\iiint_{cv} \rho_w \, dU + \iint_{cs} \rho_w \vec{v}\cdot\hat{n} \, dA$$

In the liquid state there is no flow of liquid along the control surfaces.

$$\therefore \quad \iint_{cs} \rho_w \, \vec{v}\cdot\hat{n} \, dA = 0$$

Now $$\frac{d}{dt}\iiint_{cv} \rho_w \, dU = \frac{d}{dt}\left(\rho_w A h\right)$$

$$= \rho_w A \frac{dh}{dt}$$

We can correlate the rate of fall of water in the evaporation pan w.r.t the evaporation rate.

i.e. $$E = -\frac{dh}{dt}$$

$\therefore$ The continuity eqn. becomes $-\dot{m}_v = -\rho_w A E$

or $$\underline{\underline{\dot{m}_v = \rho_w AE}}$$

Let us consider <u>Continuity equation in vapor phase</u>:

B = mass of water vapor

$\beta = q_v$

$$\frac{DB}{Dt} = \dot{m}_v = \frac{d}{dt}\iiint_{cv} q_v \rho_a \, dU + \iint_{cs} q_v \rho_a \, \vec{v}\cdot\hat{n} \, dA$$

For a steady flow of air over the evaporation pan, the time derivative of water vapor stored in the cv is zero

i.e. $$\frac{d}{dt}\iiint_{cv} q_v \rho_a \, dU = 0$$

$$\therefore \quad \dot{m}_v = \iint_{cs} q_v \rho_a \, \vec{v}\cdot\hat{n} \, dA$$

i.e. We know $\mathring{m}_v = \rho_w A E$

$$\therefore \quad E = \frac{1}{\rho_w A} \iint_{cs} q_v \rho_a \vec{v}\cdot\hat{n}\, dA$$

The energy conservation equation:

Recall the heat energy balance in a CV:

$$\frac{dH}{dt} - \frac{dW}{dt} = \frac{d}{dt}\iiint_{cv}\left(e_u + \frac{v^2}{2} + gz\right)\rho\, dU + \iint_{cs}\left(e_u + \frac{v^2}{2} + gz\right)\rho\vec{v}\cdot\hat{n}\, dA$$

where $\frac{dH}{dt}$ = rate of heat input to the system from external sources.

$\frac{dW}{dt}$ = rate of work done by the system on its surroundings.

e_u = specific internal heat energy

Now this equation can be applied for water in the evaporation pan.

In the evaporation pan:

$v = 0$, $\frac{dW}{dt} = 0$, assuming change in elevation w.r.t time is negligible.

i.e. $\frac{dz}{dt} \approx 0$.

→ The net outflow of heat energy across the control surface with the flowing water $= 0$. (Why?)

$$\therefore \quad \frac{dH}{dt} = \frac{d}{dt} \iiint_{cv} c_u \rho_w \, dU$$

$\frac{dH}{dt}$ → function of input radiation, the reflected ground heat and the water heat fluxes.

For a unit area of water surface the source of heat:

→ Net radiation flux R_n (W/m²)

→ Water supplied heat flux to air stream, H_s

→ Ground heat flux, G , $\therefore \frac{dH}{dt} = R_n - H_s - G$

Assume temperature of water within the CV is constant at any particular instant.

∴ Change in the heat energy stored in CV

= Change in internal energy of water evaporated.

$= l_v \dot{m}_v$ $\qquad$ l_v → latent heat of vaporisation.

$$\therefore \quad R_n - H_s - G = l_v \dot{m}_v$$

(Again note that this is a relation for a unit area of water surface).

Now $\quad \dot{m}_v = \rho_w AE = \rho_w E \quad$ (for unit area)

$$\therefore \quad R_n - H_s - G = l_v \rho_w E$$

$$\text{or} \quad E = \frac{1}{\rho_w l_v} \left(R_n - H_s - G \right)$$

If sensible heat flux $H_s \approx 0.0$ and ground heat flux $G \approx 0.0$, then

$$E = \frac{R_n}{l_v \rho_w}$$

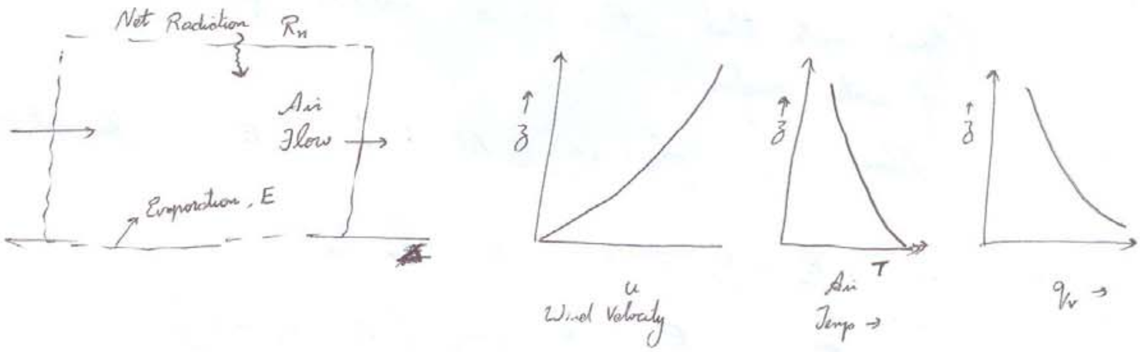
This is the energy Balance method for evaluating evaporation rate.

Aerodynamic Method

→ This is Based on the second factor for evaporation
  i.e. Ability to transport vapor away from the surface.

→ This depends on :
  ↳ humidity gradient
  ↳ Wind speed

→ One need to use equations for mass and momentum transport in air.



For a unit horizontal area:
The vapor mass flux upward by convection is $\dot{m}_v$

$$\dot{m}_v = -\rho_a K_w \frac{dq_v}{dz}$$

where $K_w \rightarrow$ vapor eddy diffusivity.

The vapor momentum flux upward is:

$$\tau = \rho_a K_m \frac{du}{dz} \quad ; \quad K_m \rightarrow \text{Momentum diffusivity.}$$

The quantities $\frac{dq_v}{dz}$ and $\frac{du}{dz}$ can be approximated with available values at two elevations

$$\therefore \frac{\dot{m}_v}{\tau} = -\frac{K_w (q_{v_2} - q_{v_1})}{K_m (u_2 - u_1)}$$

$\dot{m}_v = \rho_w E_a \times 1$, where $E_a \rightarrow$ evaporation rate by aerodynamic method.

You can rearrange terms to obtain E_a.

$\Rightarrow$ There is combined method $E = f(E_r) + g(E_a)$.

EVAPO - TRANSPIRATION

$\rightarrow$ Combination of evaporation from the soil surface and transpiration from vegetation.
Similar factors govern.

So we discussed in this chapter → The Atmospheric Water System.

→ The properties of water vapor discussed.

→ The atmospheric processes – precipitation and evaporation discussed and analysed using CV approach.

Now we will briefly cover a part of next sub-system, i.e. Subsurface Water System.