LECTURE 5
11-JAN-2013

PRECIPITATION

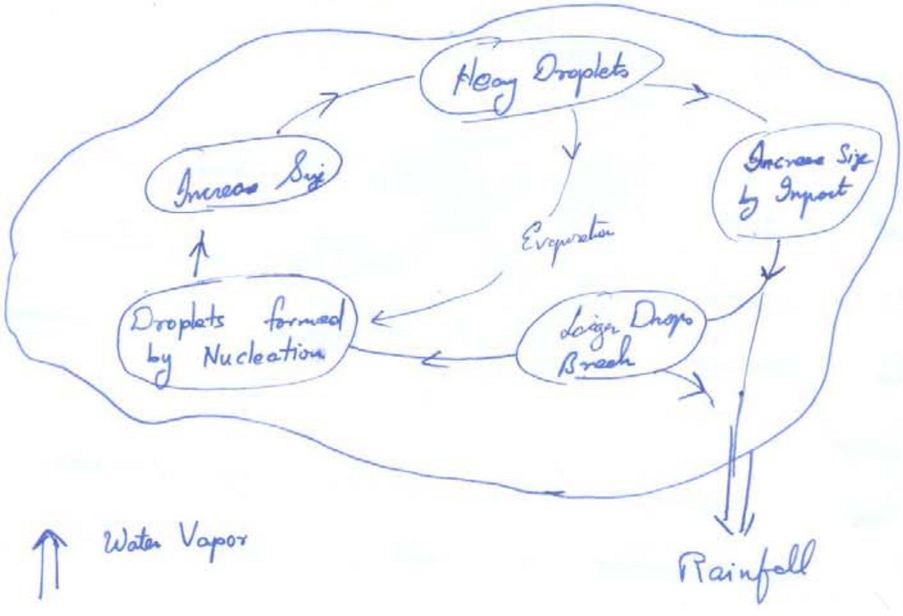
Formation of precipitation - requires lifting of air mass.

→ Frontal air lifting
→ Orographic air lifting
→ Convective air lifting

* In Frontal Lifting → warm air is lifted over cooler air by frontal passage.
* Orographic Lifting → air rises to pass over a mountain range
* Convective Lifting → Air mass is drawn up by convective action.

→ Moist air rises, by the above process.
→ It cools in the upper elevations
→ On cooling water condenses from vapor to liquid.
→ These condensations occur in presence of condensation nuclei. (dust particles, ions, etc.)

→ The tiny droplets formed by condensation grows.
→ Impacts its neighbours - cause turbulent air motion
→ As size of droplets becomes large, it begin to fall due to gravity.
→ As the drop falls, water evaporates from its surface (drop size reduces) - the drops rises.
→ The process will go on
→ This is witnessed in clouds.
→ In the process some of the water drops gain more size and will break the cloud barrier.
→ It will fall as rain



* Once a water drop falls as rain it has to move through atmosphere with some velocity.

The <u>Terminal Velocity of Rain Drop</u>

(Assume Spherical Drop)

Forces acting on the raindrop :

→ Gravity force F_g

→ Buoyancy force F_b

(This is due to displacement of air by the water drop)

→ Drag force F_d

(Due to friction b/w drop & surrounding)

Let us assume → diameter of drop = D

$$\therefore F_g = \rho_w g \frac{\pi}{6} D^3 \quad \downarrow$$

$$F_b = \rho_a g \frac{\pi}{6} D^3 \quad \uparrow$$

$$F_d = C_d \rho_a A \frac{v^2}{2} \quad \uparrow$$

C_d → dimensionless drag coefficient

A = circular area for drag = $\frac{\pi}{4} D^2$

v = drop fall velocity.

* When the drop is released from the cloud it starts from rest (assumed).
  → It will accelerate through atmosphere till it reach terminal velocity (V_t).

Q: What is the peculiarity at terminal velocity?

A: The water droplet is in dynamic equilibrium.
The net force acting on it = 0.

i.e $$F_g = F_d + F_b$$

or $$F_d = F_g - F_b$$

$$C_d \rho_a D^2 \frac{\pi}{4} \frac{V_t^2}{2} = \rho_w g \frac{\pi}{6} D^3 - \rho_a g \frac{\pi}{6} D^3$$

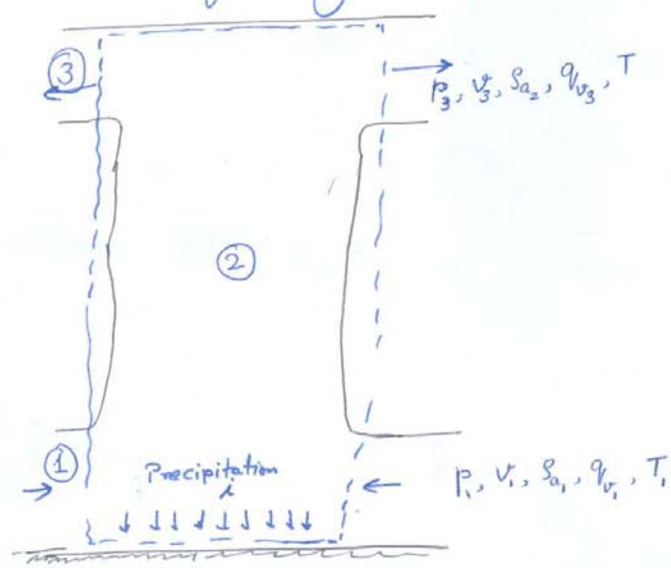
$$\therefore V_t = \sqrt{\frac{4 g D}{3 C_d}\left(\frac{\rho_w}{\rho_a} - 1\right)}$$

⇒ So the atmospheric process we described now involves lifting of air mass and precipitation.

The Thunderstorm Cell Model

In thunderstorm cell → convective air mass lifting occurs.

⇒ Consider a vertical column of air with following regions
→ Inflow region
→ Uplift "
→ Outflow region



What happen?

→ Warm moist air drawn into cell in inflow region

→ Warm air rises in the uplift region and cools and condenses.

→ In the outflow region, cooler dryer air outflows.

→ This outflow dry air may move to large area gathers moisture and may re-enter the thunderstorm cell through inflow region.

→ This will be convective cell circulation.

⇒ We can analyse thunderstorm cell using continuity equation for water vapor.

$$\mathring{m}_v = \frac{d}{dt} \iiint_{cv} q_v \rho_a \, dU + \iint_{cs} q_v \rho_a \, \vec{v} \cdot \hat{n} \, dA$$

→ Now if the precipitation intensity is "i" on the area at the bottom of thunderstorm cell, you can write

$$\mathring{m}_v = -\rho_w \, i \, A .$$

→ Let us assume a steady state condition.

Then

$$-\rho_w \, i \, A = \iint_{③} q_v \rho_a \, \vec{v} \cdot \hat{n} \, dA - \iint_{①} q_v \rho_a \, \vec{v} \cdot \hat{n} \, dA$$

Assuming cell diameter $= D$ and cylindrical shape.

Inflow region height $= \Delta z_1$

Outflow region height $= \Delta z_3$

$$\therefore \rho_w \, i \, A = (q_v \rho_a v)_1 \, \pi D \, \Delta z_1 - (q_v \rho_a v)_3 \, \pi D \, \Delta z_3$$

You know dry air + Water Vapor = Moist Air

$$\rho_d + \rho_v = \rho_a$$

or $\rho_d = \rho_a (1 - q_v)$

⇒ One can think of continuity equation for dry air

→ Dry air is not added anywhere within the control volume

$$\therefore \frac{D m_d}{Dt} = 0$$

As the mechanisms are in steady state:

$$0 = \iint_{cs} \rho_d \, \vec{v} \cdot \hat{n} \, dA$$

i.e.

$$0 = \left[\rho_a (1 - q_v) \, v \, \Delta z\right]_3 \pi D - \left[\rho_a (1 - q_v) \, v \, \Delta z\right]_1 \pi D$$

$$\therefore \left[\rho_a \, v \, \Delta z\right]_3 = \left[\rho_a \, v \, \Delta z\right]_1 \frac{(1 - q_{v_1})}{(1 - q_{v_3})}$$

Substitute in the equation for ρ_w i A to get expression for precipitation intensity.

$$i = \frac{4\, \rho_{a_1} V_1 \Delta z_1}{\rho_w D} \left(\frac{q_{v_1} - q_{v_3}}{1 - q_{v_3}} \right)$$

⇒ Now you have seen how precipitation occurs in a thunderstorm cell.

⇒ Precipitations vary in space and time.

⇒ One particular type of precipitation is Rainfall

→ We will briefly cover some topics on rainfall.

Rainfall

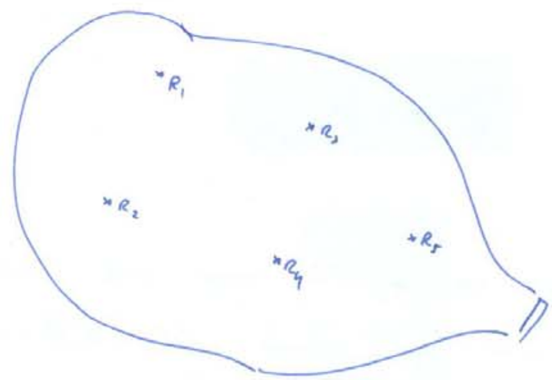
Isohyetal Maps : Maps having contours of constant rainfall.

Isohyets showing rainfall patterns. (Not Actual)

Q. How will you prepare isohyetal maps ?

→ In a watershed or geographical regions, rain gages may be established at various locations.

→



For a particular rainfall event, the amount of rainfall recorded in various gages are taken.

→ The rainfall of other locations are subsequently interpolated.

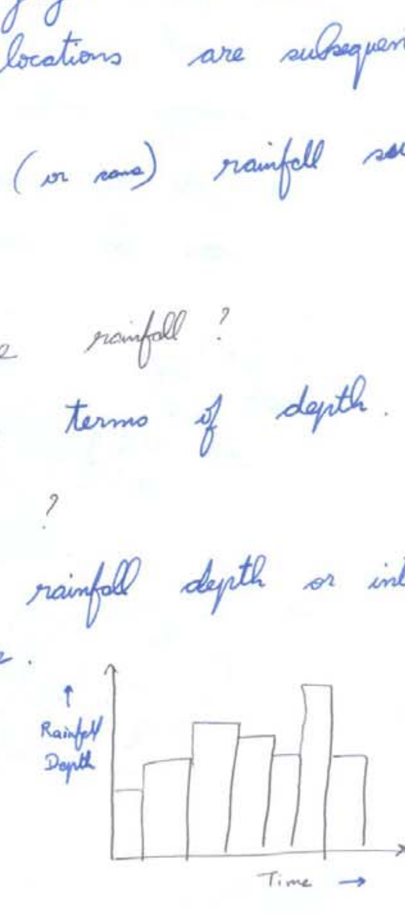
→ The contours of constant (or same) rainfall subsequently plotted.

Q. How do rain gages measure rainfall ?

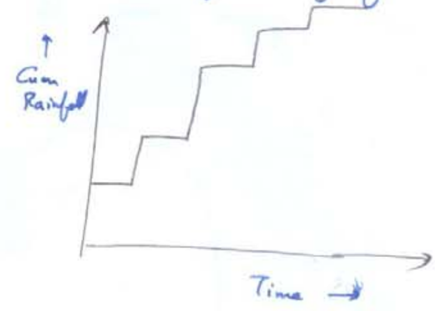
→ Measures rainfall in terms of depth.

Q. What is rainfall hyetograph ?

→ This is plot of rainfall depth or intensity as a function of time.



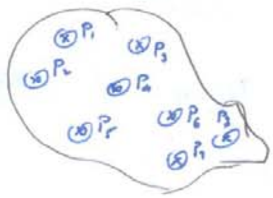
→ You can also plot cumulative rainfall hyetograph.



→ As you have seen rain gages are installed at various places and you may get different depths of rainfall.

→ If you want to lump the entire rainfall event in a watershed, you need to take average values of rainfall.

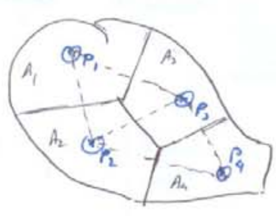
→ So the areal distribution of rainfall can be averaged.



$$\bar{P} = \frac{P_1 + P_2 + P_3 + \cdots + P_N}{N}$$

← Arithmetic mean method.

→

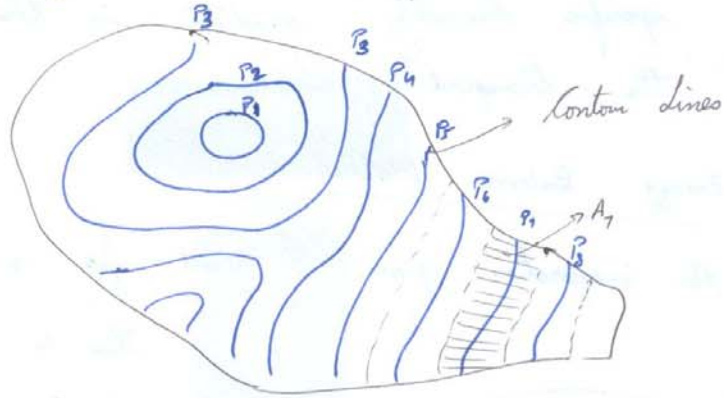


Using theissen polygon method

$$\bar{P} = \frac{P_1 A_1 + P_2 A_2 + \cdots + P_N A_N}{A}$$

← Theissen method

→ You can also find average areal rainfall by isohyetal method.



→ The average rainfall is obtained by ~~finding~~ utilizing the area under each isohyet.

$$\bar{P} = \frac{P_1 A_1 + P_2 A_2 + \cdots + P_N A_N}{A}$$

⇒ The next important atmospheric process of interest is Evaporation.

EVAPORATION

There are two main factors that influence evaporation from an open water surface.

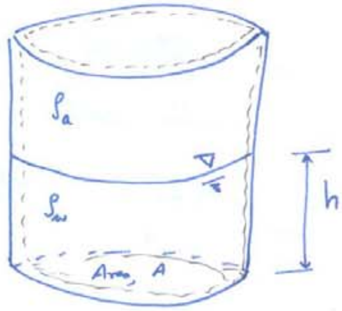
(i) Supply of energy to provide latent heat of vaporisation.

(ii) Ability to transport the formed vapor away from water surface.

→ The source of energy is solar. radiation.
→ The wind velocity in the region ~~decides~~ and specific humidity gradient in the air determines the tranport mechanisms.

Energy Balance Method

An evaporation pan is used for analysis.



→ This is a circular tank containing water.
→ The rate of evaporation is measured by inferring from rate of fall of water surface

The continuity equation for liquid phase described by RTT.

Extensive property = B = Mass of liquid water
$= \rho_w A h$

$$\frac{DB}{Dt} = -\dot{m}_v \quad (\text{Why?})$$

$$\therefore \quad -\dot{m}_v = \frac{d}{dt}\iiint_{cv} \rho_w \, dU + \iint_{cs} \rho_w \vec{V}\cdot\hat{n}\, dA$$

In the liquid state, there is no flow of liquid ~~and vapor~~ along the control surfaces.

$$\therefore \iint_{cs} \rho_w \, \vec{v} \cdot \hat{n} \, dA = 0$$

Now $\quad \dfrac{d}{dt} \iiint_{cv} \rho_w \, dU = \rho_w A \dfrac{dh}{dt}$

$$\therefore \; -\mathring{m}_v = \rho_w A \frac{dh}{dt}$$

Now we can define the term $\quad E = -\dfrac{dh}{dt}$

→ evaporation rate

$$\therefore \; \mathring{m}_v = \rho_w A E$$

<u>The continuity eqn. in vapor phase</u>

B = mass of water vapor

$\beta = q_v$

$\dfrac{DB}{Dt} = \mathring{m}_v$

$$\therefore \; \mathring{m}_v = \frac{d}{dt} \iiint_{cv} q_v \rho_a \, dU + \iint_{cs} q_v \rho_a \, \vec{v} \cdot \hat{n} \, dA$$

→ For a steady flow of air over the evaporation pan, the time derivative of water vapor stored in CV is zero.

$$\frac{d}{dt} \iiint_{cv} q_v \rho_a \, dU = 0$$

$$\therefore \; \mathring{m}_v = \iint_{cs} q_v \rho_a \, \vec{v} \cdot \hat{n} \, dA$$

$$\rho_w A E = \iint_{cs} q_v \rho_a \, \vec{v} \cdot \hat{n} \, dA$$