LECTURE 4
10-JAN-2013

ATMOSPHERIC WATER (PART II)

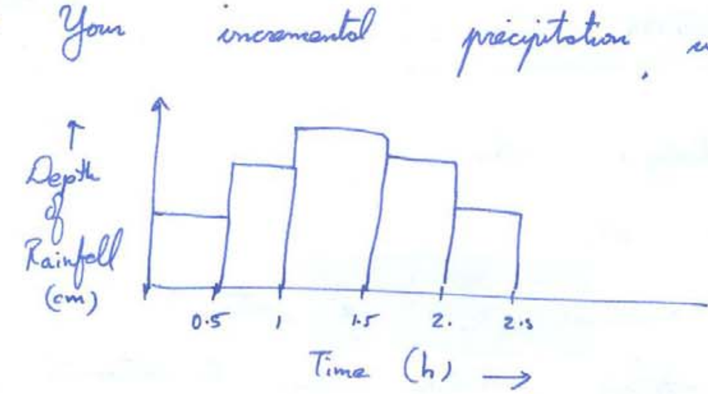
Recall yesterday's class
We discussed on:

→ Residence time of water
→ Applying continuity eqn. to determine storage
→ introduced briefly atmospheric circulation
→ RTT for water vapor in a control volume of moist air.

$$\mathring{m}_v = \frac{d}{dt}\iiint_{cv} q_v \rho_a \, dU + \iint_{cs} q_v \rho_a \, \vec{V}\cdot\hat{n} \, dA$$

I would just like to inform related to yesterday's topic:
→ Referring various literatures and internet it is observed that the term Incremental Precipitation is used to denote the amount of rainfall that is recieved during certain interval of time. In the problem the duration is ½ hour.
→ We misinterpreted the term with Cumulative Precipitation.
→ The sample problem is now worked out for the same watershed area and will be uploaded in MOODLE along with today's lecture.

→ Your incremental precipitation will look like



etc.

* Coming back into the discussion on atmospheric water:

For an extensive property as mass of water vapor in a moist air (m_v),

→ The time rate of change of water vapor in the moist air system is

$$\dot{m}_v = \frac{d}{dt}\iiint_{cv} q_v \rho_a \, dV + \iint_{cs} q_v \rho_a \, \vec{V}\cdot\hat{n} \, dA$$

<u>Vapor Pressure</u>

~~From ideal gas law:~~

Dalton's law of partial pressures

→ Pressures exerted by a gas is independent of pressures exerted by other gases.

From ideal gas law:

Water vapor pressure, $e = \rho_v R_v T$

$T \rightarrow$ absolute temperature of moist air (K)

$R_v \rightarrow$ gas constant for water vapor.

Let us consider the total pressure exerted by the moist air in the cv $= p$

By inference, $\therefore$ pressure of dry air $= p - e$

Now, we have $R_d \rightarrow$ gas constant for dry air $= 287 \; (\mathrm{J/kg\,K})$

If $\rho_d \rightarrow$ density of dry air in the cv, then

$$p - e = \rho_d R_d T \quad \text{(gas law)}$$

Also it is ~~clean~~ clear that $\rho_a = \rho_v + \rho_d$

→ From previous studies, it is observed that

* Ratio of molecular weight of water vapor to the average molecular weight of dry air is ≈ 0.622

Again, from literature it is ~~you can~~ inferred $R_v = \dfrac{R_d}{0.622}$

$$\therefore \quad p = \rho_d R_d T + \rho_v \frac{R_d}{0.622} T$$

$$= R_d T \left(\rho_d + \frac{\rho_v}{0.622} \right)$$

$\Rightarrow$ We know specific humidity $q_v = \frac{\rho_v}{\rho_a}$

Thus sp. humidity $q_v \approx 0.622 \frac{e}{p}$

$\Rightarrow$ One can also find relationships b/w gas constants of moist air and dry air.

Saturation Vapor Pressure

For a given air temperature, the moist air can hold only a certain maximum of water vapor in it. The ~~partial pressure of water vapor~~ moist air in such situation are called to be saturated with water vapor.

$\rightarrow$ The partial pressure of water vapor in such saturated conditions are called <u>Saturation Vapor Pressure</u>.

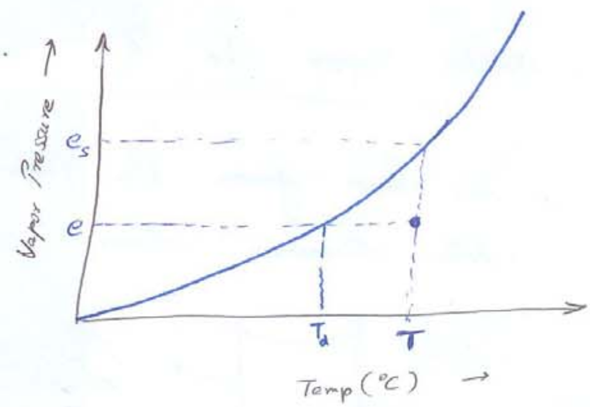
Q: What is the peculiarity in saturated moist air

A: The rate of evaporation and condensation are equal in such situations.

Raudkivi (1979) has related saturated vapor pressure with temperature and is widely used in atmospheric sciences.

$$e_s = 611 \exp\left[\frac{17.27\,T}{237.3 + T}\right]$$

where T is in °C.



→ From the plot it is clear that the gradient of e_s w.r.t temperature is not a straight line.

$$\frac{de_s}{dT} = \frac{4098\, e_s}{(237.3 + T)^2}$$

⇒ You can define a term called Relative Humidity.

R_h → Ratio of actual vapor pressure to its saturation value at a given air temp.

$$R_h = \frac{e}{e_s}$$

⇒ ~~For the same~~ You know $q_v \approx \frac{e}{p} \times 0.622$

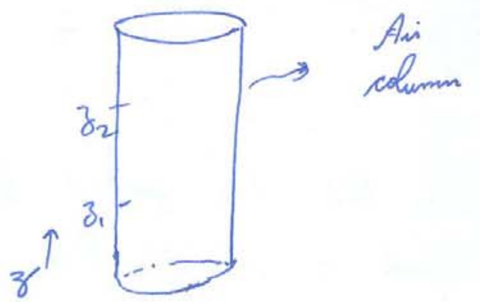
∴ For a given water vapor pressure e, you can have some specific humidity.

For this specific humidity to be ~~contain~~ in saturated conditions, you can lower the temperature (or increase)

→ The temperature at which the moist air becomes saturated is called dew-point temperature (T_d).

Water Vapor In A Static Atmospheric Column

Two laws govern the properties of water vapor in a static atmospheric column.

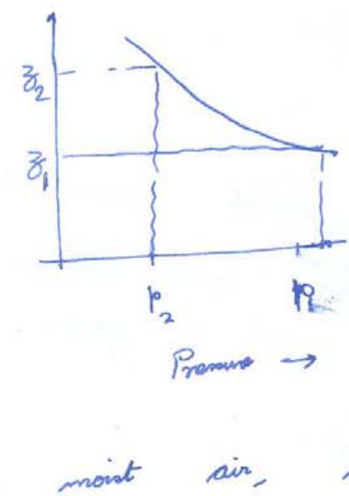


(i) Ideal gas law

$$p = \rho_a R_a T$$

(ii) Hydrostatic pressure law

$$\frac{dp}{dz} = -\rho_a g$$



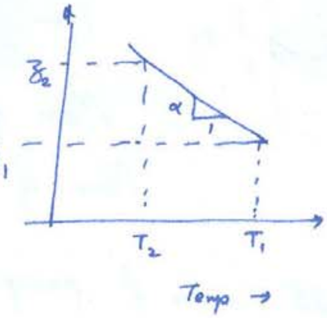
In moist air, density and specific humidity also vary non-linearly with altitude.

You know $\rho_a = \dfrac{p}{R_a T}$

Temperature in air column varies with altitude.

$$\frac{dT}{dz} = -\alpha$$

where $\alpha \rightarrow$ lapse rate (K/m) ...



Now you already have

$$\frac{dp}{dz} = -\rho_a g$$

$$\rho_a = \frac{p}{R_a T}$$

$$\therefore \frac{dp}{dz} = \frac{-pg}{R_a T}$$

(Using approximation like $dz = -\frac{dT}{\alpha}$)

$$\alpha \frac{dp}{dT} = \frac{pg}{R_a T}$$

or $$\frac{dp}{p} = \frac{g}{\alpha R_a} \cdot \frac{dT}{T}$$

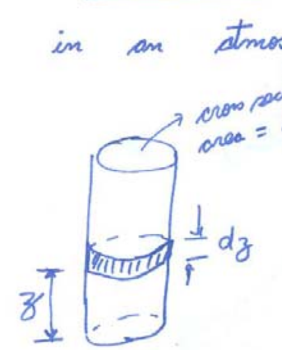
Integrating within the limits

$$\ln\left(\frac{p_2}{p_1}\right) = \frac{g}{\alpha R_a} \ln\left(\frac{T_2}{T_1}\right)$$

or $$p_2 = p_1 \left(\frac{T_2}{T_1}\right)^{g/\alpha R_a}$$

Also $$T_2 = T_1 - \alpha(z_2 - z_1)$$

Precipitable Water :→ The amount of water (or moisture) in an atmospheric column is called its precipitable water.



Consider an elemental strip of height = dz (in the air column).

Mass of air in the element = $\rho_a A \, dz$

Mass of water " " = $q_v \rho_a A \, dz$

Total mass of precipitable water between elevations z_1 & z_2

$$m_p = \int_{z_1}^{z_2} q_v \rho_a A \, dz$$

This is how you measure precipitable water in a moist air column.

Let us consider the atmospheric process - Precipitation now for discussion

Precipitation

All processes by which atmospheric water falls to the earth's surface is called precipitation.

- → Rainfall
- → Snowfall
- → Hail
- → Sleet