LECTURE 32

03-APRIL-2013

Kinematic Wave Routing

For solving kinematic wave equation, in the last class we have seen a particular finite-difference method to give an explicit solution on routing

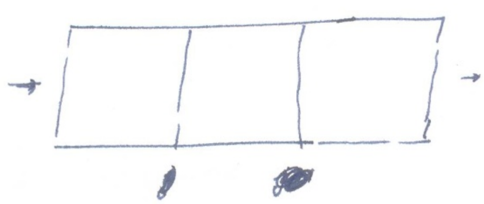
i.e. $$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q$$

$$S_0 = S_f$$

$$Q_i^{(s+1)} = Q_i^{(s)} + \frac{q_i^{(s)}}{\alpha\beta\left(Q_i^{(s)}\right)^{\beta-1}}\Delta t - \frac{\Delta t}{\Delta x}\frac{1}{\alpha\beta\left(Q_i^{(s)}\right)^{\beta-1}}\left[Q_{i+1}^{(s)} - Q_i^{(s)}\right]$$

The Muskingum-Cunge Method

We have seen Muskingum method for lumped flow routing.

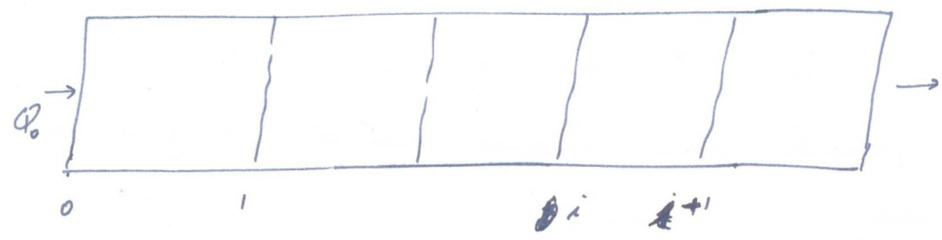


Recall $$Q_{j+1} = C_1 I_{j+1} + C_2 I_j + C_3 Q_j$$

The suffix 'j' was corresponding to time.

A similar approach for distributed flow routing can be done using Muskingum-Cunge method.

There will be variations both in space and time.



Inflow hydrograph is given at $i = 0$, $Q_0^{(\,)}$

$$Q_{i+1}^{(s+1)} = C_1 Q_i^{(s+1)} + C_2 Q_i^{(s)} + C_3 Q_{i+1}^{(s)}$$

where C_1, C_2, C_3, etc. are coefficients.

$K \rightarrow$ storage constant.

DYNAMIC WAVE ROUTING

Kinematic waves or kinematic wave routing can be applied in only very limited circumstances.

→ Rivers and channels are important in human civilization

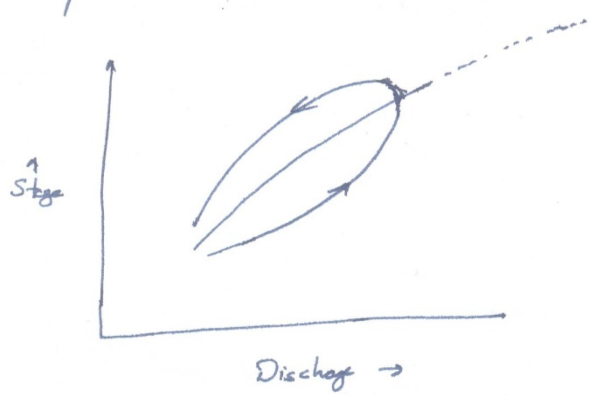
→ Many urbanisation occurred along river banks.

→ For water resources management in many areas we need to more accurately predict or route the inflow hydrographs by incorporating all influences.

Therefore dynamic wave routing is required.

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\beta \frac{Q^2}{A}\right) + gA\left(\frac{\partial y}{\partial x} - S_0 + S_f + S_e\right) - \beta q v_x + W_f B = 0$$

→ For dynamic wave situations many forces play important roles.



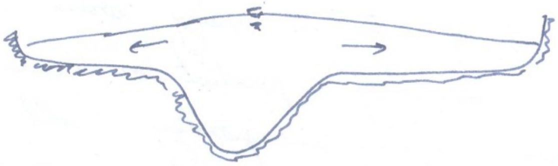
For kinematic wave we have unique discharge vs stage curve.

For dynamic wave we don't have unique stage vs discharge.

Flow in natural rivers involve:

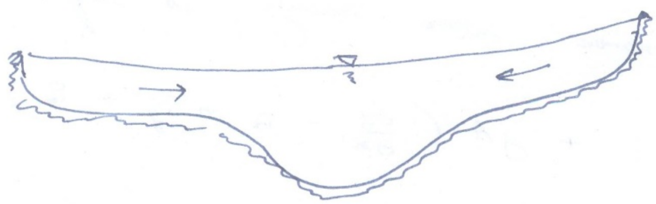
Junctions
Tributaries
Cross section variations
Variable surfaces (i.e. roughness changes)

The flood propagation



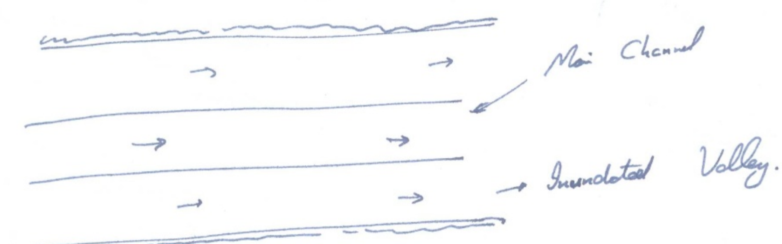
There is bank or valley storage during floods.

Rising flood



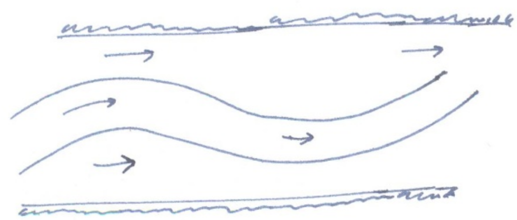
Receding flood.

→ Usually flood wave progresses slowly in inundated valley than in the main channel.



Here both the flow in main channel and flood inundated valleys are same.

There are meandering rivers where the flow gets complicated.



Again you can use numerical ~~models~~ methods to model dynamic flow routing.

e.g.: Finite - Difference Method

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\beta \frac{Q^2}{A}\right) + gA\left(\frac{\partial y}{\partial x} - S_0 + S_f + S_e\right) - \beta q v_x$$

$$+ W_f B \quad = 0 .$$

This equation is having Q and y as dependents

Now $\frac{\partial y}{\partial x} - S_0 = \frac{\partial h}{\partial x}$.

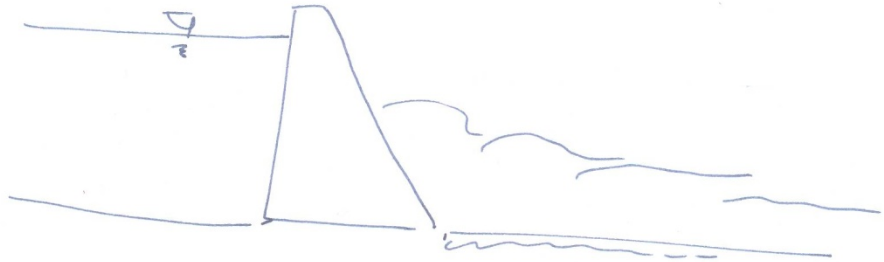
Also continuity equation:

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q$$

Apply finite-difference approximation to solve.

→ One can do flood routing for rivers, meandering rivers, etc.

→ There is an interesting problem called Dam - break problem.



On breach of the dam, how fast the waves propagate downstream is analysed by dam-break problem.

STATISTICS IN HYDROLOGY

Till now we were dealing with deterministic models — both lumped and distributed. But you know hydrological processes are rarely deterministic. Along with its change in space and time they are also random.

If you have random processes the hydrological system can be — stochastic.

If space and time dependent → respective models.

→ In hydrology : you may have extreme hydrologic events.

* Floods
* Droughts

→ For such case you may go for stochastic, time independent and space independent models.

→ Annual precipitation. — is a data averaged over long time interval.

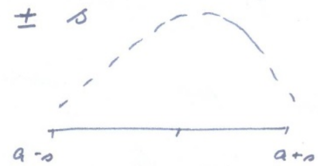
Let us first see how statistical methods are used to treat the hydrologic data :

Probability In Hydrologic Data

Q: What is meant by random variable (say X)?

Ans: A variable that is described by a probability distribution.

i.e. $X = a \pm s$



That is an observation of this random variable ~~may~~ will lie in this range of X.

e.g: If X is annual precipitation of a region then a particular observed annual precipitation of a year (say x) will lie in the given range of X.

e.g: $X = 30 \pm 10$
" $X < 30$
" $10 < X < 40$, etc.

} Defined range.

Q: What is a sample?

In a random variable X you may have a set of observations $x_1, x_2, x_3, \ldots, x_n$.

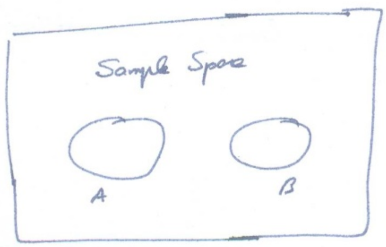
This is a sample.

e.g.: The annual precipitation of Guwahati is available from 1961 - 2012. $P_1, P_2, P_3, \ldots$ are observations.

These samples are usually drawn from hypothetical infinite population.

Sample Space: → Set of all samples that could be drawn from the population.

An <u>event</u> is a subset of the sample space.



→ e.g.: Sample space for annual precipitation
It can range from zero to infinity. But in our analyses we take some realistic values.

→ In the sample space of annual precipitation, we can define various events

e.g: Event A → the annual ppt. > 100 cm

Event B → $50 < x < 80$

Event C → $x < 50$, etc.

→ The probability of ~~the~~ an event – the chance it will occur when the random variable is measured.

e.g. $P(A)$, $P(B)$, $P(C)$, etc.

Consider a sample of n observations.

No: of observations meeting the criteria of event A $= n_A$

Then relative frequency of event A $\rightarrow \frac{n_A}{n}$

$$P(A) = \lim_{n \to \infty} \frac{n_A}{n}$$

This is objective or posterior probabilities