Distributed Flow Models

Yesterday, we discussed on the various classifications of distributed flow models.

- → Kinematic wave model
- → Diffusion wave model
- → Dynamic wave model

→ We also described what is meant by wave celerity. It is the velocity or speed with which the variation in flow parameters travels along the channel

Kinematic Wave Celerity

For kinematic waves, we have $S_0 = S_f$

Also

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q \quad \text{(Continuity)}$$

$$S_0 = S_f \quad \text{Momentum}$$

As $S_0 = S_f$, we can apply Manning's equation for discharge

$$Q = \frac{1}{n} A R^{2/3} S_0^{1/2}$$

i.e. $$Q = \frac{1}{n} \frac{A^{5/3}}{P^{2/3}} S_0^{1/2}$$

You can also express this equation as:

$$A = \left[\frac{n \; P^{2/3}}{S_0^{1/2}}\right]^{3/5} Q^{3/5} \quad \to (1)$$

In general, we can write

$$A = \alpha Q^{\gamma}$$

where $\alpha = \left[\dfrac{n P^{2/3}}{\sqrt{S_0}}\right]^{3/5}$ in this case

and $\gamma = \dfrac{3}{5} = 0.6$

If $A = \alpha Q^{\gamma}$

Then $\dfrac{\partial A}{\partial t} = \alpha \gamma Q^{\gamma-1} \dfrac{\partial Q}{\partial t}$

Hence in continuity equation:

$$\boxed{\frac{\partial Q}{\partial x} + \alpha \gamma Q^{\gamma-1} \frac{\partial Q}{\partial t} = q} \rightarrow (2)$$

This is a pde with Q as dependent variable.

That means that kinematic wave is formed due to changes in Q.

Consider a small (or elemental) discharge dQ.

As $Q \rightarrow Q(x,t)$ we can write

$$dQ = \frac{\partial Q}{\partial x} dx + \frac{\partial Q}{\partial t} dt$$

or

$$\boxed{\frac{\partial Q}{\partial x} + \frac{\partial Q}{\partial t} \frac{dt}{dx} = \frac{dQ}{dx}} \rightarrow (3)$$

Compare (2) and (3) we will get:

$$\boxed{\frac{dt}{dx} = \alpha \gamma Q^{\gamma-1}} \quad \text{and} \quad \boxed{\frac{dQ}{dx} = q}$$

i.e. $$\frac{dx}{dt} = \frac{1}{\alpha \gamma Q^{\gamma-1}}$$

as $$A = \alpha Q^{\gamma}$$

$$\frac{dA}{dA} = 1 = \alpha \gamma Q^{\gamma-1} \frac{dQ}{dA}$$

or $$\boxed{\frac{dQ}{dA} = \frac{1}{\alpha \gamma Q^{\gamma-1}}}$$

$\therefore$ Now we get $$\boxed{\frac{dQ}{dA} = \frac{dx}{dt}}$$

$\frac{dx}{dt} \rightarrow$ represents some sort of velocity.

This is kinematic wave celerity.

$$C_k = \frac{dx}{dt} = \frac{dQ}{dA}$$

Now we have

$$\left.\begin{aligned} \frac{dQ}{dx} &= q \\ C_k = \frac{dx}{dt} &= \frac{dQ}{dA} \end{aligned}\right\}$$ These are the characteristic equations for a kinematic wave.

$\Rightarrow$ In nature, kinematic and dynamic waves are present in natural floods.

Dynamic Wave Celerity

The non-conservation form of momentum equation:

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + g\frac{\partial y}{\partial x} - g(S_0 - S_f) = 0$$

Your wave celerity in kinematic wave $= \frac{dx}{dt}$

This wave celerity can be described by characteristic equations

$$\left.\begin{aligned} \frac{dx}{dt} &= V \pm c_d \\ \frac{d}{dt}(V \pm 2c_d) &= g(S_0 - S_f) \end{aligned}\right\}$$

where $c_d \rightarrow$ dynamic wave celerity

For a rectangular channel, $c_d = \sqrt{g\frac{A}{B}} = \sqrt{gy}$

$c_d \rightarrow$ velocity of dynamic wave w.r.t ~~—~~ still water.

However in moving water there are two dynamic waves.

→ One proceeding upwards $V - c_d$

→ proceeding downwards $V + c_d$

In the disturbance to move up you require $V < c_d$.

(Refer Example 9.3.1)

Solution of Kinematic Wave Equation

We will first see the analytical methods to solve Kinematic wave equation.

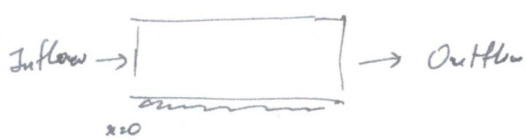
Q: What do you get by solving kinematic wave equation.

$$\frac{dQ}{dx} = q$$

$$c_k = \frac{dx}{dt} = \frac{dQ}{dA}$$

→ We will get distribution of flow (or Q) as function of x and t.

Analytical way : For a special case, when $q \approx 0.0$



Initial condition $Q(x, 0)$ is to be specified.

Boundary condition $Q(0, t)$ (i.e inflow hydrograph) is to be specified.

We have to determine now outflow hydrograph.

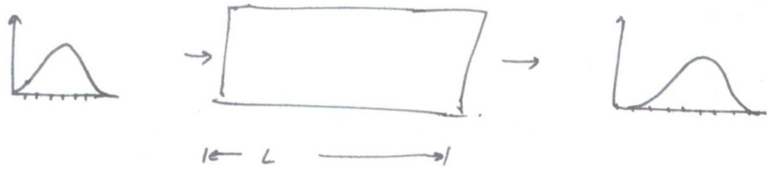
In $q \approx 0.0$, we have $\frac{dQ}{dx} = 0.0$

or Q = a constant

∴ Therefore, if Q is known at a point in time and space, this flow is propagated along the channel at kinematic wave celerity c_k.

$$c_k = \frac{dQ}{dA} = \frac{dx}{dt}$$

i.e.



We can draw the solution in an x-t plane.

From

$$c_k = \frac{dQ}{dA} = \frac{dx}{dt}$$

we have

$$\int_0^x dx = \int_{t_0}^{t} c_k \, dt$$

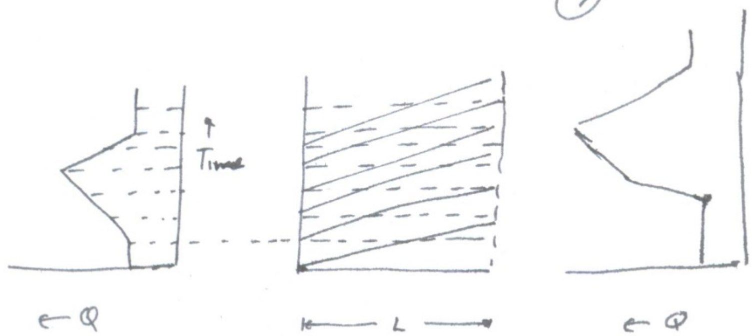
or

$$x = c_k (t - t_0).$$

Time at which discharge Q_0 enters channel at time t_0 will appear at outlet is:

$$\frac{L}{c_k} + t_0 = t$$

The slope of the characteristic line $c_k = \dfrac{dQ}{dA}$


Time
← Q
L
← Q