LECTURE 29
21-MARCH-2013

Classification of Distributed Flow Routing

As discussed, based on the number of terms involved, one can classify the distributed flow routing.

In the non-conservation form

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + g\frac{\partial y}{\partial x} - g(S_o - S_f) = 0$$

(i) We have kinematic wave model

$$-g(S_o - S_f) = 0$$

i.e. $\boxed{S_o = S_f}$

(ii) Diffusion wave models

$$g\left(\frac{\partial y}{\partial x} - S_o + S_f\right) = 0$$

or $\boxed{\frac{\partial y}{\partial x} = S_o - S_f}$

(iii) Dynamic wave model

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + g\frac{\partial y}{\partial x} - g(S_o - S_f) = 0$$

We can also have steady and unsteady distributed flow models. This is based on change in variables w.r.t time i.e. In steady state $\frac{\partial A}{\partial t} = 0$.

Wave Motion

Q: What is meant by wave motion? That too why in hydrology it is important?

→ In hydrology, we suggest wave as variation in flow.

- ★ It can be change in flow rate
- ★ water surface elevation, etc.

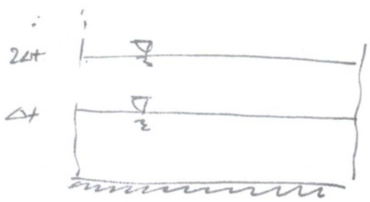
→ The wave celerity is the velocity with which this variation travels along the channel.

Kinematic wave

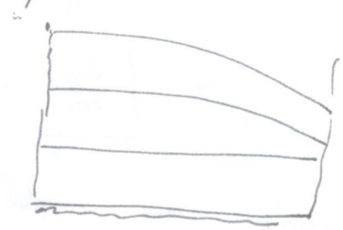
- ★ Inertial and pressure forces neglected
- ★ Gravity and friction forces are balanced
- ★ Therefore, flow does not accelerate considerably

Dynamic wave

- ★ All the force components are important
- ★ In dynamic wave, energy ~~and~~ grade line and water surface elevation are not parallel.



Kinematic wave



Dynamic wave

Kinematic Wave Celerity

In kinematic waves, $S_0 = S_f$

The celerity of wave and velocity of water are not same.

The continuity equation for kinematic wave model

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q$$

Momentum equation $S_0 = S_f$

As $S_0 = S_f$, we can apply Manning's equation for uniform flow, etc.

i.e. $Q = \frac{1}{n} A R^{2/3} S_0^{1/2}$

or $Q = \frac{1}{n} \frac{A^{5/3}}{P^{2/3}} S_0^{1/2}$

One can now express this equation as.

$$A = \left[\frac{n P^{2/3}}{\sqrt{S_0}}\right]^{3/5} Q^{3/5}$$

i.e. $A = \alpha Q^{\gamma}$

where $\alpha = \left[\frac{n P^{2/3}}{\sqrt{S_0}}\right]^{3/5}$

$\gamma = 3/5 = 0.6$ in this case.

In general one can express

$$A = \alpha Q^{\gamma}$$

For different types of flows, the values of α and γ may change.

Now

$$\frac{\partial A}{\partial t} = \alpha \gamma Q^{\gamma-1} \frac{\partial Q}{\partial t}$$

∴ In continuity equation:

$$\boxed{\frac{\partial Q}{\partial x} + \alpha \gamma Q^{\gamma-1} \frac{\partial Q}{\partial t} = q}$$

i.e. This ~~eq~~ pde is having dependent variable Q.

The kinematic waves ~~model~~ result from changes in Q.

Consider an small change in discharge i.e. dQ

$$dQ = \frac{\partial Q}{\partial x} dx + \frac{\partial Q}{\partial t} dt$$

$$\therefore \quad \frac{\partial Q}{\partial x} + \frac{\partial Q}{\partial t}\frac{dt}{dx} = \frac{dQ}{dx} \quad \rightarrow (1)$$

But we have

$$\frac{\partial Q}{\partial x} + \alpha \gamma Q^{\gamma-1} \frac{\partial Q}{\partial t} = q \quad \rightarrow (2)$$

(1) and (2) can be same, if

$$\frac{dQ}{dx} = q$$

and

$$\frac{dt}{dx} = \alpha \gamma Q^{\gamma-1}$$

∴ Rearranging

$$\frac{dx}{dt} = \frac{1}{\alpha \gamma Q^{\gamma-1}} \quad \rightarrow (3)$$

From

$$A = \alpha Q^{\gamma}$$

We have

$$\frac{dA}{dA} = \alpha \gamma Q^{\gamma - 1} \frac{dQ}{dA}$$

or
$$\frac{dQ}{dA} = \frac{1}{\alpha \gamma Q^{\gamma - 1}} \qquad \rightarrow (4)$$

From eqn. (3) and (4):

$$\boxed{\frac{dx}{dt} = \frac{dQ}{dA}}$$

$\frac{dx}{dt}$ → represents some sort of velocity.

This is kinematic wave celerity.

$$C_k = \frac{dx}{dt} = \frac{dQ}{dA}$$